On the Performance of Diversity Reception of Coded DSSS in Nakagami Multipath Fading

Salam A. Zummo, *Member, IEEE*
Electrical Engineering Department
King Fahd University of Petroleum and Minerals (KFUPM)
Dhahran 31261, Saudi Arabia
E-mail: zummo@kfupm.edu.sa

Abstract

In this paper new union bounds are derived for coded direct sequence spread spectrum (DSSS) with RAKE receivers over frequency-selective Nakagami-m fading channels. The union bounds are expressed in the product form, which makes them easily evaluated using the transfer function of the code. The bounds are general to any number of resolvable multipath components and coding scheme with a known transfer function. Results show that the proposed bounds are tight to simulation results.

1 Introduction

In spread spectrum communication systems, the channel is a frequency-selective fading channel due to the multiple received paths. This is mainly because the signal bandwidth is larger than the channel coherence bandwidth. However, if the receiver can resolve the signal paths using a RAKE receiver, the inherent diversity in the multipath signal can be exploited using the maximal-ratio combining (MRC). MRC is the optimal combining scheme in diversity systems in which the outputs of the matched filters of the RAKE fingers are added after being weighted by the fading attenuation of each finger. The resultant signal-to-noise ratio (SNR) at the output of the combiner is the sum of the SNR's of the L RAKE fingers.

In the literature, most of the performance analysis of coded DSSS is based on using the moment generating function (MGF) approach, see [5] and the references therein. However, using the MGF approach imposes the use of numerical integration to evaluate the union bound. Simple union bounds for coded DSSS were derived in [1, 6] for Rayleigh fading channels. In this paper, new union bounds on the bit error probability of coded DSSS systems over Nakagami-m fading channels are derived. The bounds are presented in the product form allowing efficient computation of the bound using the transfer function of the code.

The paper is organized as follows. In Section 2, the coded DSSS system is described. The average error performance of coded DSSS systems is derived in Section 3. The paper is concluded with main outcomes in Section 4

2 System Model

The transmitter in a coded DSSS system is generally composed of an encoder, interleaver, spreading block and a modulator. A rate-$\frac{K}{N}$ encoder encodes a block of K information bits and produces a sequence of signals $\mathbf{S} = \{s_n\}_{n=1}^{N}$. The encoder might be convolutional, turbo, trellis-coded modulation (TCM) or any other coding scheme. For each spread signal s_n transmitted at time n, there are L received resolvable paths $\mathbf{y}_n = \{y_{n,l}\}_{l=1}^{L}$. Each propagation path is characterized by its gain whose envelope is $\mathbf{a}_n = \{a_{n,l}\}_{l=1}^{L}$, which is modeled as Nakagami-m distributed random variables. The channel will be frequency selective if $L > 1$. The decoder is a Viterbi decoder which performs Maximum Likelihood (ML) decoding with perfect channel state information.

Coherent receiver is employed, and hence the phase of the channel gains affecting different received multipath fingers are assumed to be known at the RAKE receiver. For the n^{th} symbol in the codeword, the matched filter output of the l^{th} RAKE branch is given by

$$y_{n,l} = \sqrt{E_s} a_{n,l} s_n + z_{n,l}, \tag{1}$$

where E_s is the average received signal energy per RAKE finger, $\mathbf{z}_n = \{z_{n,l}\}_{l=1}^{L}$ are noise samples which are i.i.d. complex Gaussian random variables with zero-mean and noise variance of N_0. The coefficients $\mathbf{a}_n = \{a_{n,l}\}_{l=1}^{L}$ are the fading amplitudes of the L RAKE fingers modeled as correlated and non-identically distributed Nakagami random variables. The RAKE fingers are assumed to be uncor-

Proceedings of the Third International
Conference on Wireless and Mobile Communications (ICWMC'07)
0-7695-2796-5/07 $20.00 © 2007 IEEE

IEEE COMPUTER SOCIETY

related. The l^{th} finger branch is assumed to have a mean of w_i that can be different from other diversity branches such that $\sum_{l=1}^{L} w_l^2 = L$.

3 Pairwise Error Probability

The pairwise error probability (PEP) is defined as the probability of decoding a codeword $\mathbf{S}$ as another codeword $\hat{\mathbf{S}}$. In the following the PEP is written in the product form as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) = K_c \times \prod_{n=1}^{N} W(s_n, \hat{s}_n), \tag{2}$$

where $\mathbf{S}$ and $\hat{\mathbf{S}}$ are the the correct and decoded codewords, respectively. In (2), $W(s_n, \hat{s}_n)$ is the error weight profile between $\hat{s}_n$ and s_n, and K_c is a tightening constant that does not depend on the error sequence. The case of $K_c = 1$ results when the Chernoff bound is used to upper bound the pairwise error probability [4, 6]. This form enables the use of the transfer function of the code to evaluate the union bound on the bit error probability.

