

Performance Analysis of Coded Cooperation Diversity in Wireless Networks

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Abstract—Diversity is an effective technique in enhancing the link quality and increasing network capacity. When multiple antennas can not be used in mobile units, user cooperation can be employed to provide transmit diversity. In this paper we analyze the error performance of coded cooperation diversity with multiple cooperating users. We derive the end-to-end bit error probability of coded cooperation (averaged over all cooperation scenarios). We consider different fading distributions for the interuser channels. Furthermore, we consider the case of two cooperating users with correlated uplink channels. Results show that more cooperating users should be allowed under good interuser channel conditions, while it suffices to have two cooperating users in adverse interuser conditions. Furthermore, under bad interuser conditions, more cooperating users can be accommodated as the fading distribution becomes more random.

I. INTRODUCTION

Next-generation wireless communication networks will be very different from second-generation cellular systems especially in the networking architecture. The mobile radio channel suffers from multipath fading, which causes random variations of the signal levels at the mobile units during a communication session. Diversity is considered as an effective tool for combating multipath fading [1]. Diversity is achieved by effectively transmitting or processing independently faded copies of the signal. Among diversity techniques, transmit diversity relies on the principle that signals transmitted from geographically separated transmitters experience independent fading, which results in a significantly improved performance compared to systems with no diversity [2], [3]. Since most wireless networks operate in a multiuser mode, *user cooperation* [4], [5] can be employed to provide diversity. In user cooperation, mobile units share their antennas to achieve uplink transmit diversity as illustrated in Figure 1. Since signals transmitted by different users undergo independent fading paths to the base station (BS), this approach achieves spatial diversity through the partner's antenna. The basic idea of user cooperation is based on the relay channel [6], [7] and on the multiple access channel [8].

In conventional user cooperation the partner repeats the received bits (via either forwarding or hard detection). Recently, a new framework for user cooperation was proposed [9]–[11] and is called *coded cooperation*. Unlike conventional user cooperation schemes, symbols in coded cooperation are not repeated by the partner. Instead, the codeword of each user is partitioned into two parts; one part is transmitted by the user, and the other part is sent by his partner. Coded cooperation provides significant performance gains for a variety of channel conditions. In addition, by allowing different code rates through rate-compatible coding [12], coded cooperation provides a great degree of flexibility to adapt to channel conditions.

In [10] the performance of a two-user coded cooperation system was derived assuming that errors occurring in a codeword are equally distributed among the subframes sent by the cooperating users. This assumption is not necessarily true. Furthermore, the approach of [10] becomes inaccurate and complicated when the number of cooperating users exceeds two. In this paper we propose an analytical framework for deriving and evaluating the error performance of coded cooperation with multiple cooperating users. In this framework, the end-to-end probability of error averaged over different cooperation scenarios is derived. In addition, the bit error probability is derived for specific cooperation scenarios. Moreover, we consider the scenario of two cooperating users with correlated uplink channels.

The paper is organized as follows. In Section II, the system model of coded cooperation with multiple cooperating users is described. The end-to-end error performance of coded cooperation is derived in Section III. The bit error probability corresponding specific cooperation scenarios is derived in Section IV. Results are presented and discussed in Section V. The main outcomes of the paper are summarized in Section VI

II. SYSTEM MODEL

A. Network Architecture

The coded cooperation scenario is illustrated in Figure 1. Coded cooperation starts by forming *clusters* of

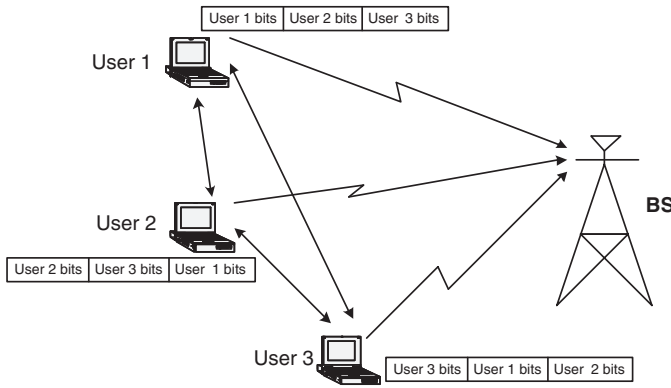


Fig. 1. Schematic diagram of a 3-user cluster employing coded cooperation.

