Design of 16-QAM Space-Time Codes for Rapid Rayleigh Fading Channels

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ABSTRACT

This paper proposes Space-Time (ST) codes for rapid fading channels using the 16-QAM signal constellation. The design of the proposed codes uses two different methods. The first method uses a high time diversity trellis encoder, and the second uses the I-Q encoding technique. Both encoding methods are expected to produce ST codes that perform better than the codes presented in the literature. Optimal and suboptimal decoding algorithms are used to decode the I-Q ST codes. The proposed codes were simulated over independent and correlated Rayleigh fading channels. Coding gains up to 3 dBs have been observed.

1. Introduction

Diversity is a popular method to improve the performance and throughput of wireless systems. Transmit time diversity can be achieved by repeating the transmission of each symbol in different time slots [1]. It can be viewed as a repetition code and consumes higher bandwidth [2]. Therefore, substantial performance improvement can be achieved using more sophisticated codes, utilizing both space and time. The concept of ST codes had appeared first in [3] as the delay diversity system, where different symbols are simultaneously transmitted via different transmit antennas. Later, ST codes were deigned explicitly for quasi-static fading channels [4,5]. Moreover, the performance criteria of ST codes were derived for quasi-static and rapid fading channels in [4,5]. In [6-8], the ST concept was applied to enhance the quality of transmission at the same bit rate of systems using single transmit antenna. So, the same error probability can be achieved at a lower signal-to-noise ratio (SNR). ST coded QPSK schemes are presented in [9,10] for rapid fading channels.

In this paper, 16-QAM ST codes are proposed using two encoding schemes. The first scheme uses conventional trellis encoders, where the second one uses the I-Q encoding technique. The general ST system model is described in the next section. Then, the proposed codes are presented. After that, the optimal and suboptimal decoding algorithms used to decode the I-Q 16-QAM ST codes are discussed. The performance of the new codes is compared to that of the 16-QAM ST code presented in [4] for the cases of rapid and correlated fading channels. Finally, conclusions are drawn from the obtained results.

2. System Model:

A typical system that employs ST coding uses *N* transmit and *M* receive antennas. In this work, *N* has been set to 2. The received signal is a noisy superposition of all transmitted symbols over all transmit antennas. The signal d_t^j received at the j^{th} antenna at time *t* is given by:

$$d_t^j = \sum_{i=1}^{N} \alpha_{ij,t} c_t^i + \eta_t^j \tag{1}$$

Where η_t^j is an AWGN modeled as independent samples of a zero-mean Gaussian random process with variance $N_o/2$ per dimension. The coefficient $\alpha_{ij,t}$ is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna at time instant *t*. The c^i_t is the transmitted symbol from the i^{th} transmit antenna.

The performance of ST codes having *N* transmit and *M* receive antennas is derived in [4] for rapid fading channels. Consider a codeword $\boldsymbol{C_l} = c^1{}_1\, c^2{}_1 \ldots c^N{}_1\, c^1{}_2 \ldots c^N{}_2 \ldots\ldots c^1{}_l \ldots\ldots c^N{}_l$, that has been transmitted over *l* time intervals and was erroneously decoded as $\boldsymbol{\hat{C}_l}$. The conditional probability of deciding $\boldsymbol{\hat{C}_l}$ in favor of $\boldsymbol{C_l}$ using maximum liklihood decoding is upper bounded as [4]:

$$P(\boldsymbol{C_l},\boldsymbol{\hat{C}_l} \mid \alpha_{ij,t},\ i=1,\ldots,N,\ j=1,\ldots,M, t=1,\ldots,l) \le \exp\left[-d^2_E(\boldsymbol{C_l},\boldsymbol{\hat{C}_l})/4N_o\right],$$

where

$$d^2_E(\boldsymbol{C_l},\boldsymbol{\hat{C}_l}) = \sum_{t=1}^{l}\sum_{j=1}^{M} E_s \left| \sum_{i=1}^{N} \alpha_{ij,t}\,(c^i_t - \hat{c}^i_t) \right|^2, \tag{2}$$

and E_s is the average signal energy at each transmit antenna. The pairwise error probability is found to be:

$$P(\boldsymbol{C_l},\boldsymbol{\hat{C}_l}) \le \prod_{t=1}^{L} \left[1 + |\boldsymbol{c_t} - \boldsymbol{\hat{c}_t}|^2 (E_s/4N_o)\right]^{-M} \tag{3}$$

