Performance Analysis of Bit-Interleaved Space-Time (BI-ST) Coded Systems Over Wireless Channels

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***Abstract*— In this paper a union bound on the bit error probability of bit-interleaved space-time (BI-ST) coded systems is derived. The derivation is based on the uniform interleaving assumption of the coded sequence prior to transmission over the multiple antennas. The performance of a BI-ST coded system is a function of how the bit errors are distributed over the signals in the codeword. In this paper, we derive this distribution as well as the corresponding pairwise error probability. The bound is a function of the distance spectrum of the code, the signal constellation used and the space-time (ST) encoding scheme. The bound is derived for a general BI-ST coded system and applied to two specific examples; namely, the BI space-time coded modulation (BI-STCM) and the BI space-time block codes (BI-STBC). Results show that the analysis provides a close approximation to the performance for a wide range of signal-to-noise ratios (SNR).**

I. Introduction

One standard approach to mitigate fading and achieve bandwidth efficiency is transmit diversity in which multiple antennas are used at the transmitter. Simple and elegant space-time block codes (STBC) were proposed in [1], [2] to provide diversity at the transmitter. Coded modulation [3] is an efficient technique that provides high transmission rates at good quality by combining error control coding and modulation. The basic idea in coded modulation is to partition the signal space into signal subsets and use coding to maximize a distance measure between the coded signals. In perfectly-interleaved fading channels the symbol-wise Hamming distance between signals in different subsets has to be maximized by the code designer. This Hamming distance can be increased by interleaving the coded bits prior to mapping them onto the signal constellation [4], [5]. This method is referred to as bit-interleaved coded modulation (BICM). BICM was applied to multi-input multi-output (MIMO) systems in [6], [7], in which the coded bits are bit-interleaved and each group of bits are mapped onto signals that are transmitted over multiple transmit antennas. Two approaches to mapping the coded bits onto the signals are considered in this paper; namely, the BI ST block code (BI-STBC) [8] and the BI ST coded modulation (BI-STCM) [9], [10].

The original motivation behind proposing ST coded systems is to provide diversity to systems operating in quasi-static fading environments. However, it is also of interest to study the performance of ST systems over rapidly varying fading channels. For example, multiple antennas can be used at base stations to provide receive diversity to the uplink as well as transmit diversity to the downlink. When the speed of a mobile unit increases, the fading between the base station and the mobile unit becomes rapidly varying and can not be modeled as a quasi-static fading channel. In delay-tolerant applications, interleaving with large depth can be used to imitate the fully-interleaved channels, which results in almost uncorrelated fading attenuations of neighboring symbols within a codeword. Therefore it is of great interest to analyze the performance of ST coded systems over rapidly varying fading channels.

Because of the interleaver used in the transmitter, each signal vector (transmitted over the multiple transmit antennas) is composed of coded bits that are randomly located in the coded sequence (from the decoder point-of-view). Thus a symbol error of the modulation mostly will not cause consecutive bit errors in the codeword, which enhances the performance dramatically. However, the random nature of distributing the error bits over different symbols causes the performance analysis to be difficult. A union bound on the bit error probability of BICM systems was presented in [5], [11]. The bound was based on the assumption that every symbol error causes only one bit error among the bits associated with the symbol. However, due to

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the interleaving used, symbol errors result in multiple bit errors that are randomly distributed over the coded sequence. This fact was used in [12] to derive a union bound for BICM. For quasistatic fading environments, union bounds for BI-STBC and BI-STCM were derived in [8] and [9], respectively.

In this paper we derive a union bound on the bit error probability of general BI-ST coded systems over rapid fading channels. Both BI-STBC and BI-STCM systems are considered as specific examples. The new bound is based on the uniform (random) interleaving of the coded bits prior to mapping them onto modulation symbols that are transmitted over the transmit antennas. The distribution of the error bits in a received vector is derived and the corresponding pairwise error probability is evaluated. Simulation results show that the proposed bound is tight for different signal constellations, ST coding schemes and channel models.

