Analysis of Coded FHSS Systems With Multiple Access Interference in Nakagami Fading

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Abstract— **This paper studies the effect of Multiple access interference (MAI) on the performance of coded FHSS systems. This is achieved by modelling the physical channel in these networks as a block fading channel with the hopping rate as a network parameter. From this, error performance of convolutionally coded FHSS systems is derived by finding the exact interference statistics and averaging over it.**

I. Introduction

A serious challenge to having good communication quality in wireless networks is the time-varying multipath fading environments, which causes the received signal-to-noise ratio (SNR) to vary randomly. One solution to fading is the use of spread spectrum (SS) techniques, which randomizes the fading effect over a wide frequency band. The main types of SS are the direct sequence SS (DSSS) and the frequency hopping SS (FHSS).

In FHSS, each user starts transmitting his data over a narrowband during a time slot (called dwell time), and then hops to other bands in the subsequent time slots according to a pseudo-random (PN) code (sequence) assigned to the user. The main advantages of FHSS is the robust performance under multipath fading, interference and jamming conditions. Furthermore, data sent over a jammed frequency can be easily corrected by employing error correcting codes with FHSS systems [1]. In particular, convolutional codes are considered to be practical for short-delay applications because the performance is not affected significantly by the frame size.

In cellular networks, multiple access interference (MAI) may arise when more than one user transmit over the same frequency band at the same time in the uplink. This happens when users in closely located cells are assigned PN codes that are not perfectly orthogonal. Also, MAI may be caused by the lake of synchronization between users transmitting in the same cell [2,3]. The performance of channel coding with fast FHSS and partial-band interference is well studied in the literature as in [4–6]. However, no much work was done to investigate the performance of coding with slow FHSS and partial-band interference. The error performance of coded systems was derived [7] for the case of block fading channels with a small number of hops not exceeding $J = 4$. Applying this method for systems with a large number of hops (as is the case for FHSS systems at hand) results in a prohibitive computational complexity due to the J-dimensional integration used in the method.

In this paper, we derive new union bounds on the bit error probability of coded FHSS systems under MAI. We consider systems with perfect channel estimation over Nakagami fading channels. We accomplish this by modelling the FHSS effective channel as a block interference channel as in [8]. The error probability is derived by conditioning over the number of interfering users in the network and then by averaging over this number.

The outline of the paper is as follows. The system model is described next. In Section III, a union bound on the bit error probability for coded FHSS systems is derived. Results are discussed in Section IV and conclusions are presented in Section V.

II. System Model

The transmitter consists of a binary encoder (e.g., convolutional or turbo), an interleaver, a modulator and a FHSS transmission scheme. A packet is composed of K information bit encoded into N bits using a rate-$R_c = K/N$ convolutional code. Each coded bit is modulated using coherent BPSK. Each packet is transmitted using FHSS after being bit-interleaved, where the transmitter hops J times during the transmission of a packet. Thus the packet undergoes J independent fading realizations, where blocks of $m = \lceil \frac{N}{J} \rceil$ bits undergo the same fading. Effectively, each packet undergoes a block fading channel [8].

In this paper, only slow FHSS is considered, where the number of symbols transmitted during each hop is $m = \lceil \frac{N}{J} \rceil$ symbols, and is referred to as the *hop length*. Thus each frame undergoes J independent fading blocks. Furthermore, we consider a multiple-access FHSS network of K users. The frequency band is divided into Q bands and users transmit their data by hopping from one band to another randomly. Throughout this paper, we assume synchronous transmission with the probability of a hit given by $p_h = 1/Q$. Given that only k users (among the total of K users operating in the network) interfere with the user of interest, the matched filter sampled

