EE 407
Microwave Engineering

Lecture 5 to 7

Impedance Transformers and Smith chart

Dr. Sheikh Sharif Iqbal

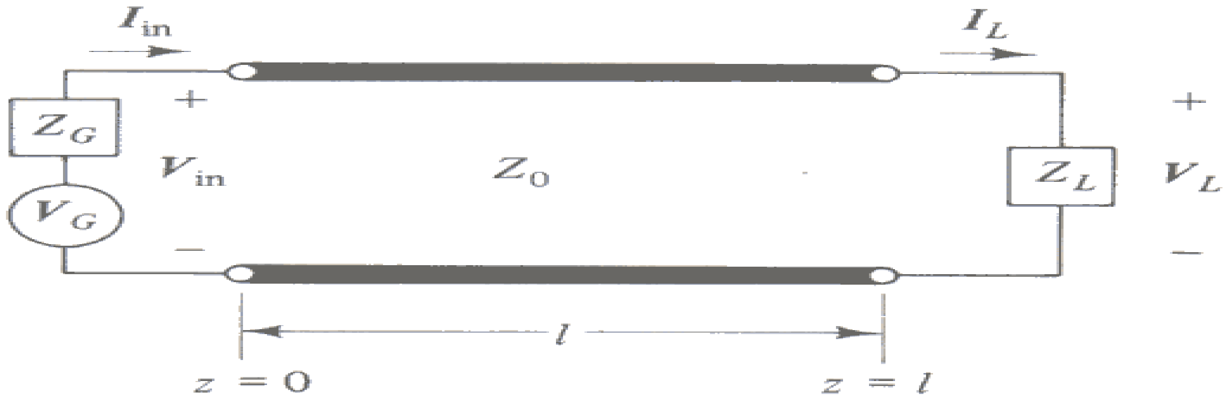
References: Text and reference books

- Elec. length, $\theta = \beta$**l**

  where,

  l = physical length

  <u>β = phase constant</u>



<u>Impedance Transformers</u>: (for α=0 and Z_0 & Z_L are real no's)

- For T.L. with <u>**l=λ/4**</u> has: $\beta l=(2\pi/\lambda)(\lambda/2)= \pi/2$ & $\tan \pi/2=\infty$

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tan\beta l}{Z_0 + Z_L \tan\beta l} = Z_0 \frac{\frac{Z_L}{\tan\beta l} + Z_0}{\frac{Z_0}{\tan\beta l} + Z_L} = \frac{Z_0^2}{Z_L}$$