The outline of the paper is as follows. The model for BI-ST coded system is described in Section II. In Section III, the proposed union bound is derived. The characteristic function required to evaluate the union bound is derived for the BI-STCM and BI-STBC systems in Section IV. Analytical and simulation results are presented in Section V. Conclusions are discussed in Section VI.

II. System Model

Consider the BI-ST coded system shown in Figure 1. The encoder receives an information block $\mathbf{u}$ of K bits and generates an N-bit codeword $\mathbf{c}$ resulting in a code rate $R_c = \frac{K}{N}$. After encoding, the codeword $\mathbf{c}$ is bit interleaved to generate the interleaved codeword $\pi(\mathbf{c}) = (\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_L)$ that consists of L blocks each of qm bits. Each of the L blocks is referred to as a ST block (STB). Note that $N = qmL$. The STB $\mathbf{b}_l$ is mapped onto q symbols $(s_{l,1}, s_{l,2}, \cdots, s_{l,q})$ by a ST mapper. Each of the q symbols are drawn from an M-ary complex signal constellation that consists of $M = 2^m$ signal points with average symbol energy equal to E_s. Every q symbols are mapped by the ST encoder into p column vectors of length n_T for transmission by n_T transmit antennas.

The ST code is characterized by an $n_T \times p$ transmission matrix, where $p = 1$ or an integer that satisfies $p \geq n_T$. In the case of $p = 1$, the system is called the ST coded modulation (STCM) [13], whereas the case of $p \geq n_T$ results in the well-known ST block code (STBC) [1] as will be clarified in the examples presented below. The rows of the transmission matrix consists of entries that are linear combinations of $s_{l,1}, s_{l,2}, \cdots, s_{l,q}$ and $s^*_{l,1}, s^*_{l,2}, \cdots, s^*_{l,q}$. Denote the transmission matrix by $\mathbf{x}_l = [\ \mathbf{x}_{l,1}{}^T, \mathbf{x}_{l,2}{}^T, \cdots, \mathbf{x}_{l,p}{}^T\ ]$, where $\{\mathbf{x}_{l,t}{}^T\}$ are column vectors of dimension $n_T \times 1$. The ST encoder maps the vector $(s_{l,1}, s_{l,2}, \cdots, s_{l,q})$ onto the column vectors $\mathbf{x}_{l,1}{}^T, \mathbf{x}_{l,2}{}^T, \cdots, \mathbf{x}_{l,p}{}^T$, and the vectors

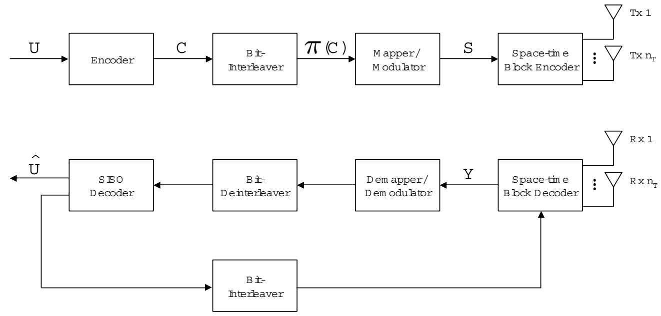

Fig. 1. Block diagram of the general BI-ST coded system.

$\{\mathbf{x}_{l,t}{}^T\}$ are transmitted by n_T antennas one at a time over p transmission intervals. The p transmission intervals constitute one STB. The code rate of the ST encoder is $R_s = \frac{qm}{p}$, and the overall rate of the BI-ST coded system is $R = R_s R_c = \frac{qmK}{pN}$. In the case of STBC, i.e., $p \geq n_T$, the rows of transmission matrix are constructed to be orthogonal in order to enable a linear-complexity receiver [1]. Specific examples of ST codes are presented below.