The parameter *L*, which is the length of the shortest error event with *L* time intervals, can be referred to as the Space-Time Minimum Time Diversity (ST-MTD) of the code. It can be visualized as the "branch-wise" Hamming distance (HD) or the MTD in conventional trellis codes, by considering the whole codeword $\boldsymbol{c_t}$ as one symbol. The quantity multiplied by the SNR term can be referred to as the Space-Time Minimum Square Product Distance (ST-MSPD) and defined as:

$$\prod_{t\in\eta} |\boldsymbol{c_t} - \boldsymbol{\hat{c}_t}|^2 = \prod_{t\in\eta}\sum_{i=1}^{N} |c^i{}_t - \hat{c}^i{}_t|^2 \le \prod_{t=1}^{L}\sum_{i=1}^{N} |c^i{}_t - \hat{c}^i{}_t|^2 \tag{4}$$

The ST-MTD and ST-MSPD are referred to in [4] as Distance and Product criteria, respectively. So, maximizing both of them yields good ST codes suitable for rapid fading channels. The proposed ST codes are presented in the following.

3. The Proposed Codes:

Different ST codes were designed in [4] for the quasi-static fading channel. The ST coded QAM scheme, referred to QAM1 here, uses a rate-4/8 trellis encoder to encode the incoming 4 bits to 8 output bits. The

8 bits at the output of the encoder are mapped onto two 16-QAM signals and transmitted over two antennas. The ST-MTD of this code is 2 and its ST-MSPD is 0.16.

The first proposed scheme, called QAM2, uses a rate-4/8 trellis encoder. However, it is designed so that both the ST-MTD and the ST-MSPD of the code are maximized. To be able to do this, the allowed pair of 16-QAM signals to appear at branches departing or emerging into the same state should be different in both symbols. This can be done by applying the permutation method used for the 4-dimensional MPSK signal space in [11] with slight modifications.

At the beginning, all the possible 16-QAM symbols are listed in order, starting by s_0 and ending with s_{15} in a 16x1 vector. Then a 16x2 vector is formed by listing all the pairs of same first and second symbols and denoted by A_0^0. The vector, that has the second column of A_0^0 shifted by i rows is denoted by A_i^0. Similarly, when the vector A_i^0 is shifted by j rows, it is denoted by A_i^j. Figure 1-a shows the vectors A_0^0, A_1^0 and A_0^1 as examples of the permuted vectors. The labels of branches leaving each state are taken as the rows of the vectors having the maximum HD from each other. The trellis diagram of the 16-state QAM2 code is shown in Figure 1-b.

Since the MTD of a trellis code is inversely proportional to the number of input bits of the encoder, then using different encoders in parallel, such as I-Q encoding, can increase the ST-MTD. I-Q trellis codes with different throughputs were presented in [12]. These codes show significant coding gains over conventional trellis codes having the same complexity. The proposed structure of the encoder/decoder employing the ST concept is shown in Figure 2. It uses two rate-2/4 encoders, where each one encodes two bits per signaling interval. Each encoder outputs two 4-AM signals: the first symbols from both encoders are mapped onto a 16-QAM signal to be transmitted over the first antenna. The second symbols are mapped onto the second 16-QAM symbol and transmitted over the second antenna.