In the following, the work of [1] is extended to the case of Nakagami fading channels. The conditional PEP for MRC diversity can be written as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}|\mathbf{A}) = P\Big(\sum_{n=1}^{N}\sum_{l=1}^{L}\big(|y_{n,l} - \sqrt{E_s}a_{n,l}s_n|^2 - |y_{n,l} - \sqrt{E_s}a_{n,l}\hat{s}_n|^2\big) \geq 0\Big|\mathbf{A}\Big), \tag{3}$$

where $\mathbf{A}$ is a vector that contains the fading gains affecting a codeword. The conditional PEP can be simplified as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}|\mathbf{A}) = P\left(\xi \geq E_s \sum_{n=1}^{N}\sum_{l=1}^{L} a_{n,l}^2 |s_n - \hat{s}_n|^2 \Big| \mathbf{A}\right), \tag{4}$$

The conditional PEP for a RAKE receiver can be written as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}|\mathbf{A}) = \frac{1}{2}\text{erfc}\left(\sqrt{\sum_{n=1}^{N} d_n \gamma_n}\right), \tag{5}$$

where $d_n = E_s|s_n - \hat{s}_n|^2/4N_0$, E_s is the received signal energy, $N_0/2$ is the noise variance affecting each signal, and $\gamma_n = \sum_{l=1}^{L} a_{n,l}^2$ represents the effective channel at the output of the RAKE receiver. As assumed in [6], the differences between the delays of the resolvable multipath components are larger than the inverse of the signal bandwidth, so the propagation paths can be approximately uncorrelated, resulting in i.i.d. Nakagami random variables, γ_n has a Gamma distribution with parameter mL.

Since the fading affecting different RAKE fingers are assumed to be i.i.d. and a_l's are Nakagami random variables, the probability density function (pdf) of γ_n is given by

$$f_{\gamma_n}(\gamma) = \frac{m^{mL}}{\Gamma(mL)}\gamma^{mL-1}e^{-m\gamma}, \quad \gamma \geq 0,\ m \geq 0.5, \tag{6}$$

where $\Gamma(\cdot)$ is the Gamma function. The unconditional PEP is found by averaging (5) over the statistics of the γ_n's as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) = \frac{1}{2}\int_0^\infty \ldots \int_0^\infty \text{erfc}\left(\sqrt{\sum_{n=1}^{N} d_n\gamma_n}\right) \times f_\gamma(\gamma_1)\ldots f_\gamma(\gamma_N)d\gamma_1\ldots d\gamma_N. \tag{7}$$

Define

$$\delta_n = \frac{d_n}{1 + d_n/m} \quad \text{and} \quad \omega_n = \gamma_l(1 + d_n/m). \tag{8}$$

Then, the PEP becomes

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) = \frac{1}{2}\prod_{l\in\eta}\frac{1}{(1 + d_n/m)^{mL}} \times \int_0^\infty \ldots \int_0^\infty \text{erfc}\left(\sqrt{\sum_{n\in\eta}\delta_n\omega_n}\right)\exp\left[\sum_{n=1}^{N_\eta}\delta_n\omega_n\right] \times f_\omega(\omega_1)\ldots f_\omega(\omega_{N_\eta})d\omega_1\ldots d\omega_{N_\eta}, \tag{9}$$

where $\eta = \{n : s_n \neq \hat{s}_n\}$ and $N_\eta = |\eta|$ is the minimum time diversity of the code. Define $\Omega = \sum_{n=1}^{N_\eta}\omega_n$, then the pdf of Ω is given by

$$f_\Omega(\Omega) = \frac{m^{mLN_\eta}}{\Gamma(mLN_\eta)}\Omega^{mLN_\eta-1}e^{-m\Omega}, \quad \Omega \geq 0,\, m \geq 0.5. \tag{10}$$

Let $\delta_m = \min\{\delta_n, n \in \eta\}$, and note that $\sum_{n=1}^{N_\eta}\delta_n\omega_n \geq \delta_m\Omega$. Since $\text{erfc}(\sqrt{x})e^x$ is a monotonically decreasing function for $x \geq 0$, then the PEP can be upper bounded as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) \leq \frac{J}{2}\left[\prod_{n=1}^{N_\eta}\frac{1}{(1 + d_n/m)^{mL}}\right], \tag{11}$$

where

$$J = \frac{m^{mLN_\eta}}{\Gamma(mLN_\eta)}\int_0^\infty \text{erfc}\left(\sqrt{\delta_m\Omega}\right) \times \Omega^{mLN_\eta-1}e^{\Omega(\delta_m-m)}d\Omega. \tag{12}$$

In the following, the integral in (12) is simplified using two approaches resulting in two upper bounds on the PEP. In the following, the integral in (12) is simplified using two approaches resulting in two upper bounds on the PEP.