Example 1: When $p = 1$ and $q = n_T$, the resulting system is a STCM system. In STCM, every n_T symbols are transmitted over the n_T transmit antennas during one symbol duration. In this case the fading processes affecting consecutive symbols are assumed to be independent.

Example 2: When $p = q = n_T = 2$, the resulting system is the Alamouti STBC. The Alamouti STBC is characterized by the 2×2 complex matrix $\mathbf{x}_l$ presented in [1]. In this case the fading process should stay constant for at least two symbols to enable simple detection. This is a full-rate STBC.

In general, the receiver is assumed to have n_R antennas. However, in order to simplify notation we derive the results for systems with a single receive antenna. Note that all the results are easily generalized to multiple-receive antennas and the result will be summarized separately later. The channel from the n_T transmit antennas to the receive antenna is represented by an $1 \times n_T$ channel vector. The fading channel is assumed to be constant during one STB to enable low-complexity receivers for the STBC case [1]. The channel vector of the t^{th} transmission interval in the l^{th} STB is denoted by $\mathbf{h}_{l,t} = (h^1_{l,t}, h^2_{l,t}, \cdots, h^{n_T}_{l,t})$, where $h^i_{l,t}$ denotes the fading attenuation of the channel from the i^{th} transmit antenna to the receive antenna in the l^{th} STB. The fading channels from different transmit antennas are assumed to be independent and identically distributed (i.i.d.) Rayleigh random variables.

The received signal vector corresponding to a codeword $\mathbf{c}$ is denoted as $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_L)$ where $\mathbf{r}_l =$

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$(r_{l,1}, r_{l,2}, \cdots, r_{l,p})$ and

$$r_{l,t} = \mathbf{h}_{l,t}\mathbf{x}_{l,t}{}^T + \mathbf{n}_{l,t}, \quad (1)$$

where $\mathbf{n}_{l,t}$ is a length-p AWGN vector at the receiver during the t^{th} transmission period in the l^{th} STB modeled as $\mathcal{CN}(\mathbf{0}_p, N_0\mathbf{I}_p)$, where $\mathbf{I}_p$ denotes the $p\times p$ identity matrix and $\mathbf{0}_p$ is the $1 \times p$ zero matrix. The receiver is assumed to have perfect channel state information (CSI) and the decoding is done by minimizing the decision metric

$$\sum_{l=1}^{L}\sum_{t=1}^{p} ||r_{l,t} - \mathbf{h}_{l,t}\mathbf{x}_{l,t}{}^T||^2, \quad (2)$$

which can be closely achieved via iterative ST detection and decoding [14]. Since the design of iterative detection and decoding is beyond the scope of this paper, the reader is referred to [14] for further information. Note that when multiple-receive antennas are used at the receiver, the decision metric in (2) is replaced by the corresponding maximal-ratio combining (MRC) metric.

III. The Union Bound

For the sake of analysis, we make the uniform interleaving assumption, i.e., if we feed a codeword $\mathbf{c}$ with a Hamming weight d to the interleaver, then the output could be any weight-d bit sequence of length N with an equal probability given by $\frac{1}{\binom{N}{d}}$. The bit error probability for a convolutional code is upper bounded [15] by

$$P_b \lesssim \sum_{d=d_{min}}^{N}\sum_{j=1}^{K}\frac{j}{K}\; w_{j,d}\; P_u(d), \quad (3)$$

where $d_{\min}$ is the minimum Hamming distance of the convolutional code, $w_{j,d}$ denotes the number of convolutional codewords with input Hamming weight j and total weight d, and $P_u(d)$ is the pairwise error probability defined as the probability of decoding a received sequence as a weight-d codeword $\hat{\mathbf{c}}$ given that the codeword $\mathbf{c}$ was transmitted, i.e., the Hamming distance is $d_H(\mathbf{c}, \hat{\mathbf{c}}) = d$. Throughout the paper, for any variable defined for $\mathbf{c}$, the corresponding variable defined for $\hat{\mathbf{c}}$ is denoted by using " ˆ ". The subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. Clearly $P_u(d)$ depends on the squared Euclidean distance $d_E^2 \triangleq [\ d_E(\mathbf{c}, \hat{\mathbf{c}})\ ]^2$ between the received sequences corresponding to the codewords $\mathbf{c}$ and $\hat{\mathbf{c}}$, which is a function of the distribution of the d nonzero bits over the L STBs in the codeword. Using the integral expression [16] of the Q-function, $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{(-x^2/2\sin^2\theta)}d\theta$, we have

