Relation between C/N Ratio and S/N Ratio

In our discussion in the past few lectures, we have computed the C/N ratio of the received signals at different points of the satellite transmission system. The C/N generally is related to the quality of the received signal but it does not give an exact measure of how good the quality of the received signal has been preserved. What determines this quality exactly is the Signal to Noise Ratio (S/N). The S/N ratio and C/N ratio are related to each other by a relationship that is determined by the type of modulation that is used. While the carrier power (C) represents the total amount of power that is transmitted, which includes the power carrying the information in addition to possibly other transmitted power that may carry no information at all, the signal power (S) represents only the power that carries the information. In the following discussion, we learn the relation between C/N and S/N for different types of modulations:

S/N vs. C/N for DSBSC Modulation

Double Side Band Suppressed Carrier (DSBSC) is characterized by transmitting a frequency shifted version of the information signal obtained by multiplying the information signal by a carrier signal. The transmitted signal does not contain any unmodulated component of the carrier, ion signal multiplied by a carrier and no component of the unmodulated carrier, hence the naming of "suppressed carrier". Because the transmitted signal is purely the information signal shifted in frequency to some carrier frequency, the carrier power (C) is purely equal to the signal power (S). So, the C/N and S/N ratios are equal for that type of analog modulation:

$$\left(\frac{S}{N}\right)_{DSBSC} = \left(\frac{C}{N}\right)$$

S/N vs. C/N for SSB Modulation

Similar to DSBSC, Single Side Band (Suppressed Carrier) (SSB) modulation transmits one sideband of the DSBSC signal. Similar to the DSBSC modulation, the transmitted signal in SSB modulation does not contain any unmodulated component of the carrier (except a very small pilot carrier that has power much less than the information power and is used for extracting the phase of the carrier). Because of the lack of an unmodulated carrier, the C/N and S/N ratios are equal for that type of analog modulation too:

$$\left(\frac{S}{N}\right)_{SSB} = \left(\frac{C}{N}\right)$$

S/N vs. C/N for Full AM (DSB with Carrier) Modulation

The case for the full AM modulation is different from the DSBSC or SSB modulations. The reason is that a significantly large unmodulated carrier is transmitted in full AM modulation. In fact, the power of the unmodulated carrier in the full AM is at least 2/3 the total power of the transmitted signal (i.e., the information power is at most 1/3 of the transmitted signal power). So,

$$\left(\frac{S}{N}\right)_{Full\,AM} \le \frac{1}{3}\left(\frac{C}{N}\right)$$

S/N vs. C/N for FM Modulation

Background of FM Modulation

Let us have a very quick review of Fm modulation first. An FM signal has the form

$$g_{FM}(t) = A \cdot \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

where A is the amplitude of the FM signal, f_c is the carrier frequency, k_f is called the frequency sensitivity and has the units of Hz/Volt, and $m(t)$ the information signal in units of Volts. The frequency sensitivity determines how many Hz the frequency of the FM signal changes as a result of increasing the input signal by 1 V.

The instantaneous frequency $f_i(t)$ of the FM signal changes in the range

$$f_c - k_f m_p \le f_i(t) \le f_c + k_f m_p \,.$$

where m_p is amplitude (or maximum value of the information signal).

This means that the instantaneous frequency changes over a range of $\Delta f = k_f \cdot m_p$ on each side around the carrier frequency f_c.

The approximate bandwidth of the of original FM signal $g_{FM}(t)$ we use the **<u>Carson's rule</u>**, which states that the bandwidth of the FM signal is given by (we also use the fact that $\Delta f = k_f \cdot m_p$)

$$\begin{aligned} BW_{FM} &= 2k_f \cdot m_p + 2B_m \quad Hz \\ &= 2 \cdot \Delta f + 2B_m \quad Hz \\ &= 2 \cdot \left(\Delta f + B_m \right) \quad Hz \end{aligned} \,.$$

where B_m is bandwidth of message signal $m(t)$ in Hz. If we define the quantity β such that:

$$\beta = \frac{\Delta f}{B_m} = \frac{k_f m_p}{B_m}$$

which is known as the modulation index of the FM signal, the bandwidth becomes

$$\begin{aligned} BW_{FM} &= 2 \cdot B_m \left(\frac{\Delta f}{B_m} + 1 \right) \quad Hz \\ &= 2 \cdot B_m \left(\beta + 1 \right) \quad Hz \end{aligned}$$

<u>Relation between S/N and C/N Ratios for FM Modulation</u>

One important feature of FM is that it trades performance (or quality of received signal in terms of S/N ratio) with bandwidth used for transmission. That is, the higher the bandwidth of the transmitted signal relative to the input signal, the higher the S/N ratio of the received signal. The relation between the S/N ratio and C/N ratio is given by

$$\left(\frac{S}{N}\right)_{FM} = \left(\frac{C}{N}\right)\cdot\frac{3}{2}\cdot\frac{BW_{FM}}{B_{\mathrm{m}}}\cdot\left(\frac{\Delta f}{B_{\mathrm{m}}}\right)^2$$

or in dB form as

$$\left(\frac{S}{N}\right)_{FM} \mathrm{dB} = \left(\frac{C}{N}\right) \mathrm{dB} + 1.8\ \mathrm{dB} + 10\log_{10}\left(\frac{BW_{FM}}{B_{\mathrm{m}}}\right) + 20\log_{10}\left(\frac{\Delta f}{B_{\mathrm{m}}}\right)$$

where

BW_{FM} is the bandwidth of the FM signal obtained using Carson's rule

Δf is the peak frequency deviation, which is equal to $\Delta f = k_f \cdot m_p$

B_{m} is the bandwidth of the information signal.