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output at time l in the j^{th} hop is given by

$$y_{j,l} = \sqrt{E_s} h_j s_{j,l} + z_{j,l} + \sum_{f=1}^{k} \sqrt{E_I} h_f, \quad (1)$$

where E_s is the average received energy, $s_{f,l} = (-1)^{c_{f,l}}$, where $c_{f,l}$ is the corresponding coded bit out of the channel encoder, and $z_{f,l}$ is a zero-mean AWGN sample a with variance $\frac{N_0}{2}$. The coefficient h_j is the channel gain in hop j modeled as $\mathcal{CN}(0,1)$ and written as $h_j = a_j \exp(j\theta_j)$, where θ_j is uniformly distributed and a_j is the channel amplitude distributed according to Nakagami distribution. The term E_I is the average received energy for each of interfering user and h_f is the channel gain affecting the f^{th} interfering user and modeled as $\mathcal{CN}(0,1)$.

We define the signal-to-interference ratio (SIR) as the ratio $\Delta = E_s/E_I$. The SIR indicates the relative received energy of each of the interfering signals to the received energy of the desired signal. The average signal-to-interference-and-noise ratio (SINR) given k interfering users is defined as

$$\Gamma(k) = \frac{E_s}{N_0/2 + kE_I} = \frac{R_c \gamma_b}{1/2 + k\frac{\gamma_b}{\Delta}}, \quad (2)$$

where $\Delta = \frac{E_s}{E_I}$ is the average SIR defined above and $\gamma_b = \frac{E_s}{R_c N_0}$ is the SNR per information bit.

Given that k users are interfering with the user of interest, the instantaneous signal-to-interference-and-noise ratio (SINR) in the j^{th} hop is written as

$$\beta_j = \frac{a_j^2 \gamma_b}{1/2 + \frac{\gamma_b}{\Delta}\sum_{f=1}^{k} a_f^2}, \quad (3)$$

where a_f is the fading gain of the signal arriving from the f^{th} interfering user. In (3), we assumed that the desired and interfering signals have the different average received energies related by the constant Δ. As the value of Δ increases the performance improves since the interfering energy decreases relative to the energy of the desired user.

III. Bit Error Probability

Throughout the paper, the subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. For linear convolutional codes with r input bits, the bit error probability is upper bounded [9] as

$$P_b \leq \frac{1}{r} \sum_{d=d_{\min}}^{N} w_d P_u(d), \quad (4)$$

where $d_{\min}$ is the minimum distance of the code, $P_u(d)$ is the unconditional pairwise error probability (PEP), and w_d is the number of codewords with output weight d.

In FHSS systems, $P_u(d)$ in (4) is a function of the distribution of the d nonzero bits over the J hops. This distribution is quantified assuming uniform channel interleaving of the coded bits over the hops [10]. Denote the number of hops with weight v by j_v and define $w = \min(m,d)$, then the hops are distributed according to the pattern $\mathbf{j} = \{j_v\}_{v=0}^{w}$ if

$$J = \sum_{v=0}^{w} j_v, \qquad d = \sum_{v=1}^{w} v j_v. \quad (5)$$

Denote by $L = J - j_0$ the number of hops with nonzero weights. Then $P_u(d)$, determined by averaging over all possible block patterns, is given by

$$P_u(d) = \sum_{L=\lceil d/m \rceil}^{d} \sum_{j_1=0}^{L_1} \sum_{j_2=0}^{L_2} \cdots \sum_{j_w=0}^{L_w} P_u(d|\mathbf{j}) p(\mathbf{j}|d), \quad (6)$$

where

$$L_v = \min\left\{ L - \sum_{r=1}^{v-1} j_r, \frac{d - \sum_{r=1}^{v-1} r j_r}{v} \right\}, \qquad 1 \leq v \leq w. \quad (7)$$

The probability of a block pattern for a specific codeword weight d is computed using combinatorics as

$$p(\mathbf{j}|d) = \frac{\binom{m}{1}^{j_1}\binom{m}{2}^{j_2}\cdots\binom{m}{w}^{j_w}}{\binom{mJ}{d}} . \frac{J!}{j_0! j_1! \ldots j_w!}. \quad (8)$$

Substituting (6)-(8) in (4), results in the union bound on the bit error probability of convolutional coded FHSS systems.