$$P_u(d) = \mathrm{E}_{d_E^2|d}\left[Q\left(\sqrt{\frac{R\gamma_b}{2}\cdot d_E^2}\right)\right]$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Psi_{d_E^2|d}\left(\frac{R\gamma_b}{4\sin^2\theta}\right)\, d\theta, \quad (4)$$

where $\gamma_b = \frac{E_b}{N_0}$ is the signal-to-noise ratio (SNR) per information bit and $\Psi_{d_E^2|d}(z) \triangleq \mathrm{E}_{d_E^2|d}\left[e^{-zd_E^2}\right]$ is the conditional characteristic function of the random variable d_E^2 given d.

Since the combination of the signal constellation mapping with the ST encoding may not have a symmetric structure for all codewords, the Euclidean distance $d_E(\mathbf{c}, \hat{\mathbf{c}})$ may not be the same for different choices of $\mathbf{c}$ and $\hat{\mathbf{c}}$ even if the Hamming distance $d_H(\mathbf{c}, \hat{\mathbf{c}})$ is fixed at d. Hence we have to take the expectation in (4) with respect to the distribution of d_E^2 given d. Thus the task is to find the conditional distribution of d_E^2 given d. Denote the error vector between two codewords $\pi(\mathbf{c})$ and $\pi(\hat{\mathbf{c}})$ by $\mathbf{e}(\mathbf{c}, \hat{\mathbf{c}}) = (\ \mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_L\ )$, where $\mathbf{e}_l = (\mathbf{e}_{l,1}, \mathbf{e}_{l,2}, \cdots, \mathbf{e}_{l,p})$ and $\mathbf{e}_{l,t} = \mathbf{x}_{l,t} - \hat{\mathbf{x}}_{l,t}$. The squared Euclidean distance d_E^2 can be expressed as

$$d_E^2 = \sum_{l=1}^{L}\sum_{t=1}^{p} ||\mathbf{h}_{l,t}\mathbf{e}_{l,t}{}^T||^2 = \sum_{l=1}^{L} d_l^2, \quad (5)$$

where $d_l^2 = \sum_{t=1}^{p} ||\mathbf{h}_{l,t}\mathbf{e}_{l,t}{}^T||^2$ is the squared Euclidean distance between the received signal vectors associated with the l^{th} STB, $\mathbf{b}_l$. Since the total number of bit errors in the codeword is d, the distribution of d_l^2 depends on how many bit errors exist in the STB $\mathbf{b}_l$. Thus it is necessary to find the conditional distribution of d_l^2 given f_l, where f_l denotes the number of bit errors in $\mathbf{b}_l$. Due to the uniform interleaving and the independent fading assumptions, the conditional distributions of $\{d_l^2|f_l\}$ are identical and the characteristic function of d_E^2 given d can be obtained as

$$\Psi_{d_E^2|d}(z) = \mathrm{E}_{f_1,\cdots,f_L}\left[\prod_{l=1}^{L}\Psi_{d_l^2|f_l}(z)\right]$$

$$= \mathrm{E}_{j_1,\cdots,j_w}\left[\prod_{v=1}^{w}\left[\phi_v(z)\ \right]^{j_v}\right], \quad (6)$$

where j_v denotes the number of STB's with v bit errors, $w = \min\{d, qm\}$ and $\phi_v(z)$ is given by

$$\phi_v(z) \triangleq \Psi_{d_l^2|f_l}(z|f_l = v) = \mathrm{E}_{d_l^2|f_l}\left[e^{-zd_l^2}|f_l = v\right]. \quad (7)$$

