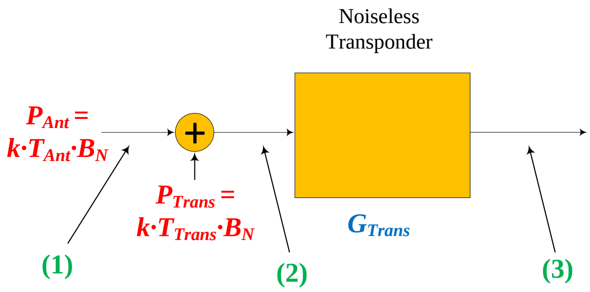
Carrier to Noise Ratio (C/N or CNR)

As mentioned before, what matters in determining the performance of a satellite communication system is not the signal (or carrier) power but the carrier power to the noise power ratio (C/N) of the received signal, because this ratio is what determines the quality of the transmitted information and whether it can be retrieved properly or not. There is a difference between the term Carrier to Noise Ratio (C/N) and the Signal to Noise Ratio (S/N). This will be discussed later. But now, let us determine the relation between the carrier to noise ratio of the different satellite links (uplink and downlink) and the carrier to noise ratio of the whole communication system.

C/N Before and After a Block

What happens to the carrier to noise ratio before and after a specific block? To get the answer, let us use consider a noisy transponder block shown below, where the noise has been transferred to its input. Let us compute the C/N ratio at different points in this system indicated by (1), (2), and (3) as indicated in the figure:



At (1): $\left(\frac{C}{N}\right)_{(1)} = \frac{C}{k \cdot T_{Ant} \cdot B_N}$

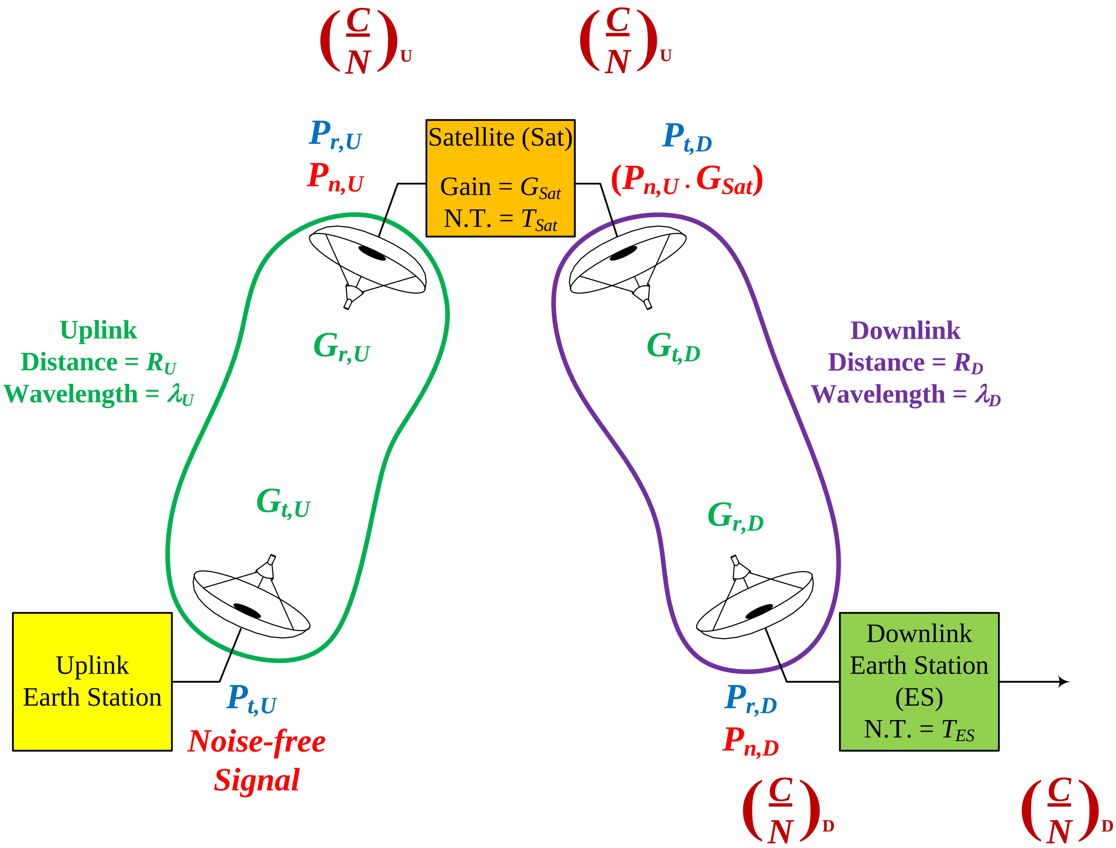
At (2): $\left(\frac{C}{N}\right)_{(2)} = \frac{C}{k \cdot (T_{Ant} + T_{Trans}) \cdot B_N}$