users, where users in a cluster cooperate to transmit their information to a common BS. The users within a cluster are called *partners*. Let J be the number of cooperating users in a cluster. For each user, a frame is formed by encoding K bits into $L = K/R$ bits, where R is the code rate. Partners cooperate by dividing their L -bit frames into J subframes containing L_1, L_2, \dots, L_J bits, where $L = L_1 + L_2 + \dots + L_J$. In the first $N_1 T$ seconds of each frame, each user transmits his first subframe composed of $N_1 = K/R_1$ coded bits, where R_1 is the code rate of the codeword in the first subframe, obtained by puncturing N -bit codeword. Upon the end of the first subframe, each user decodes the rate- R_1 codewords of his partners. The partitioning of the coded bits in the J subframes may be achieved using a rate-compatible punctured convolutional (RCPC) codes [9] as in [12].

In the remaining $J - 1$ subframes, each user in the cluster transmits one subframe for each of his $J - 1$ partners. Each of these subframes contains parity bits of one of his partners which were not sent yet to the BS. Figure 1 shows the contents of the J subframes of each user in a 3-user cluster, i.e., $J = 3$. If a user was not able to decode the first subframe of his partner, whom he should send his parity in a given subframe, then he sends his next parity subframe, i.e., the parity subframe that was not yet sent by any of his partners. Thus each user transmits a total of N bits per source block over the J subframes. The *cooperation level* is defined as the percentage of the total bits per each source block that each user transmits for his partners, i.e., $\frac{N - N_1}{N}$.

B. Physical Link

After encoding the information block, the coded bits are modulated using BPSK. The matched filter output at user k due to user l in the time interval t in the first subframe is modeled by

$$y_{l,k}(t) = \sqrt{E_i} a_{l,k} s_l(t) + z_k(t), \quad (1)$$

where $s_l(t)$ is the signal transmitted from user l in time instance t in the first subframe and $z_k(t)$ is an AWGN sample at user k with a Normal distribution given by $\mathcal{N}(0, \frac{N_0}{2})$. Here, E_i is the average received energy through the interuser channel and the average interuser signal-to-noise ratio (SNR) is $\gamma_i = \frac{E_i}{N_0}$. The coefficient $a_{l,k}$ is the gain of the interuser channel between user l and user k . The interuser channels are assumed to be independent and identically distributed (i.i.d) with a Rayleigh distribution.

When $k = 0$, the signal model in (1) represents the uplink channel from user l to the BS, where the received average energy is denoted by E_s and the average uplink SNR is $\gamma_s = \frac{E_s}{N_0}$. The uplink channels from different users are assumed to be i.i.d with a Rayleigh distribution. Moreover, the interuser channels and the uplink channels are assumed to be mutually independent and slow enough such that the fading process stays fixed within a subframe. This is a reasonable assumption for slowly moving mobile units that are separated enough in the space [13]. In addition, we assume that the interuser channels are reciprocal as in [4], [5]. At the receivers of users and the BS, coherent detection is employed using perfect channel side information.

III. END-TO-END PROBABILITY OF ERROR

In this section we derive the end-to-end bit error probability for users in a coded cooperation network. Throughout the paper, the subscripts c , u and b are used to denote conditional, unconditional and bit error probabilities, respectively. In a cluster, each user acts independently from his partners, not knowing whether his partners have decoded successfully his first subframe. Hence, there are different scenarios for the transmission in the subsequent $J - 1$ subframes for each user in the cluster. The end-to-end error probability is obtained by averaging the error probability (of a specific cooperation scenario) over the different cooperation scenarios, which was derived for the case of two cooperating users in [10].

In a cluster of size J , there are J^2 possible cooperation scenarios. The end-to-end error probability of a user is obtained by averaging the probability of error over two random variables. The first random variable, U indicates the number of partners who were able to decode the first subframe of the user. The second variable, V indicates the number of partners whose first subframes were decoded successfully by the user. In order to simplify analysis, we assume that the effect of duplicate reception of subframes (from the user and one of his partners) is negligible, i.e., subframes are transmitted once through the cluster.