Clearly, the form of $\phi_v(z)$ depends on the fading distribution, which will be derived in Section IV. Since $d_H(\mathbf{c}, \hat{\mathbf{c}}) = d$, the components of the vector $\mathbf{j} = \{j_0, j_1, ..., j_w\}$ are constrained by the conditions

$$L = \sum_{v=0}^{w} j_v, \qquad d = \sum_{v=1}^{w} v j_v. \quad (8)$$

The joint pdf of $\mathbf{j}$ given d can be derived using combi-

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natorics as

$$p(\mathbf{j}|d) = \frac{\binom{qm}{1}^{j_1}\binom{qm}{2}^{j_2}\cdots\binom{qm}{w}^{j_w}}{\binom{N}{d}} \cdot \frac{L!}{j_1!j_2!\ldots j_w!}. \quad (9)$$

The left factor of $p(\mathbf{j}|d)$ in (9) is the probability of distributing d nonzero bits over L error vectors with j_v error vectors having v bits for possible values of v. The right term of $p(\mathbf{j}|d)$ is the number of combinations of $\mathbf{j} = \{j_v\}_{v=0}^{w}$ among the L error vectors. The expectation in (6) is computed as

$$\Psi_{d_E^2|d}(z) = \sum_{j_w=0}^{L_w}\sum_{j_{w-1}=0}^{L_{w-1}}\cdots\sum_{j_1=0}^{L_1}\left(p(\mathbf{j}|d)\prod_{v=1}^{w}[\ \phi_v(z)\ ]^{j_v}\right), \quad (10)$$

where

$$L_v = \max\left\{0, \left\lfloor\frac{d-\sum_{r=v+1}^{v} rj_r}{v}\right\rfloor\right\}, \quad 1 \le v \le w. \quad (11)$$

Substituting (7)-(11) into (4) results in the final form of the unconditional pairwise error probability. The rest of the paper is devoted to deriving expressions of the characteristic function $\phi_v(z)$ for BI-STCM ($p = 1$) and BI-STBC ($p \ge n_T$) systems with different fading statistics.

IV. The Characteristic Function

A. BI-STCM

In BI-STCM systems, $p = 1$ and $q = n_T$, and thus we use the notations $\mathbf{e}_l = \mathbf{e}_{l,1}$ and $\mathbf{h}_l = \mathbf{h}_{l,1}$. In this case, the distance is given by $d_l^2 = |\mathbf{h}_l\mathbf{e}_l{}^T|^2$. Going through the derivation in [13], the distance d_l^2 simplifies to

$$d_l^2 = ||\mathbf{e}_l||^2 \cdot |\beta_l(\mathbf{e}_l)|^2, \quad (12)$$

where

$$\beta_l(\mathbf{e}) \triangleq \frac{\mathbf{h}_l\ \mathbf{e}^T}{||\mathbf{e}||}, \quad (13)$$

is a random variable whose distribution depends on the fading distribution which will be derived later. This implies that $\{\beta_l(\mathbf{e}_l)\}$ are independent random variables. Given a realization of the error vector $\mathbf{e}_l$, the conditional characteristic function of d_l^2 given $\mathbf{e}_l$ becomes

$$\Psi_{d_l^2|\mathbf{e}_l}(z) = \Psi_{|\beta_l(\mathbf{e}_l)|^2}\left(z||\mathbf{e}_l||^2\right). \quad (14)$$