At (1): $\boxed{\left(\frac{C}{N}\right)_{(3)} = \frac{C \cdot G_{Trans}}{k \cdot (T_{Ant} + T_{Trans}) \cdot B_N \cdot G_{Trans}} = \frac{C}{k \cdot (T_{Ant} + T_{Trans}) \cdot B_N} = \left(\frac{C}{N}\right)_{(2)}}$

The conclusion, the C/N ratio before and after a noiseless device are the same. So, once the noise of a system is moved to its input, it is enough to compute the C/N ratio before the remaining noiseless device or after because they are basically the same.

C/N for a Complete Satellite System

Let us try to find the C/N ratio for the complete satellite system shown below that involves an uplink and a downlink. We will assume at first that the Uplink Earth station that transmits to the satellite is transmitting a noise-free signal towards the satellite. We will consider the complete system's C/N ratio and try to relate it to the C/N ratios of the uplink and downlink separately. This will allow us to study the uplink and downlink of a satellite separately and then combine them to get the overall C/N ratio. Note that since we are bringing the noise of the satellite and the Downlink Earth Station (ES) to their inputs, we see that the C/N ratio before and after each of these blocks is the same.



Uplink C/N Ratio

Considering first that the uplink Earth Station is transmitting noise-free signal, the received signal at the Satellite is

$$P_{r,U} = \frac{P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2}$$

The uplink noise at the input of the Satellite (resulting from the satellite receiving antenna and satellite transponder noise) is given as below. Note that the bandwidth of all carrier signals and noise signals are equal to B_n.

$$P_{n,U} = k \cdot T_{Sat} \cdot B_n$$

So, the Uplink C/N ratio is:

$$\left(\frac{C}{N}\right)_U = \frac{\frac{P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2}}{k \cdot T_{Sat} \cdot B_n} = \frac{P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2 k \cdot T_{Sat} \cdot B_n}$$

Downlink C/N Ratio

Considering now that the satellite transmits in the downlink a noise-free signal, the received signal at the downlink Earth station is

$$P_{r,D} = \frac{P_{t,D} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2}$$

The downlink noise at the input of the Earth Station (ES) (resulting from the ES receiving antenna and ES receiver blocks noise) is given as below. Again, the bandwidth of all carrier signals and noise signals are equal to B_n.

$$P_{n,D} = k \cdot T_{ES} \cdot B_n$$

So, the Uplink C/N ratio is:

$$\left(\frac{C}{N}\right)_U = \frac{\frac{P_{t,D} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2}}{k \cdot T_{ES} \cdot B_n} = \frac{P_{t,D} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2 k \cdot T_{ES} \cdot B_n}$$