Proceedings of the Third International
Conference on Wireless and Mobile Communications (ICWMC'07)
0-7695-2796-5/07 $20.00 © 2007 IEEE

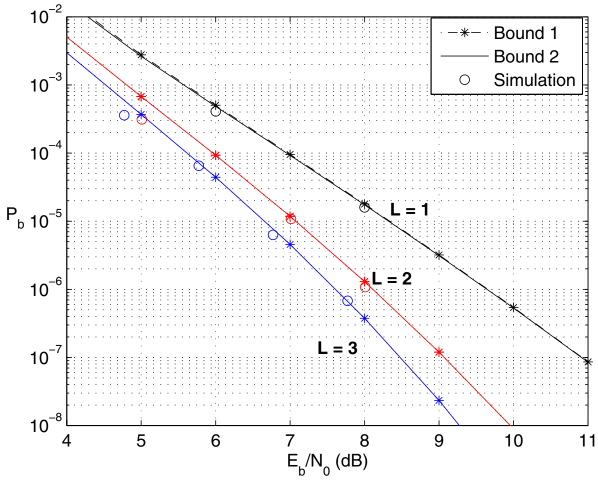

Figure 1. Bit error probability of a convolutionally coded DSSS with a RAKE receiver in Nakagami fading with $m = 2$ and different number of resolvable multipath fingers.

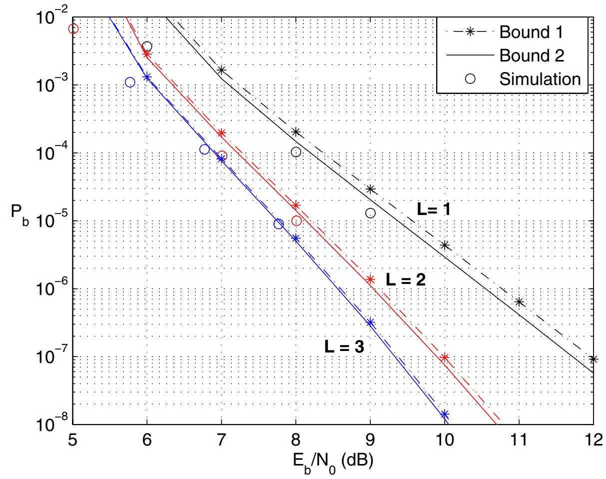

Figure 2. Bit error probability of an 8PSK TCM coded DSSS with a RAKE receiver in Nakagami fading with $m = 4$ and different number of resolvable multipath fingers.

Using Eq. (6.286) of [2], the integral in (12) can be evaluated as

$$J = \frac{m^{mLN_\eta}\Gamma(mLN_\eta + 0.5)}{\sqrt{\pi} mLN_\eta \Gamma(mLN_\eta)\delta_m^{mLM_\eta}} \times {}_2F_1\left(mLN_\eta, mLN_\eta + 0.5; mLN_\eta + 1; 1 - \frac{m}{\delta_m}\right), \quad (13)$$

where ${}_2F_1(.,.;.;.)$ is the Gaussian confluent hypergeometric function [2]. Defining $x = 1 - \frac{m}{\delta_m}$ and using the relation ${}_2F_1(\alpha, \beta; \gamma; z) = (1-z)^{-\alpha}\, {}_2F_1(\alpha, \gamma - \beta; \gamma; z/(z-1))$ results in

$$J = \frac{m^{mLN_\eta}\Gamma(mLN_\eta + 0.5)}{\sqrt{\pi}\Gamma(mLN_\eta + 1)\delta_m^{mLN_\eta}}(1-x)^{-mLN_\eta} \times {}_2F_1\left(mLN_\eta, 0.5; mLN_\eta + 1; \frac{x}{x-1}\right). \quad (14)$$