To find $\phi_v(z) = \Psi_{d_l^2|f_l}(z)$, we first consider all $\binom{qm}{2}$ possible STB combinations of $\mathbf{b}_l$ and $\hat{\mathbf{b}}_l$. For each pair, we feed them to the STBC encoder to find the corresponding $\mathbf{x}_l, \hat{\mathbf{x}}_l$, and the error vector $\mathbf{e}_l$. Classify these STB pairs into groups according to $d_H(\mathbf{b}_l, \hat{\mathbf{b}}_l)$. Suppose in the group of $d_H(\mathbf{b}_l, \hat{\mathbf{b}}_l) = v$ bits, the STB pairs of the group generates error vectors $\mathbf{e}_{v,1}, \mathbf{e}_{v,2}, \cdots$ each with multiplicity $\mu_{v,1}, \mu_{v,2}, \cdots$, respectively. Then the conditional joint pdf of $\mathbf{e}_l$ given f_l can be written as

$$p_{\mathbf{e}_l|f_l}(\mathbf{e}|v) = \sum_k \chi_{v,k}\ \Delta(\mathbf{e} - \mathbf{e}_{v,k}), \quad (15)$$

where $\chi_{v,k} = \frac{\mu_{v,k}}{\sum_k \mu_{v,k}}$ is the probability for an error vector $\mathbf{e}_{v,k}$ to occur, and $\Delta(\mathbf{e}) \triangleq 1$ if $\mathbf{e} = \mathbf{0}$; and 0 otherwise. By (14) and (15), we have

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \chi_{v,k}\ \Psi_{|\beta_l(\mathbf{e}_{v,k})|^2}\left(z||\mathbf{e}_{v,k}||^2\right). \quad (16)$$

Clearly, the form of $\Psi_{|\beta_l(\mathbf{e}_{v,k})|^2}(z)$ is a function of the fading distribution. For the case of Rayleigh fading, $\phi_v(z)$ is given by

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \frac{\chi_{v,k}}{1 + z||\mathbf{e}_{v,k}||^2}. \quad (17)$$

B. BI-STBC

In BI-STBC systems the fading gain of each channel remains constant during each STB, i.e., $\mathbf{h}_{l,1} = \mathbf{h}_{l,2} = \cdots = \mathbf{h}_{l,p} = \mathbf{h}_l = \{h_l^i\}$. Recall that $\mathbf{e}_{l,t}$ is a vector of dimension $1 \times n_T$ and denoted by $\mathbf{e}_{l,t} = (e_{l,t}^1, e_{l,t}^2, \cdots, e_{l,t}^{n_T})$. Due to the orthorgonality of the row vectors of STBC transmission matrix, we have

$$d_l^2 = \sum_{t=1}^{p} ||\mathbf{h}_l\ \mathbf{e}_{l,t}{}^T||^2 = \sum_{i=1}^{n_T} |h^i|^2 \cdot \xi_l^i,$$

where $\xi_l^i = \sum_{t=1}^{p} |e_{l,t}^i|^2$. Since $\{h_l^i\}$ are i.i.d. random variables, then the random variables $\{|h_l^i|^2\}$ are also i.i.d. with a characteristic function given by $\Psi_{|h|^2}(z)$. Since $\{|h_l^i|^2\}$ are independent, we can obtain the characteristic function of d_l^2 given $\mathbf{e}_l'$ given a realization of $\mathbf{e}_l' = (\xi_l^1, \xi_l^2, \cdots, \xi_l^{n_T})$ as

$$\Psi_{d_l^2|\mathbf{e}_l'}(z) = \prod_{i=1}^{n_T} \Psi_{|h|^2}\left(z\ \xi_l^i\right). \quad (18)$$

Again we feed all $\binom{qm}{2}$ possible STB pairs of $\mathbf{b}_l$ and $\hat{\mathbf{b}}_l$ to the ST encoder to get the vector $\mathbf{e}_l'$. Using a similar approach of finding (15) in Section IV-A, we can obtain the conditional joint pdf of $\mathbf{e}_l'$ given f_l

$$p(\mathbf{e}_l'|f_l = v) = \sum_k \chi_{v,k}'\ \Delta(\mathbf{e} - \mathbf{e}_{v,k}'). \quad (19)$$