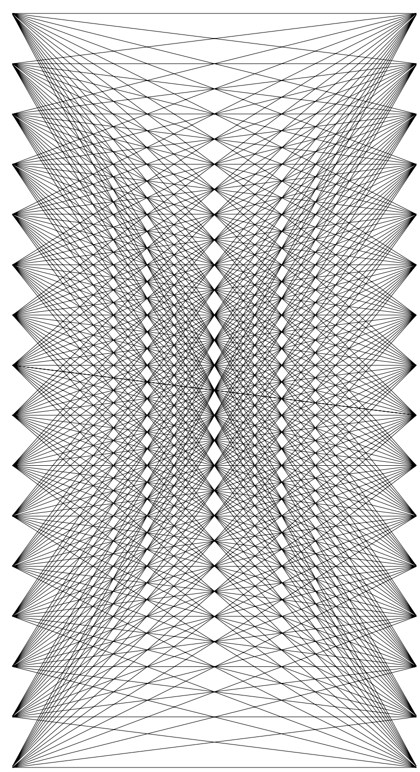
In order to design codes with the highest possible MTD and MSPD, the 2-dimensional 4-AM signal space is partitioned. The partitioning process is done so that the HD and the squared product distance (SPD) in the generated subsets are higher each time the partitioning is performed. The set partitioning of the 2-dimensional 4-AM signal space is shown in Figure 3. The trellis diagrams of the 4 and 32-state codes are shown in Figure 4. It can be observed that the labels of branches departing or emerging at the same state differ in both symbols. This maximizes the ST-MTD of the 4, and 32-state codes to 2 and 3, respectively. The ST-MSPD is 16.6 for the 4-state code and 106 for the 32-state code. The large difference in both the ST-MTD and the ST-MSPD, compared to those of the previous two codes, and resulting from using the I-Q encoding scheme is clearly observed.

The complexity of a trellis code is equal to the total number of branches leaving all states divided by the associated information bits with each transition [12]. The complexity of the 16-state QAM1 and QAM2 codes is 64. The I-Q code that has a similar complexity is the 32-state code. So, for fair comparison, the

$$A_0{}^0 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ 4 & 4 \\ 5 & 5 \\ 6 & 6 \\ 7 & 7 \\ 8 & 8 \\ 9 & 9 \\ 10 & 10 \\ 11 & 11 \\ 12 & 12 \\ 13 & 13 \\ 14 & 14 \\ 15 & 15 \end{bmatrix} \qquad A_1{}^0 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 9 \\ 9 & 10 \\ 10 & 11 \\ 11 & 12 \\ 12 & 13 \\ 13 & 14 \\ 14 & 15 \\ 15 & 0 \end{bmatrix} \qquad A_0{}^1 = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ 4 & 4 \\ 5 & 5 \\ 6 & 6 \\ 7 & 7 \\ 8 & 8 \\ 9 & 9 \\ 10 & 10 \\ 11 & 11 \\ 12 & 12 \\ 13 & 13 \\ 14 & 14 \\ 15 & 15 \\ 0 & 0 \end{bmatrix}$$

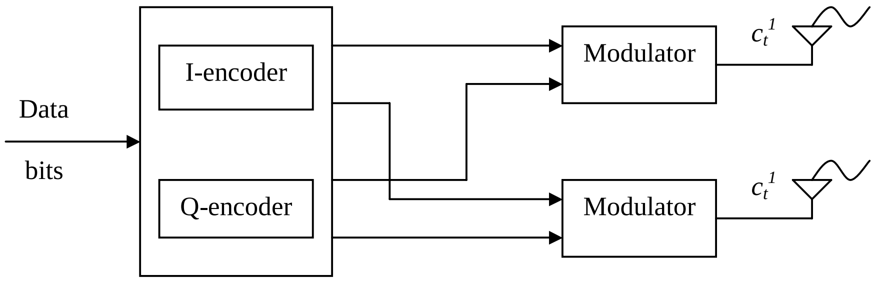
(a)