Overall C/N Ratio

Now, let us try to evaluate the C/N ratio of the overall system and try to relate it to the C/N ratio of the uplink and C/N of the downlink. Let us again assume that transmitted signal by the uplink ES is noise-free. The received signal at the Satellite is

$$P_{r,U} = \frac{P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2}$$

This signal gets amplified by the satellite that has a gain of G_{Sat} such that the transmitted signal by the satellite becomes

$$P_{t,D} = G_{Sat} \cdot P_{r,U} = \frac{G_{Sat} \cdot P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2}$$

This results in the received signal at the downlink earth station (ES) becoming:

$$\begin{aligned} P_{r,D} &= \frac{P_{t,D} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2} \\ &= \frac{G_{Sat} \cdot P_{r,U} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2} \\ &= \frac{\dfrac{G_{Sat} \cdot P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2} \end{aligned}$$

For the noise, the signal transmitted from the uplink earth station is assumed to be noise free, so the noise at the satellite is noise that is generated by the satellite and is equal to what we found previously:

$$P_{n,U} = k \cdot T_{Sat} \cdot B_n$$

This noise passes through the satellite, and therefore gets amplified by the gain of the satellite to produce an amount equal to

$$G_{Sat} \cdot P_{n,U} = G_{Sat} \cdot k \cdot T_{Sat} \cdot B_n$$

This noise is transmitted to the downlink ES and experiences the same behavior that the information signal experiences. So, the noise power at the ES becomes two components: (1) the component that was generated by the satellite and got amplified and transmitted to the ES, and (2) the noise generated by the ES itself. So, the total noise at the receiver because of the two components becomes:

$$P_{n,D} = \frac{G_{Sat} \cdot P_{n,U} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2} + k \cdot T_{ES} \cdot B_n$$

$$= \underbrace{\frac{G_{Sat} \cdot k \cdot T_{Sat} \cdot B_n \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2}}_{\text{Component (1)}} + \underbrace{k \cdot T_{ES} \cdot B_n}_{\text{Component (2)}}$$

Given the above carrier and noise powers, the overall C/N ratio becomes:

$$\left(\frac{C}{N}\right)_{Overall} = \frac{\dfrac{\dfrac{G_{Sat} \cdot P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2} \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2}}{\dfrac{G_{Sat} \cdot k \cdot T_{Sat} \cdot B_n \cdot G_{t,D} \cdot G_{r,D}}{\left(\frac{4\pi R_D}{\lambda_D}\right)^2} + k \cdot T_{ES} \cdot B_n}$$

This can be written as follows:

$$\left(\frac{C}{N}\right)_{Overall} = \frac{\dfrac{G_{Sat} \cdot P_{t,U} \cdot G_{t,U} \cdot G_{r,U}}{\left(\frac{4\pi R_U}{\lambda_U}\right)^2} \cdot G_{t,D} \cdot G_{r,D}}{G_{Sat} \cdot k \cdot T_{Sat} \cdot B_n \cdot G_{t,D} \cdot G_{r,D} + \left(\frac{4\pi R_D}{\lambda_D}\right)^2 k \cdot T_{ES} \cdot B_n}$$

Multiplying both the numerator and denominator by the inverse of the numerator gives:

$$\left(\frac{C}{N}\right)_{Overall} = \frac{1}{\dfrac{\left(\frac{4\pi R_U}{\lambda_U}\right)^2 G_{Sat} \cdot k \cdot T_{Sat} \cdot B_n \cdot G_{t,D} \cdot G_{r,D} + \left(\frac{4\pi R_U}{\lambda_U}\right)^2 \left(\frac{4\pi R_D}{\lambda_D}\right)^2 k \cdot T_{ES} \cdot B_n}{G_{Sat} \cdot P_{t,U} \cdot G_{t,U} \cdot G_{r,U} \cdot G_{t,D} \cdot G_{r,D}}}$$