The end-to-end bit error probability averaged over

all cooperation scenarios is given by

$$P_b = \sum_{v=0}^{J-1} \sum_{u=0}^{J-1} \binom{J-1}{v} \binom{J-1}{u} p_{v,u} P_b(v, u), \quad (2)$$

where $P_b(v, u)$ is the conditional bit error probability of a user given that u partners decoded his first subframe successfully, and he decoded v of his partners, and $p_{v,u}$ is the probability of such event and given by

$$p_{v,u} = E_{h_i} \{ [1 - P_B(h_i)]^{v+u} P_B(h_i)^{2J-2-v-u} \}, \quad (3)$$

where h_i is the gain of the interuser channel and $P_B(h_i)$ is the packet error probability of the first subframe, which is upper bounded [14] as

$$P_B(h_i) \leq 1 - [1 - P_E(h_i)]^B, \quad (4)$$

where B is the number of trellis branches in the rate- R_1 codeword of the first subframe. In general, for a rate- $1/n$ convolutional code (or obtained by puncturing a rate- $1/n$ code), B is equal to the source block length K [15]. In (4), $P_E(h_i)$ is the error event probability that is evaluated using the *limiting-before-averaging* approach [16] as

$$P_E(h_i) \leq \min \left\{ 1, \sum_{d=d_{\min}}^{N_1} a_d P_c(d|h_i) \right\}, \quad (5)$$

where a_d is the number of error events with a Hamming distance d from the all-zero codeword and $P_c(d|h_i) = Q(\sqrt{2d|h_i|^2})$ is the conditional pairwise error probability of a weight- d codeword over the interuser channel with a channel gain of h_i . Note that $P_c(d|h_i)$ is the probability of decoding a received sequence as a weight- d codeword in a rate- R_1 code given that the all-zero codeword was transmitted.

Among the different cooperation scenarios, it was found that the two extreme scenarios of *no cooperation* and *full cooperation* have the largest probabilities, denoted as $p_{0,0}$ and $p_{J-1,J-1}$, respectively. Thus the performance of coded cooperation is dominated by the performance of these two cooperation scenarios. The probabilities $p_{0,0}$ and $p_{J-1,J-1}$ are listed in Table I for different cluster sizes and interuser SNR values. We observe that for a fixed interuser channel quality, the probability of no cooperation increases as the cluster size increases, which causes the performance of large-size clusters to be worse than that of small-size clusters. As the uplink quality improves for a fixed interuser quality, small-size clusters are expected to outperform large-size clusters. This is because small-size clusters has a smaller probability of no cooperation which has a clear effect on the performance especially at high uplink SNR as will be shown through the results in Section V.

TABLE I

THE PROBABILITIES OF *no cooperation* AND *full cooperation* SCENARIOS FOR A J -USER CLUSTER OVER RAYLEIGH INTERUSER CHANNELS WITH AN INTERUSER SNR OF γ_i .

γ_i (dB)	$p_{v,u}$	$J = 2$	$J = 3$	$J = 4$
0	$p_{0,0}$	0.5950	0.7249	0.8987
	$p_{J-1,J-1}$	0.3491	0.1769	0.0481
10	$p_{0,0}$	0.0869	0.1216	0.2053
	$p_{J-1,J-1}$	0.8992	0.8389	0.7345
20	$p_{0,0}$	0.0088	0.0124	0.0223
	$p_{J-1,J-1}$	0.9897	0.9830	0.9698
∞	$p_{0,0}$	0	0	0
	$p_{J-1,J-1}$	1	1	1

IV. BIT ERROR PROBABILITY

In this section we derive the bit error probability corresponding to a specific cooperation scenario. Given $U = u$ and $V = v$ for a user in a cluster, the bit error probability of the corresponding convolutional code is upper bounded [15] as

$$P_b(v, u) \leq \sum_{d=d_{\min}}^{N(v,u)} c_d P_u(v, u; d), \quad (6)$$

where d_{\min} is the minimum distance of the code and c_d is the number of information bit errors corresponding to codewords with output weight d . In (6), $P_u(v, u; d)$ is the unconditional pairwise error probability for a weight- d codeword given that u partners decoded correctly the first subframe of this user and he decoded the first subframe of v of his partners. Furthermore, $N(v, u)$ is the codeword length corresponding to $V = v$ and $U = u$.

Conditioning on $U = u$ and $V = v$ has two consequences on the error performance of a user. First, the received codeword at the BS has a rate R_ξ , where $\xi = \max(J-v, u+1)$. This is due to the negligible effect of duplicate transmission of subframes because of the dominant performance of the no and full cooperation scenarios as discussed above. In this case, $\{c_d\}$ used in (6) are for the rate- R_ξ code. Second, given that $U = u$, each codeword is transmitted over $u+1$ subframes, whose lengths are $\{N_j\}_{j=1}^{u+1}$ bits. Recall that each subframe is transmitted over an independent fading channel via one of the partners in a cluster. Thus, the pairwise error probability $P_u(v, u; d)$ is a function of the distribution of the d error bits over the $u+1$ subframes transmitted by the $u+1$ partners. Since the coded bits of each subframes may not be consecutive bits due to the puncturing used, this distribution is quantified assuming uniform distribution of the coded bits over the subframes [17], [18] and is derived as follows.