Conditioning on the number of interfering users and the fading, the conditional PEP for coherent detection is given by

$$P_c(d|\mathbf{j},k) = \mathrm{Q}\left(\sqrt{\frac{R_c \gamma_b \sum_{v=1}^{w} v \sum_{i=1}^{j_v} a_i^2}{1/2 + \frac{\gamma_b}{\Delta}\sum_{f=1}^{k} a_f^2}}\right) = \mathrm{Q}\left(\sqrt{\sum_{v=1}^{w} v \sum_{i=1}^{j_v} \beta_i^2}\right), \quad (9)$$

where β_i is the SINR defined in (3). An exact expression of the PEP is found using the integral form of the Q-function resulting in

$$P_u(d|\mathbf{j}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{E}_{\{\beta\}}\left[\exp\left(-\alpha_\theta \sum_{v=1}^{w} v \sum_{i=1}^{j_v} \beta_i\right)\right] d\theta = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left[\Phi_\beta\left(v\alpha_\theta\right)\right]^{j_v} d\theta, \quad (10)$$

where $\alpha_\theta = \beta/\sin^2\theta$ and

$$\Phi_\beta(s) = \mathrm{E}_\beta\left[e^{-s\beta}\right], \quad (11)$$

is the moment generating function (MGF) of the random variable β and the product in (10) results from the independence

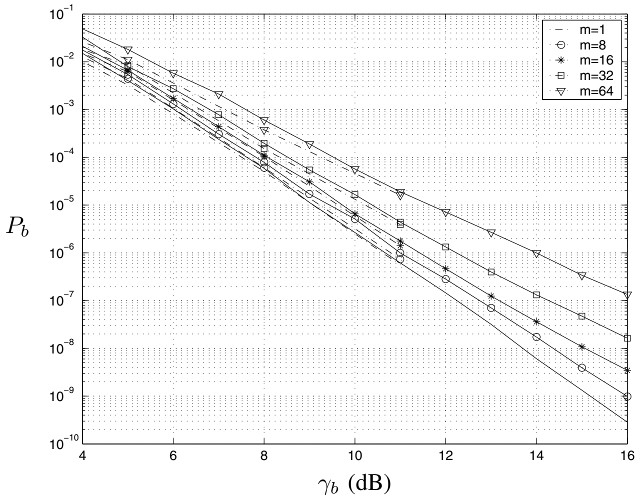

Fig. 1. Performance of a rate-$\frac{1}{2}$ (23,35) convolutionally coded FHSS system for 10 users ($K = 10$) and different hop lengths $m = 1, 8, 16, 32, 64$ (solid: approximation using the union bound, dash: simulation).

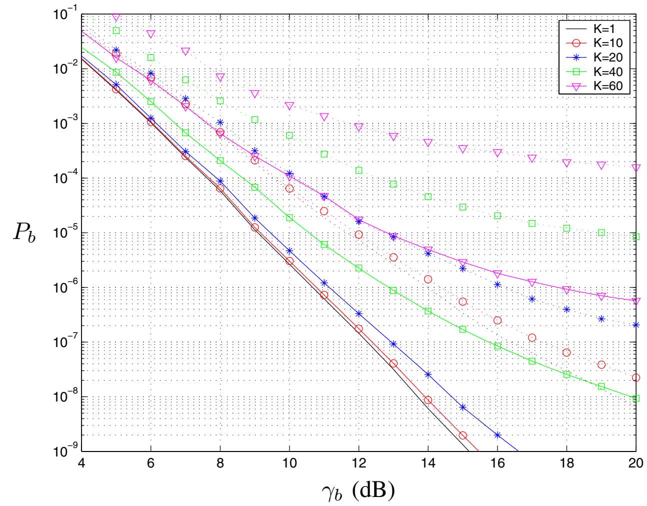

Fig. 2. Performance of a convolutionally coded FHSS system over Rayleigh fading for different number of users K and SIR = 5 dB, (solid: $m = 1$, dashed: $m = 64$).

of the fading processes affecting different hops in a codeword.