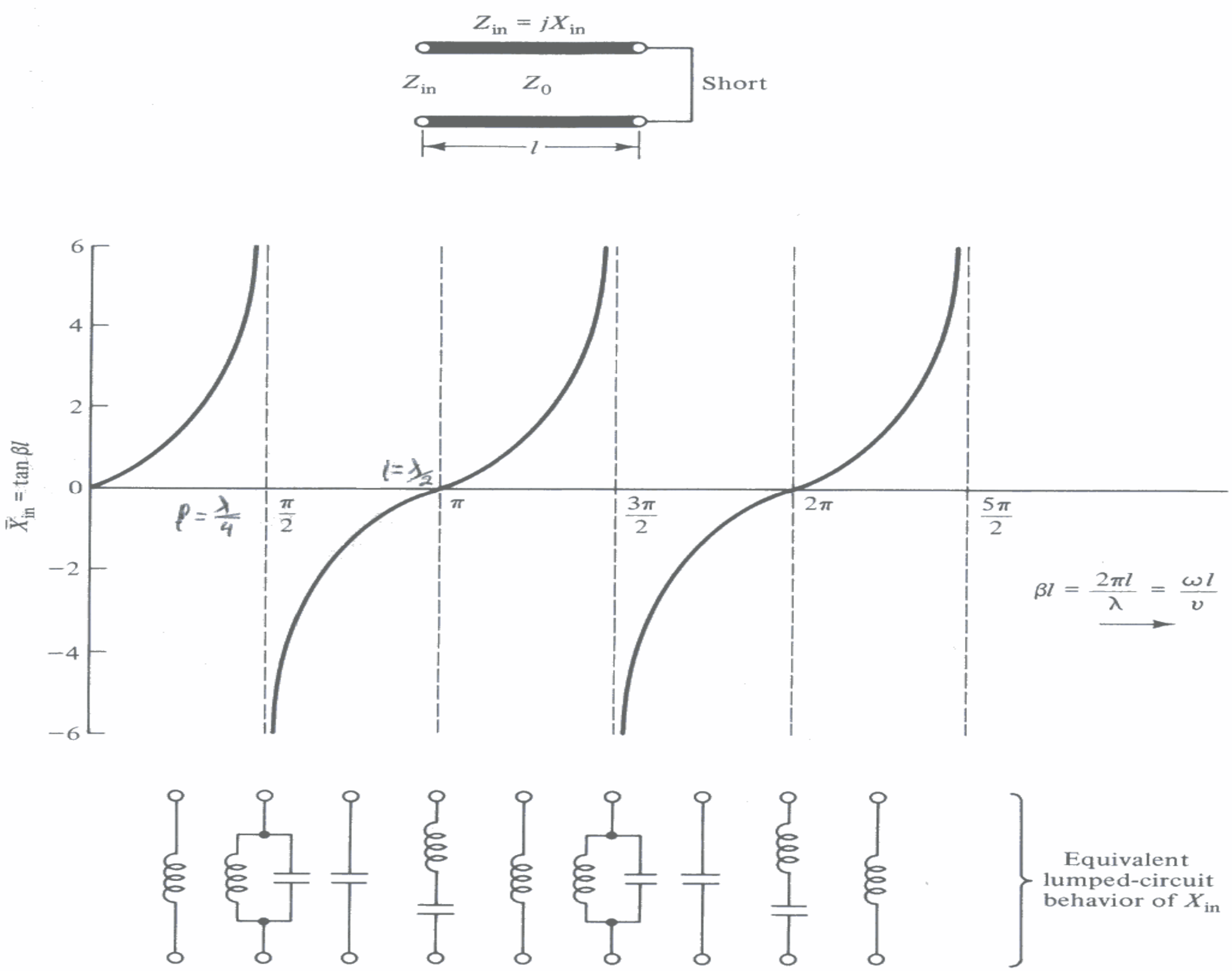
Thus, $\begin{cases} Z_0 = \sqrt{Z_{in} Z_L} \\ \overline{Z}_{in} = \dfrac{1}{\overline{Z}_L} \end{cases}$

This transformer is used impedance inversion and matching purposes.