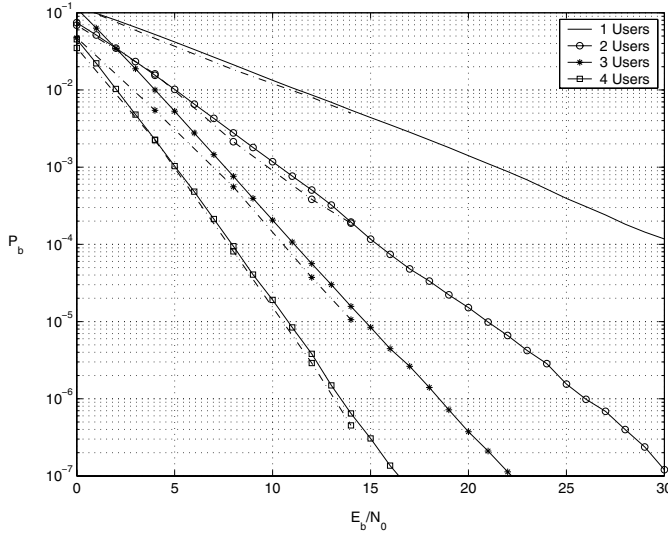


Fig. 2. Bit error probability of coded cooperation with Rayleigh uplink channels and perfect Rayleigh interuser channels, solid: approximation, dashed: simulation.

A. Uncorrelated Uplink Channels

Denote the weight of the j^{th} subframe in the codeword by w_j such that $\sum_{j=1}^{u+1} w_j = d$, then the pairwise error probability averaged over the weight patterns $\mathbf{w} = \{w_j\}_{j=1}^{u+1}$ is given by

$$P_u(v, u; d) = \sum_{w_1, w_2, \dots, w_{u+1}} \frac{\binom{N_1}{w_1} \binom{N_2}{w_2} \dots \binom{N_{u+1}}{w_{u+1}}}{\binom{N}{d}} P_u(v, u; d|\mathbf{w}). \quad (7)$$

The pairwise error probability $P_u(v, u; d|\mathbf{w})$ is found by averaging $P_c(v, u; d|\mathbf{w})$ over the fading gains. The conditional pairwise error probability for BPSK with coherent detection is given by

$$P_c(v, u; d|\mathbf{w}) = Q \left(\sqrt{2\gamma_s \sum_{j=1}^{u+1} w_j a_j^2} \right), \quad (8)$$

where $a_j = |h_j|$. An exact expression of the pairwise error probability can be found by using the integral expression of the Q -function, $Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-x^2/2 \sin^2 \theta} d\theta$ [19] as

$$\begin{aligned} P_u(v, u; d|\mathbf{w}) &= \frac{1}{\pi} E_{\mathbf{a}} \left[\int_0^{\frac{\pi}{2}} \exp \left(-\beta_{\theta} \sum_{j=1}^{u+1} w_j a_j^2 \right) d\theta \right] \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{j=1}^{u+1} \frac{1}{1 + w_j \beta_{\theta}} d\theta, \end{aligned} \quad (9)$$

where $\mathbf{a} = \{a_j\}_{j=1}^{u+1}$, $\beta_{\theta} = \gamma_s / \sin^2 \theta$ and the product results from the independence of the fading processes affecting different subframes. Note that due to the summation in (7), the union bound in (6) becomes complicated when d is large. Thus an approximation