$A_0{}^0$	00, 11, 22, 33, 44, 55, 66, 77, 88, 99, 10 10, 11 11, 12 12, 13 13, 14 14, 15 15
$A_8{}^1$	15 7, 08, 19, 2 10, 3 11, 4 12, 5 13, 6 14, 7 15, 80, 91, 10 2, 11 3, 12 4, 13 5, 14 6
$A_4{}^2$	14 2, 15 3, 04, 15, 26, 37, 48, 59, 6 10, 7 11, 8 12, 9 13, 10 14, 11 15
$A_{12}{}^3$	13 9, 14 10, 15 11, 0 12, 1 13, 2 14, 3 15, 40, 51, 62, 73, 84, 95, 10 6, 11 7, 12 8
$A_1{}^4$	12 13, 13 14, 14 15, 15 0, 01, 12, 23, 34, 45, 56, 78, 89, 9 10, 10 11, 11 12
$A_9{}^5$	11 4, 12 5, 13 6, 14 7, 15 8, 09, 1 10, 2 11, 3 12, 4 13, 5 14, 6 15, 70, 81, 92, 10 3
$A_5{}^6$	10 15, 11 0, 12 1, 13 2, 14 3, 15 4, 05, 16, 27, 38, 49, 5 10, 6 11, 7 12, 8 13, 9 14
$A_{13}{}^7$	96, 10 7, 11 8, 12 9, 13 10, 14 11, 15 12, 0 13, 1 14, 2 15, 30, 41, 52, 63, 74, 85
$A_2{}^8$	8 10, 9 11, 10 12, 11 13, 12 14, 13 15, 14 0, 15 1, 02, 13, 24, 35, 46, 57, 68, 79
$A_{10}{}^9$	71, 82, 93, 10 4, 11 5, 12 6, 13 7, 14 8, 15 9, 0 10, 1 11, 2 12, 3 13, 4 14, 5 15, 60
$A_6{}^{10}$	6 12, 7 13, 8 14, 9 15, 10 0, 11 1, 12 2, 13 3, 14 4, 15 5, 06, 17, 28, 39, 4 10, 5 11
$A_{14}{}^{11}$	53, 64, 75, 86, 97, 10 8, 11 9, 12 10, 13 11, 14 12, 15 13, 0 14, 1 15, 20, 31, 42
$A_4{}^{12}$	48, 59, 6 10, 7 11, 8 12, 9 13, 10 14, 11 15, 12 0, 13 1, 14 2, 15 3, 04, 15, 26, 37
$A_{11}{}^{13}$	3 14, 4 15, 50, 61, 72, 83, 94, 10 5, 11 6, 12 7, 13 8, 14 9, 15 10, 0 11, 1 12, 2 13
$A_7{}^{14}$	29, 3 10, 4 11, 5 12, 6 13, 7 14, 8 15, 90, 10 1, 11 2, 12 3, 13 4, 14 5, 15 6, 07, 18
$A_5{}^{15}$	15, 26, 37, 48, 59, 6 10, 7 11, 8 12, 9 13, 10 14, 11 15, 12 0, 13 1, 14 2, 15 3, 04

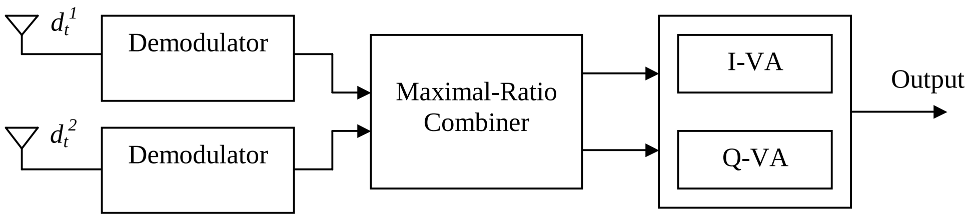


(b)

Figure 1: (a) Examples of vectors used in the permutation method of the 4-dimensional 16-QAM signal space, (b) Trellis diagram of the QAM2 ST code, 4 bits/s/Hz.



(a)



(b)

Figure 2: The structure of the I-Q ST system (a) Encoder, (b) Decoder.

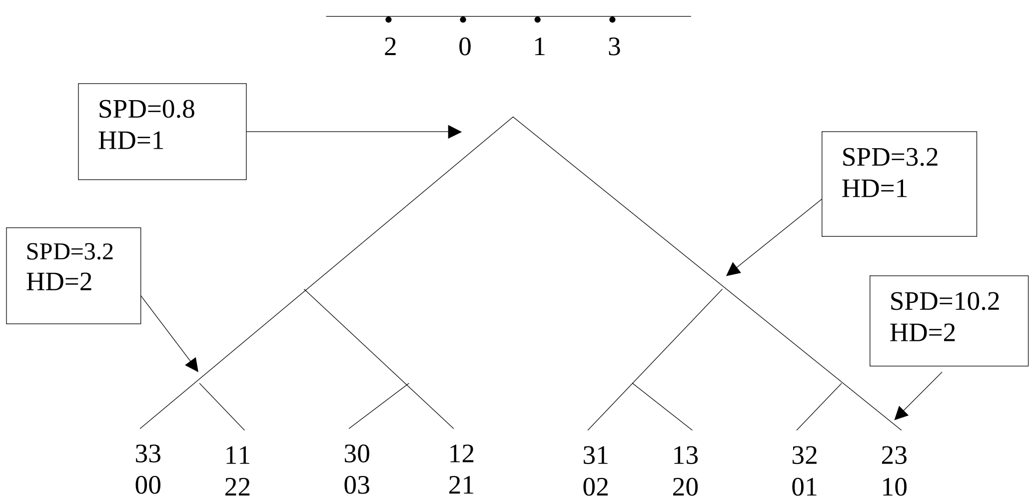


Figure 3: 2-dimensional 4-AM set partitioning.