Denote the i^{th} component of $\mathbf{e}_{v,k}'$ by $\mathbf{e}_{v,k}'(i)$. By (18) and (19), we have

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \chi_{v,k}' \prod_{i=1}^{n_T} \Psi_{|h|^2}\left(z\ \mathbf{e}_{v,k}'(i)\right). \quad (20)$$

Clearly, $\Psi_{|h|^2}(z)$ depends on the fading distribution of the channel. If $|h|$ is a Rayleigh distributed random variable, we have

$$\phi_v(z) = \sum_k \chi_{v,k}' \left[\prod_{i=1}^{n_T} \frac{1}{1 + z\ \mathbf{e}_{f,v}'(i)}\right]. \quad (21)$$

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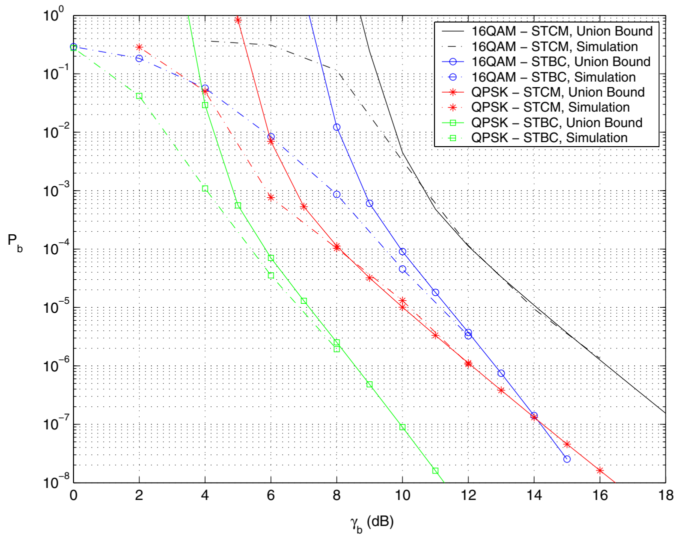

Fig. 2. Bit error probability of convolutionally encoded BI-ST systems using $n_T = 2$ over a Rayleigh fading channel.

V. Numerical Results

As an illustrative example, we use BI-ST coded systems employing a rate-$\frac{1}{2}$ (5,7) convolutional code with two transmit antennas, i.e., $n_T = 2$. Throughout the results, the interleaver size is set to $N = 1024$ coded bits. It is expected that increasing the interleaver size improves the performance. However, due to the space limitation this point is not addressed in this paper. The modulation techniques used are quadrature-phase shift keying (QPSK) and quadrature-amplitude modulation 16-QAM. Note that the derivation above applies to any signal constellation. The throughputs of the BI-STBC and BI-STCM systems are mR_c and mn_TR_c bits/s/Hz, respectively.

The performance of BI-STBC and BI-STCM over Rayleigh fading channels is shown in Figure 2. We observe that the bound is tight to simulation curves at medium-to-high SNR values. Note that the union bound becomes loose for SNR values lower than the cutoff rate of the system [15]. We observe that the performance of BI-STBC is better than that of the BI-STCM. This is because the throughput of BI-STCM is n_T time larger than that of BI-STBC. Furthermore, in BI-STBC there are n_T observations available to detect the transmitted n_T signals, whereas it is only one observation in BI-STCM. Note that the slope of the error probability curves achieved by BI-STBC is larger than that achieved by BI-STCM. This indicates that the time diversity of BI-STBC is larger than that of BI-STCM.

VI. Conclusions

In this paper we derived a union bound on the bit error probability of BI-ST coded systems over rapidly varying fading channels. The derivation is based on the uniform interleaving of coded bits prior to the ST mapping and encoding. The bound is a function of the distance spectrum of the channel code, the signal constellation and the ST encoding scheme. Results show that the proposed bound is tight in medium to high SNR regions.

VII. Acknowledgements

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