Using the relation ${}_2F_1(\alpha, \gamma - \beta; \gamma; z) = \alpha z^{-\alpha} B_z(\alpha, \gamma - \beta)$ and substituting (14) in (11), the PEP can be finally simplified to

$$P(\mathbf{S} \to \hat{\mathbf{S}}) \leq \frac{\Gamma(mLN_\eta + 0.5)}{2\sqrt{\pi}\Gamma(mLN_\eta)} x^{-mLN_\eta}(x-1)^{mLN_\eta} \times B_{x/(x-1)}(mLN_\eta, 0.5) \prod_{n=1}^{N_\eta} \frac{1}{(1 + d_n/m)^{mL}}, \quad (15)$$

where $B_x(.,.)$ is the incomplete Beta function defined in [2]. Bound 1 can be finally expressed as

$$P_b \leq \frac{1}{k} \frac{\Gamma(mLN_\eta + 0.5)}{2\sqrt{\pi}\Gamma(mLN_\eta)} x^{-mLN_\eta}(x-1)^{mLN_\eta} \times B_{x/(x-1)}(mLN_\eta, 0.5) \left.\frac{\partial T(D, I)}{\partial I}\right|_{I=1, D=(1+d_n/m)^{-mL}}, \quad (16)$$

where $T(D, I)$ is the transfer function of the code.

Another way to simplify the term J is as follows. Making the change of variable $\xi = \Omega(m - \delta_m)$ and using the integral form of the Q(.) function, the integral in (12) can be written for integer Nakagami parameter, m as

$$J = \frac{1}{(1 - \delta_m/m)^{mLN_\eta}} \int_0^\infty \left(\sqrt{\frac{2}{\pi}} \int_{\sqrt{2\nu\xi}}^\infty e^{-\tau^2/2} d\tau\right) \times \frac{\xi^{mLN_\eta - 1} e^{-\xi}}{(mLN_\eta - 1)!} d\xi, \quad (17)$$

where $\nu = \frac{\delta_m}{m - \delta_m}$. Changing the order of integration and using the properties of the number of arrivals in a Poisson random process as in [3], (17) simplifies to

$$J = \frac{\sqrt{\delta_m/m}}{(1 - \delta_m/m)^{mLN_\eta}} \sum_{r=mLN_\eta}^\infty \left(\frac{1 - \delta_m/m}{4}\right)^r \binom{2r}{r}. \quad (18)$$

Proceedings of the Third International
Conference on Wireless and Mobile Communications (ICWMC'07)
0-7695-2796-5/07 $20.00 © 2007 IEEE

Following [3] and substituting (18) in (11) and using the transfer function of the code, Bound 2 can be expressed as

$$P_b \leq \frac{1}{k}\frac{4^{-mLN_\eta}}{2\sqrt{\delta_m/m}}\binom{2mLN_\eta}{mLN_\eta} \times \frac{\partial T(D,I)}{\partial I}\bigg|_{I=1,D=(1+d_n/m)^{-mL}}. \quad (19)$$

The proposed union bounds are shown in Figures 1 and 2 for a rate-1/2 (5,7) convolutionally coded BPSK and the 8-state 8PSK TCM [4], respectively. It is worth mentioning that MATLAB package was used in the computation of the proposed bounds. In the figures, E_b/N_0 represents the SNR per information bit where $E_s = R_c E_b$ and the performance is shown for channels with different number of resolvable multipath components. We observe that the proposed bounds are tight to simulation results for a wide range of SNR, resolvable multipath components and Nakagami parameters.

4 Conclusions

Union bounds on the bit error probability of coded coherent DSSS systems over Nakagami-m fading channel were derived. Results show that the proposed bounds are tight to simulation results. Furthermore, proposed bounds are expressed in closed forms and simple to evaluate using the transfer function of the code.

5 Acknowledgements

The author acknowledges the support provided by KFUPM.

References

[1] S. Al-Semari. On the error probability of convolutionally encoded spread spectrum bpsk in frequency selective rayleigh fading. *IEEE International Symposium on Information Theory, ISIT*, page 459, August 1998.

[2] I. Gradshteyn and I. Ryzhik. *Table of Integrals, Series, and Products*. Academic Press Inc., London, UK, 4th edition, 1965.

[3] E. Ince, N. Kambo, and S. Ali. Efficient Expression and Bound for Pairwise Error Probability in Rayleigh Fading Channels, With Application to Union Bounds for Turbo Codes. *IEEE Communications Letters*, 19:25–27, January 2005.

[4] S. H. Jamali and T. Le-Ngoc. *Coded-Modulation Techniques for Fading Channels*. Kluwer Academic Publisher, Boston, MA, USA, 1994.

[5] L. Jianxia, J. Zeidler, and J. Proakis. Error probability performance for w-cdma systems with multiple transmit and receive antennas in correlated nakagami fading channels. *IEEE Transactions on Vehicular Technology*, 51:15021516, November 2002.

[6] P. Schramm. Tight upper bound on the bit error probability of convolutionally encoded spread spectrum communication over requency-selective rayleigh-fading channels. *IEEE International Conference on Communication, ICC*, pages 1727–1731, 1995.