16-state QAM1 and QAM2 codes are compared to the 32-state I-Q code. The decoding algorithms used to decode the I-Q ST codes are presented in the following.

4. Suboptimal Decoding Algorithm

Two decoding algorithms are used to decode the I-Q 16-QAM ST codes: the optimal and a sub-optimal decoding algorithms. The suboptimal decoding algorithm was proposed in [9,13] to decode the I-Q ST coded QPSK systems and is based on estimating the Q/I components in the I/Q decoders.

This is performed by partitioning the 2-dimensional signal space available at the output of the ST encoder. The signal space to be partitioned is a 4-dimensional 16-QAM space that consists of 16^2 possible signal pairs. The partitioning is performed so that all pairs in one subset have the same in-phase components. In other words, they are caused by the same 4-AM symbol pair at the output of the I-encoder. Hence, for each 4-AM symbol pair, there are 16 possible 16-QAM signal pairs that could be transmitted from both transmit antennas.

The notation S^{lk} denotes the possible 16-QAM pairs that can appear at the output of the I-Q ST encoder given that the 4-AM symbols at the output of the I-encoder are l and k. In addition, the signal s_j^i is the 16-QAM signal whose label in the constellation is j, and is transmitted over the i^{th} transmit antenna. The set partitioning yields 16 subsets and one set is presented for illustration:

$$\begin{aligned} S^{00} = \{ & (s_0^1,s_0^2),(s_0^1,s_1^2),(s_0^1,s_2^2),(s_0^1,s_3^2),(s_1^1,s_0^2),(s_1^1,s_1^2),(s_1^1,s_2^2),(s_1^1,s_3^2), \\ & (s_2^1,s_0^2),(s_2^1,s_1^2),(s_2^1,s_2^2),(s_2^1,s_3^2),(s_3^1,s_0^2),(s_3^1,s_1^2),(s_3^1,s_2^2),(s_3^1,s_3^2) \} \end{aligned}$$

Now, the following 16 metrics are computed at the I-decoder before the trellis:

$$M_{lk} = \min_{(s^1,s^2)\in S^{lk}} \sum_{j=1}^{M} \left(r_I^j - \sum_{i=1}^{N} \alpha_{Iij} x^i + \sum_{i=1}^{N} \alpha_{Qij} y^i \right)^2 + \left(r_Q^j - \sum_{i=1}^{N} \alpha_{Qij} x^i + \sum_{i=1}^{N} \alpha_{Iij} y^i \right)^2 \qquad (5)$$

Where x^i and y^i are the in-phase and quadrature components of the 16-QAM signal s_j^i. In each M_{lk}, 16 metrics are compared and the minimum is found accordingly, ending with 16 different metrics. Each of them is associated with one of the 4-AM signal pairs that may be at the I-encoder's output. Since each encoder has two input bits, there are four possible metrics to be compared at each state in the trellis of the I-decoder. The same principle is applied to the Q-decoder case.

The results of the I-Q ST codes show that the suboptimal decoding algorithm used does not perform well because it is trying to guess for the best Q/I components from the received signals in the I/Q decoder. For a decision in the I-decoder, there are 16 different combinations of the Q component to be compared, which is a large number. This algorithm is called suboptimal because the I-decoder uses sequence decoding for the I components and symbol-by-symbol decoding for the Q components. In order to get the maximum performance of the I-Q ST codes, an optimal but complex decoding algorithm is used and discussed in the following section.