In order to find the MGF of β, we need to derive its pdf which is a function of the number of interfering users. The conditional pdf of the SINR given the number of interfering users is k for integer values of the Nakagami parameter M [11] is found to be

$$f_{\beta|k}(\beta) = \frac{M^{M(1+k)}\beta^{M-1}e^{-M\beta}}{\Gamma(M)\Gamma(kM)} \times \sum_{h=0}^{M}\binom{M}{h}\frac{\Gamma(kM+h)}{(M\beta+M)^{kM+h}}, \quad \beta > 0. \quad (12)$$

Since the users collide with probability p_h and the total number of users is K, the number of interfering users is a Binomial random variable with parameters p_h and K. Hence, the MGF of the SINR, β is found by averaging (12) over the number of interfering users as follows

$$\Phi_\beta(s) = \sum_{k=0}^{K}\binom{K}{k}p_h^k(1-p_h)^{K-k}\Phi_{\beta|k}(s), \quad (13)$$

where $\Phi_{\beta|k}(s)$ is the conditional MGF of the SINR, β. For integer Nakagami parameter μ [11], it is given by

$$\Phi_{\beta|k}(s) = \frac{\mu^\mu}{\Gamma(k\mu)}\sum_{h=0}^{\mu}\binom{\mu}{h}\frac{\Gamma(k\mu+h)}{\mu^h} \times U\left(\mu;\mu(1-k)-h+1;1+\frac{\mu}{s}\right), \quad (14)$$

where $U(.;.;.)$ is the confluent hypergeometric function of the second kind defined in [12]. The MGF required to evaluate (10) is found by substituting (14) in (13) and expressing $U(.;.;.)$ as

$$U(a;b;x) = \frac{\pi}{\sin(\pi b)}\left[\frac{{}_1F_1(a,b;x)}{\Gamma(a-b+1)\Gamma(b)} - \frac{x^{1-b}}{\Gamma(a)\Gamma(2-b)}\,{}_1F_1(a-b+1,2-b;x)\right], \quad (15)$$

where ${}_1F_1(.,.;.)$ is the confluent hypergeometric function that is available in any numerical package such as MATLAB. Once the MGF is evaluated, the PEP is evaluated by substituting (13) in (10). Note that due to the summation in (6), the union bound in (4) becomes complicated when d is large. Thus an approximation to the bit error probability is obtained by truncating (4) to a distance $d_{\max}$.

IV. Results and Discussion

Throughout this subsection, the proposed performance analysis is applied to coded FHSS systems employing a rate-$\frac{1}{2}$ (23,35) convolutional code with a frame size of $N = 2 \times 512$ coded bits. Figure 1 shows the performance of a FHSS network with 10 users and perfect CSI for different hop lengths. We observe that the obtained analytical results closely approximate the simulation results. Thus the proposed analytical approach provides an accurate measure of the performance of coded systems over systems that can be modeled by a block fading channel model. In the rest of the paper, only analytical results are shown in order to make the presentation of the results clear.