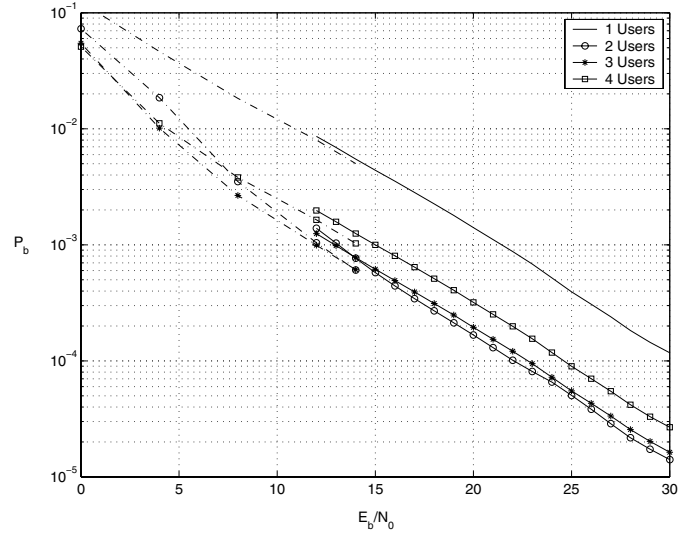


Fig. 3. Bit error probability of coded cooperation with Rayleigh uplink channels and 10-dB Rayleigh interuser channels, solid: approximation, dashed: simulation.

to the bit error probability is obtained by truncating (6) to a distance d_{\max} .

B. Correlated Uplink Channels

The mobile units might be located closely in the space which causes the uplink channels to be correlated. The effect of correlation in the uplink channels is investigated below. The conditional pairwise error probability in (8) can be rewritten as

$$P_c(v, u; d|\mathbf{w}) = Q \left(\sqrt{2\gamma_s \tilde{\mathbf{h}}^* \tilde{\mathbf{h}}} \right), \quad (10)$$

where $\tilde{\mathbf{h}} = [\sqrt{w_1}h_1, \sqrt{w_2}h_2, \dots, \sqrt{w_{u+1}}h_{u+1}]^T$. If the fading gains of the uplink channels are complex Gaussian (i.e., fading magnitude is Rayleigh distributed), the vector \mathbf{h} is a correlated complex Gaussian random vector with a covariance matrix $K_{\tilde{\mathbf{h}}}$ whose $(i, j)^{\text{th}}$ element is given by

$$K_{\tilde{\mathbf{h}}}(i, j) = E[\tilde{h}_{i,j} \tilde{h}_{i,j}^*] = \rho_{ij} \sqrt{w_i w_j}, \quad (11)$$

where ρ_{ij} is the correlation coefficient between the uplink channels of the i^{th} and the j^{th} cooperating users. Clearly this probability is a function of the inner product $\sum_{j=1}^{u+1} |\tilde{h}_j|^2 = \tilde{\mathbf{h}}^* \tilde{\mathbf{h}}$. The unconditional error probability is found by averaging (10) over the joint pdf of $\tilde{\mathbf{h}}$ as in [20]. Thus the unconditional pairwise error probability becomes

$$P_u(v, u; d|\mathbf{w}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{j=1}^{u+1} \frac{1}{1 + \lambda_j \beta_{\theta}} d\theta, \quad (12)$$

where $\{\lambda_i\}_{i=1}^{u+1}$ are the eigenvalues of $K_{\tilde{\mathbf{h}}}$. When the SNR becomes high, the pairwise error probability ap-

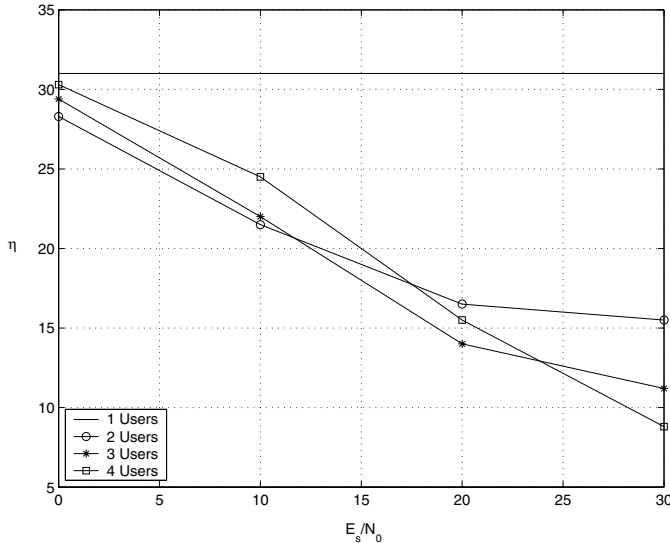


Fig. 4. Uplink SNR required to achieve $P_b = 10^{-4}$ versus the interuser SNR for Rayleigh interuser channels.

proaches

$$P_u(v, u; d|\mathbf{w}) \sim \prod_{j=1}^{u+1} \frac{1}{\beta_\theta \lambda_j}. \quad (13)$$

From (13) it is clear that the diversity order of the coded cooperation with correlated uplink channels is maintained with a reduction in the SNR.