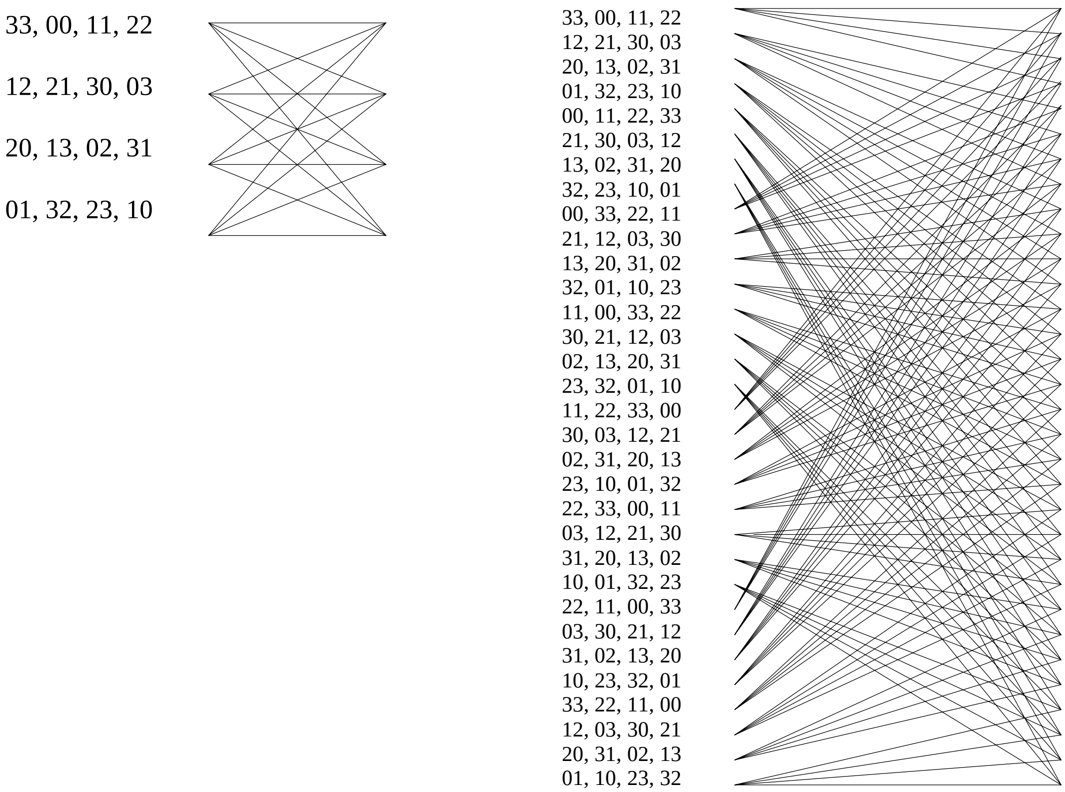


Figure 4: Trellis diagrams of I-Q 16-QAM ST code (4 and 32-state), 4 bit/s/Hz.

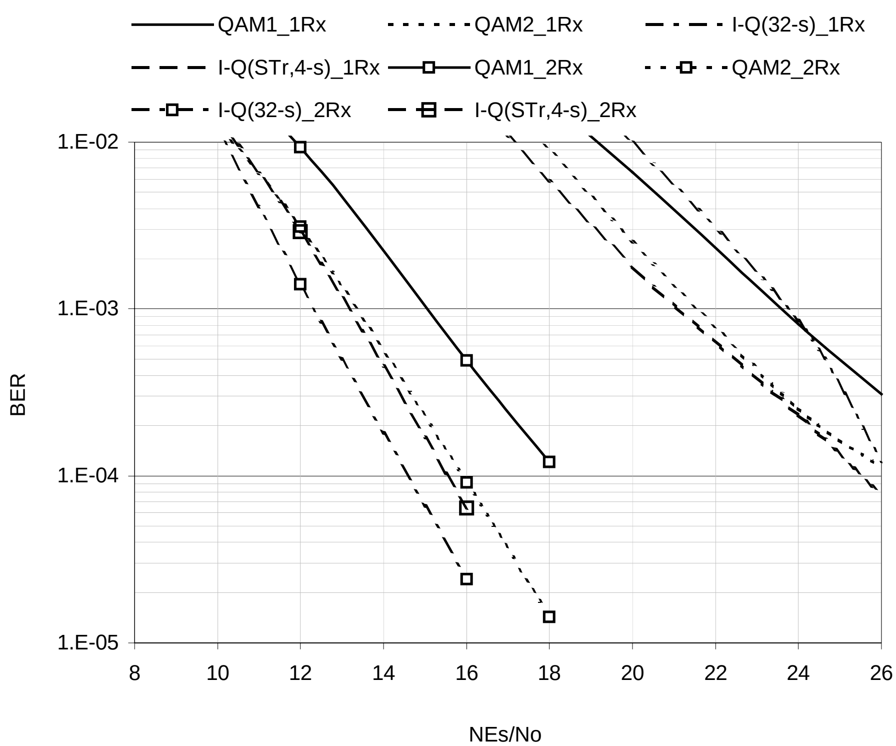


Figure 5: Performance of the 16-QAM codes for 1-Rx and 2-Rx antenna over rapid fading channel. STr: Super Trellis.