Dividing the denominator into two parts and cancelling some quantities produces:

$$\left(\frac{C}{N}\right)_{Overall}=\frac{1}{\dfrac{\left(\dfrac{4\pi R_U}{\lambda_U}\right)^2 \cancel{G_{Sat}}\cdot k\cdot T_{Sat}\cdot B_n\cdot \cancel{G_{t,D}}\cdot \cancel{G_{r,D}}}{\cancel{G_{Sat}}\cdot P_{t,U}\cdot G_{t,U}\cdot G_{r,U}\cdot \cancel{G_{t,D}}\cdot \cancel{G_{r,D}}}+\underbrace{\left[\dfrac{\left(\dfrac{4\pi R_U}{\lambda_U}\right)^2}{G_{Sat}\cdot P_{t,U}\cdot G_{t,U}\cdot G_{r,U}}\right]}_{\frac{1}{P_{t,D}}}\cdot\dfrac{\left(\dfrac{4\pi R_D}{\lambda_D}\right)^2 k\cdot T_{ES}\cdot B_n}{G_{t,D}\cdot G_{r,D}}}$$

$$=\frac{1}{\underbrace{\dfrac{\left(\dfrac{4\pi R_U}{\lambda_U}\right)^2 k\cdot T_{Sat}\cdot B_n}{P_{t,U}\cdot G_{t,U}\cdot G_{r,U}}}_{\frac{1}{\left(\frac{C}{N}\right)_U}}+\underbrace{\dfrac{\left(\dfrac{4\pi R_D}{\lambda_D}\right)^2 k\cdot T_{ES}\cdot B_n}{P_{t,D}\cdot G_{t,D}\cdot G_{r,D}}}_{\frac{1}{\left(\frac{C}{N}\right)_D}}}$$

This gives us

$$\boxed{\left(\frac{C}{N}\right)_{Overall}=\frac{1}{\dfrac{1}{\left(\frac{C}{N}\right)_U}+\dfrac{1}{\left(\frac{C}{N}\right)_D}}}$$

The above is an important conclusion that states that knowing the uplink C/N ratio and the downlink (C/N) ratio (as it is the case when the two links are designed separately) allows us to compute the overall C/N of the system from them. This relationship is similar to the equivalent resistance that is obtained when connecting two resistors in parallel. Recall that the equivalent resistance of two resistors connected in parallel is smaller than any of the two resistors. The same thing holds here. The overall C/N ratio is smaller than the either of the C/N ratios of the uplink or downlink. If any of these C/N ratios is much smaller than the other, then it is the one that dominates the overall C/N ratio resulting in an overall C/N ratio is slightly smaller than small C/N ratio.

The above relation can be extended for cases where multiple uplinks and multiple downlinks are used. A general formula for the case where a signal is transmitted over m uplinks and m downlinks would be:

$$\boxed{\left(\frac{C}{N}\right)_{Overall}=\frac{1}{\dfrac{1}{\left(\frac{C}{N}\right)_{U1}}+\dfrac{1}{\left(\frac{C}{N}\right)_{D1}}+\dfrac{1}{\left(\frac{C}{N}\right)_{U2}}+\dfrac{1}{\left(\frac{C}{N}\right)_{D2}}+\cdots+\dfrac{1}{\left(\frac{C}{N}\right)_{Um}}+\dfrac{1}{\left(\frac{C}{N}\right)_{Dm}}}}$$

In many cases, in addition to the noise that a link suffers from, the link would suffer from interference from other transmitting earth stations or satellites at the same frequency. For an uplink-downlink case where there is interference in the uplink and interference in the downlink, the above formula can be modified to be become $\left(\frac{C}{N+I}\right)_{Overall}$, which is given by:

$$\boxed{\left(\frac{C}{N+I}\right)_{Overall} = \frac{1}{\frac{1}{\left(\frac{C}{N}\right)_U} + \frac{1}{\left(\frac{C}{I}\right)_U} + \frac{1}{\left(\frac{C}{N}\right)_D} + \frac{1}{\left(\frac{C}{I}\right)_D}}}$$