Figure 2 shows the performance of a convolutionally coded FHSS system over Rayleigh fading with perfect CSI for different number of users and hop lengths of $m = 1$ and $m = 64$. Comparing the sets of curves corresponding to the cases of $m = 1$ and $m = 16$, we observe that the performance loss due to interference increases as the hop length increases (or in

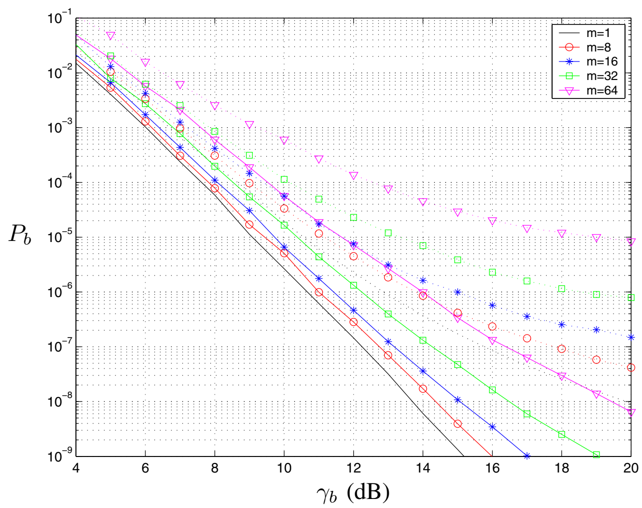

Fig. 3. Performance of a convolutionally coded FHSS system over Rayleigh fading for different hop lengths m and SIR = 5 dB, (solid: $K = 1$, dashed: $K = 40$).

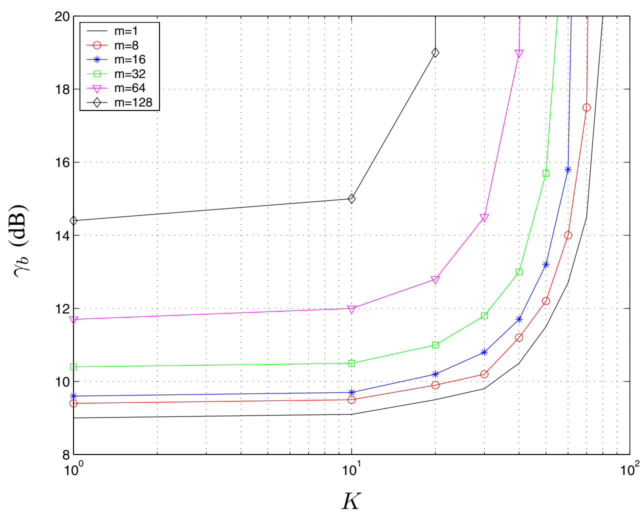

Fig. 4. SINR required for a convolutionally coded FHSS system to achieve $P_b = 10^{-5}$ over Rayleigh fading versus the number of users K for $m = 1, 8, 16, 32, 64, 128$ and SIR = 5 dB.

other words as the number of hops decreases). For example, for the case of $m = 1$, 20 users is worse than 1 user by almost 0.5 dB, whereas this difference is almost 1 dB for the case of $m = 64$. This is also clear in Figure 3, which shows the performance of the coded FHSS system over Rayleigh fading with perfect CSI for different hop lengths m and for one and 40 users. The reason behind this phenomenon is that increasing the hop length decreases the diversity order provided to the coded system, which increases the impact of interference on the performance of the system.

Figure 4 shows the SINR required for the coded FHSS system to achieve $P_b = 10^{-5}$ over Rayleigh fading versus the number of users K with perfect CSI for different hop lengths. In the figure we observe that as the hop length increases the required SINR increases up to a maximum number of users beyond which the required performance can not be achieved. For example, a coded FHSS system with $m = 64$ can achieve a $P_b = 10^{-5}$ with a SINR of 12 when only 10 users exists in the system. However, it can not achieve the same performance whatsoever if the number of users in the system exceeds 40 users. Therefore, if more than 40 users need to be supported at a $P_b = 10^{-5}$, then the hop length has to be decreased, i.e., the number of hops per frame has to be increased to increase the diversity order in the coded system.

V. Conclusions

In this work we derived a union bound approximation of the performance of coded FHSS systems under MAI conditions. Results show that the performance loss due to interference increases as the hop length increases (or in other words as the number of hops in FHSS systems decreases). Furthermore, this performance loss increases more as the number of users increases.

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