5. Optimal Decoding Algorithm

To mitigate for suboptimality of the suboptimal algorithm, the use of the super-trellis decoding is proposed to decode the I-Q ST codes. In this algorithm, the I and Q components are decoded using one trellis whose size is the square of that of the individual I and Q trellises.

The super-trellis is used to decode I-Q ST codes optimally by decoding (not guessing) the I and Q components simultaneously using sequence decoding. The optimal decoding algorithm is used to decode the 4-state 16-QAM I-Q ST code only. The resultant super-trellis decoding complexity of this code is 64, which is the same as the complexities of the QAM1, QAM2 codes.

6. Results

The above ST codes are simulated over independent fading channels. Figure 5 shows the performance of the 16-state QAM1, QAM2 and I-Q codes for the cases of one and two receive antennas. The 4-state I-Q code with super-trellis decoding is the best followed by QAM2, QAM1 codes and the 32-state I-Q code with suboptimal decoding. This is expected for the 4-state I-Q code since the main controlling parameter of the code in rapid fading channels is the ST-MTD of the code that is highest in the I-Q code.

It can be seen that the 32-state I-Q code with suboptimal decoding does not perform well in the single receive antenna case. Also, degradation decreases as the SNR in increased, because the guessing process is done in a less noisy environment. This degradation becomes less in the case of two receive antennas, where the 32-state I-Q with suboptimal decoding is the best. In the case of one receive antenna, the gains of the I-Q code, decoded by the super-trellis method, over the QAM1 and QAM2 codes are 2.5 and 0.5 dBs, respectively. In the two receive antennas case, the above gains become around 2 and 0.3 dB, respectively. On the other hand, gains of the 32-state I-Q code, using suboptimal decoding algorithm, over QAM1, QAM2 and the 4-state code with super-trellis decoding are 2.5, 1 and 0.5, respectively.

The same codes are tested over a correlated fading channel with a fade rate ($f_D T$) of 0.01, and the results are shown in Figure 6. A 25x16 block interleaver is used to break the memory of the channel. The same trends observed in the rapid fading channel case are observed in this case. The gains of the best code over worse codes are less in the one receive antenna case. In the two receive antennas case, they do not change much because the presence of two receive antennas makes it less dependent on the codes' parameters.

Figure 7 shows the performance of the codes for a slower channel, with a fade rate $f_D T$ of 0.005. The above interleaver is used which is improper for this channel. The performance trends for the codes are the same as the previous two channels with decreased gains.

7. Conclusions and Discussion

Two new ST codes based on the 16-QAM signal constellation for rapid fading channels are proposed. The results showed that the new codes are better in terms of the design criteria. Also, they were tested