Therefore, the S/N ratio for FM can be written as

$$\left(\frac{S}{N}\right)_{FM} = \left(\frac{C}{N}\right)\cdot 3\cdot(\beta+1)\cdot(\beta)^2$$

where β is the modulation index of the FM signal. It is clear from the last relation that for large values of β , the S/N ratio becomes equal to 3 times the C/N ratio multiplied by the modulation index cubed, which indicates that the improvement of the S/N ratio over the C/N ratio becomes huge. To show the performance improvement of FM, we can write the dB relationship above as

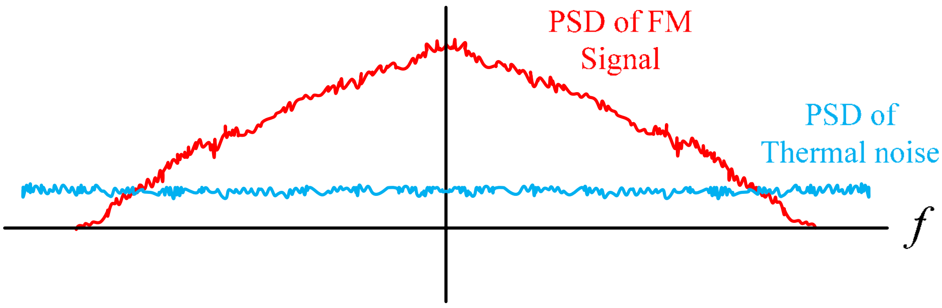
$$\left(\frac{S}{N}\right)_{FM} \mathrm{dB} = \left(\frac{C}{N}\right) \mathrm{dB} + (\text{FM Improvment})\ \mathrm{dB}$$

It is this FM improvement that makes the use of FM attractive especially in applications where the C/N ratio of the received signal is relatively low and a reasonably higher S/N ratio is needed to demodulate the received signal properly. Often, the needed improvement may be around 20 dB to 30 dB.

An important point that is worth mentioning is that increasing the bandwidth of the FM signal improves the S/N ratio over the C/N ratio but also reduces the C/N ratio of the received signal because a larger bandwidth of the transmitted signal (the FM signal) results in a larger noise power since the noise power is proportional to the transmitted signal bandwidth and hence the added noise bandwidth.

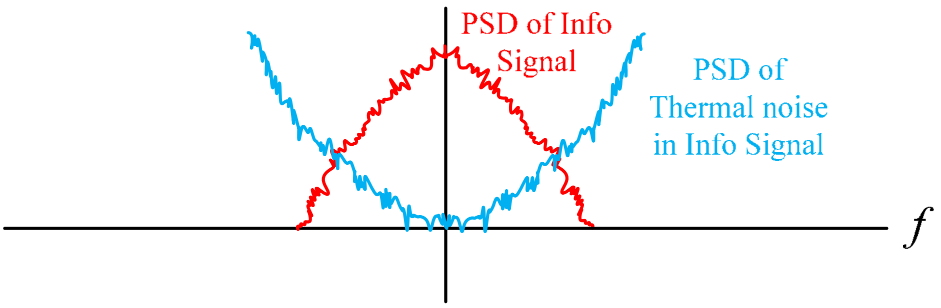
Pre-Emphasis and De-Emphasis

One important feature of FM modulation that is not seen with other types of AM modulation is related to the noise that gets added the message signal as it gets transmitted. Although noise power that gets added to the transmitted FM signal is almost flat in spectrum (i.e., different frequencies of the transmitted FM signal get equal amounts of noise power), the noise power that effectively gets added to the demodulated message signal is not flat. This noise is low at low frequencies and increases as the frequency increases. This is illustrated in the following figure.



In the above figure, the transmitted FM signal is accompanied by thermal noise at the receiver. Thermal noise does not distinguish between different type of signal as they are transmitted, and therefore, the power spectrum of thermal noise that gets added to the FM signal is almost flat.

When demodulating the FM signal to get the original information signal, it is found that (because of the nature of the FM modulation), the power spectrum density of thermal noise in the demodulated signal is not flat but appears to be as shown below:



The noise in the demodulated signal is found to be low at low frequencies and it increases as the frequency increases. Since low frequencies of the infoamtion signal are currepted by small amounts of noise while high frequencies are corrupted by large amounts of noise, the S/N ratio of the demodulated signal can be improved by amplifying the frequency components of the information signal that will experience high amounts of noise power and reduce the power of the frequency components of the information signal that will experience low amounts of power. This process is known as PRE-EMPHASIS. Clearly, this process introduces some controlled distortion to the information signal that would have to be reversed at the receiver side. At the receiver, the signal components that were amplified are attenuated and the signal components that were attenuated are amplified. This process is called DE-EMPHASIS. The process of pre-emphasis at the transmitter and de-emphasis at the receiver can improve the S/N ratio of the received signal by an amount of 5 to 10 dB without the need to transmit higher power or do any other modifications to the system. Therefore,

$$\left(\frac{S}{N}\right)_{FM} \text{ dB} = \left(\frac{C}{N}\right) \text{ dB} + \left(\text{FM Improvment}\right) \text{ dB} + \left(\text{Pre- \& De-Emphasis Improvment}\right) \text{ dB}$$