V. NUMERICAL RESULTS

In this section we present numerical results based on the analysis derived above. We consider coded cooperation with cluster sizes $J = 1, 2, 3, 4$. Each user in the cluster employs a RCPC code from [12] with four memory elements, a puncturing period $P = 8$ and a mother code of rate of $R = R_J = \frac{1}{4}$. In all cases, the source block is $K = 128$ information bits. Analytical results are obtained by truncating the union bound by including terms in (6) up to $d_{\max} = 20$.

In Figure 2 the bit error probability is shown versus the uplink SNR assuming perfect Rayleigh interuser channels, i.e., infinite interuser SNR. We observe that increasing the cluster size by one user results in significant performance gains, where the achieved performance gains decrease as the cluster size increases. Note that the performance gains of coded cooperation appears in the slope of the error probability curve versus the SNR. This is because more cooperating users increases the diversity order of the coded system.

Figure 3 shows the bit error probability for Rayleigh interuser channels with an SNR of 10 dB, where the approximation is shown for $\gamma_s > 10$ to reduce confusion resulting from the overlapping curves in the low-SNR region. We observe that the performance of clusters with four users is the best for low-to-medium SNR values, where the situation gets reversed as the as the

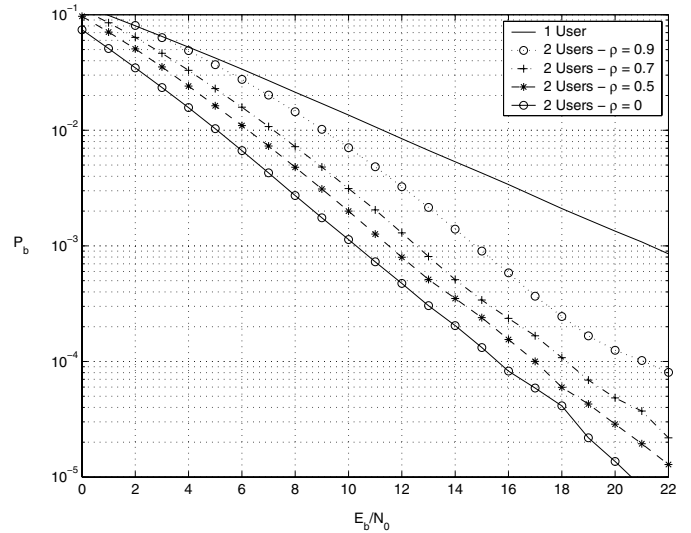


Fig. 5. Analytical bit error probability of a two-user coded cooperation with correlated Rayleigh uplink channels and perfect Rayleigh interuser channels.

uplink SNR increases. This is because at high SNR the performance becomes limited by the performance of the no cooperation scenario, whose probability increases with the cluster size as shown in Table I. For example, when the interuser SNR is 10 dB, four users provide the best performance for an uplink SNR lower than 7 dB. For an uplink SNR between 7 dB and 14 dB, three users perform the best, where two users become the best for an uplink SNR greater than 14 dB. Figure 4 shows the uplink SNR, required to achieve $P_b = 10^{-4}$ over Rayleigh interuser channels versus the interuser SNR. We observe that two users perform the best for low interuser channel SNR, and the situation gets reversed as the interuser channel SNR increases. Note that the quality of the interuser channel is usually better than that of the uplink channels because the BS is usually located far away relative to the users within a cluster.

In Figure 5 we show the effect of correlated uplink channels for a two-user cluster in a Rayleigh environment. We observe that the diversity order is maintained even in highly correlated uplink channels ($\rho = 0.9$). For example, coded cooperation with a correlation coefficient $\rho = 0.7$ provides an SNR gain of 9 dB over the single-user case at $P_b = 10^{-3}$, where it encounters an SNR loss of 2 dB compared to the uncorrelated case. This shows that coded cooperation is a powerful technique even when the mobile units are closely located.

VI. CONCLUSIONS

In this paper we analyzed the performance of coded cooperation diversity with multiple cooperating users. We derived a union bound on the end-to-end bit er-

ror probability averaged over different cooperation scenarios. We considered uncorrelated and correlated Rayleigh uplink channels. The effect of the interuser channel quality was investigated analytically. Results show that as the interuser channel quality improves, large clusters outperform small clusters.

VII. ACKNOWLEDGEMENTS

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