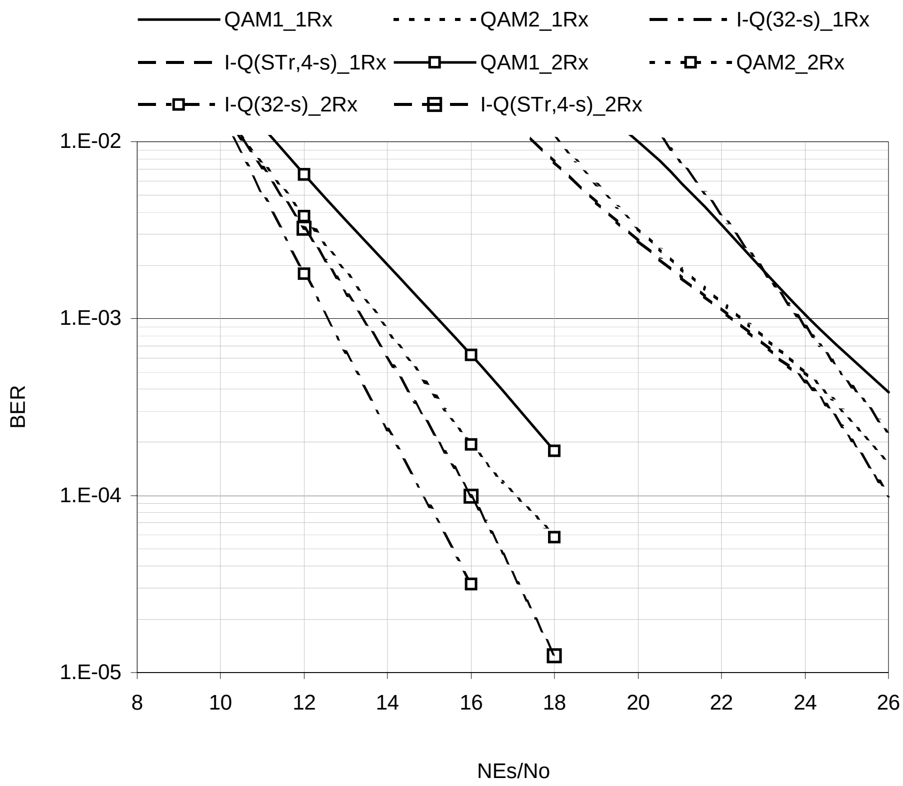


Figure 6: Performance of the codes for 1-Rx and 2-Rx antenna over correlated fading channel with $f_D T$=0.01.

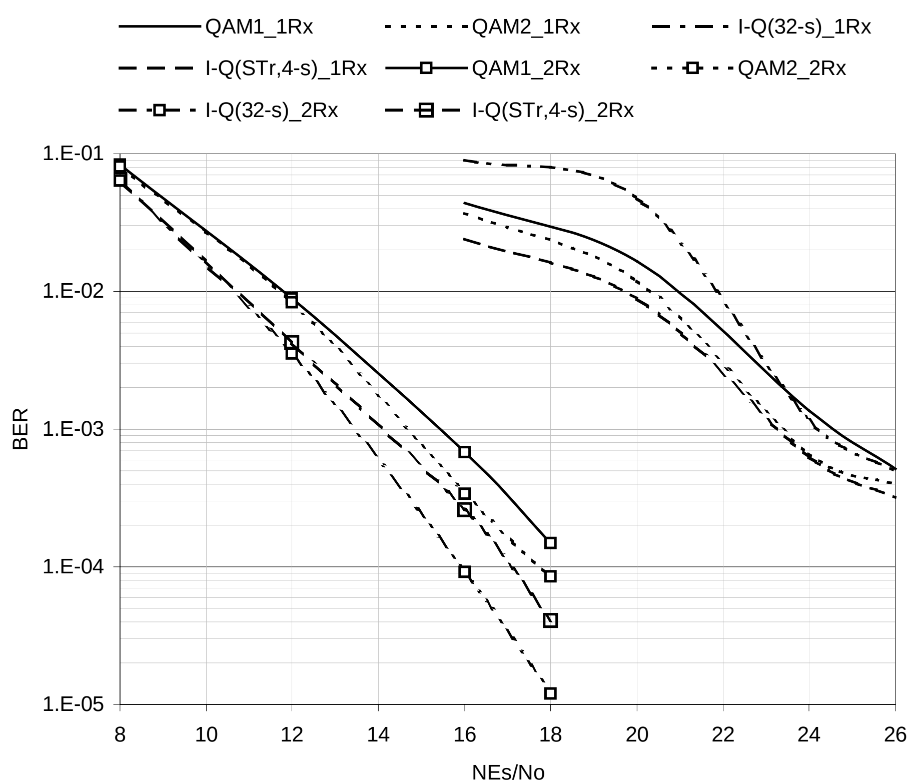


Figure 7: Performance of the codes for 1-Rx and 2-Rx antenna over correlated fading channel with $f_D T$=0.005.

over different fading rates and they showed to be robust in such environments. The optimal and suboptimal decoding algorithms were used to decode I-Q ST codes. Results indicated that the suboptimal algorithm is less complex than the optimal one at the expense of degradation in the performance, especially for the case of one receive antenna. More gains are expected from the I-Q ST codes if better and simple decoding algorithms are used.

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