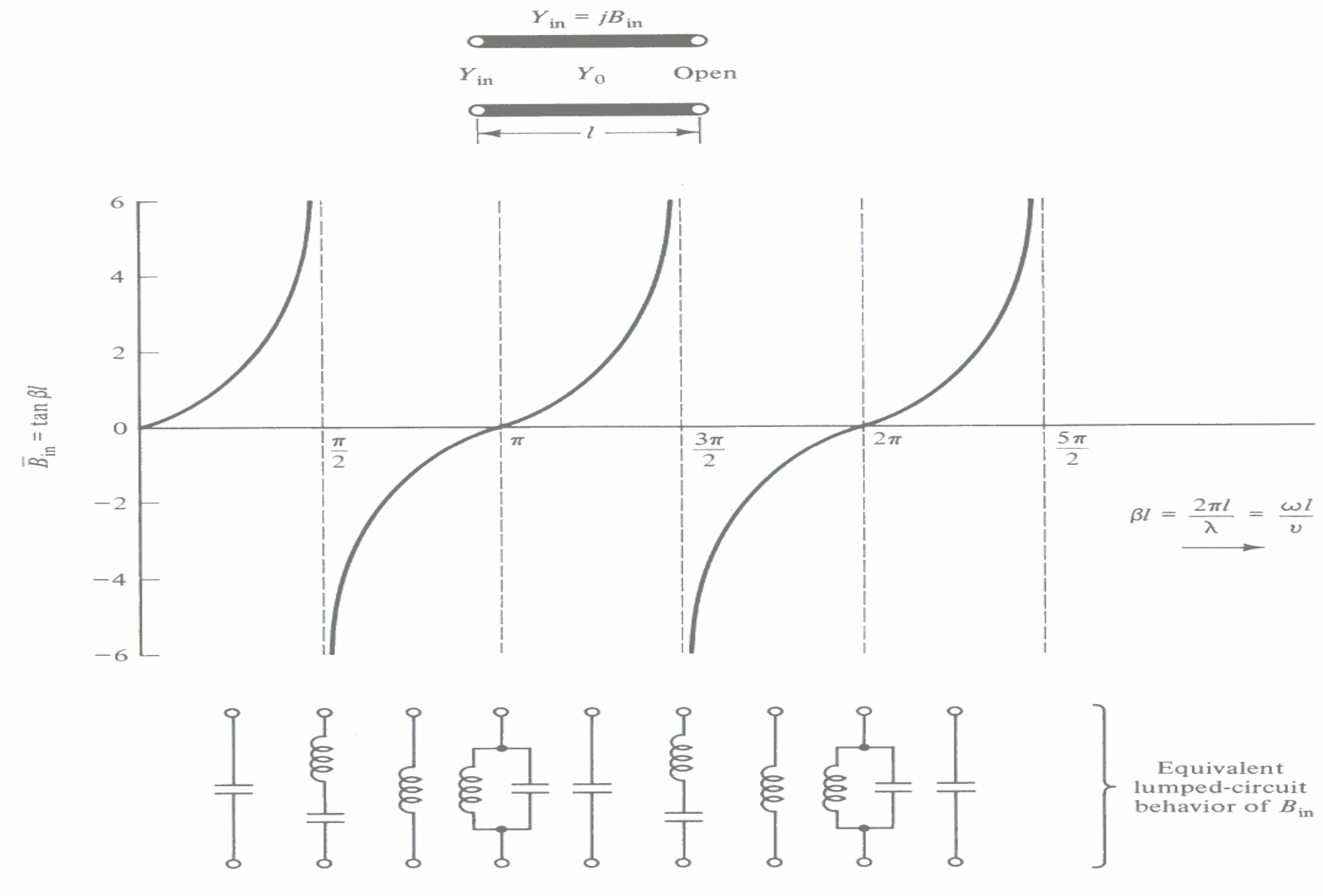
- For T.L. with <u>**l=λ/2**</u>: $\beta l=(2\pi/\lambda)(\lambda/2)= \pi$; $Z_{in} = Z_0 \dfrac{Z_L + Z_0 \tan\beta l}{Z_0 + Z_L \tan\beta l} = Z_L$

This transformer is used impedance point extension and matching.

- <u>For T.L. with Z_L=0 or S/C</u>; we know that $Z_{in} = jZ_0 \tan\beta l$ *or* $\mathbf{X_{in}} = \tan\beta l$;
- Here if $0 < l < \lambda/4$, X_{in}=>'+' and Input impedance (Z_{in}) is Inductive
- But if $\lambda/4 < l < \lambda/2$, X_{in}=>'-' and Input impedance (Z_{in}) is Capacitive



- T.L. with Z_L=∞ or Y_L=0 ; we know that Y_{in}= jY_0tanβl *or* B_{in}= tanβl;
- Here if $0 < l < \lambda/4$, B_{in}=>'+' and Input impedance (Z_{in}) is Capacitive
- But if $\lambda/4 < l < \lambda/2$, B_{in}=>'-' and Input impedance (Z_{in}) is Inductive



- These O/C or S/C lines can be used as STUBS for matching TL's

Smith Chart and its Applications: *(Invented by P.Smith in 1939)*

- 'Γ' to 'Z *or* Y' converter; Simplifies analysis of complex TL *or* LE prob

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\overline{Z_N} - 1}{\overline{Z_N} + 1} \quad \text{or} \quad U + jV = \frac{r + jx - 1}{r + jx + 1} \qquad as, \begin{cases} \Gamma = U + jV \\ \overline{Z_N} = r + jx \end{cases}$$

$$U = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \quad \text{and} \quad V = \frac{2x}{(r+1)^2 + x^2}$$

- eq.1:eliminating ***x***; $\left(U - \frac{r}{r+1}\right)^2 + V^2 = \left(\frac{1}{r+1}\right)^2$

center → $(U_0, V_0) = \left(\frac{r}{r+1}, 0\right)$

radius → $R = \left(\frac{1}{r+1}\right)$

- eq.2:eliminating ***r***; $(U - 1)^2 + \left(V - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \rightarrow (U_0', V_0') = \left(1, \frac{1}{x}\right); \quad R' = \left(\frac{1}{|x|}\right)$

- Eq.1 ⇒ Resistance circles

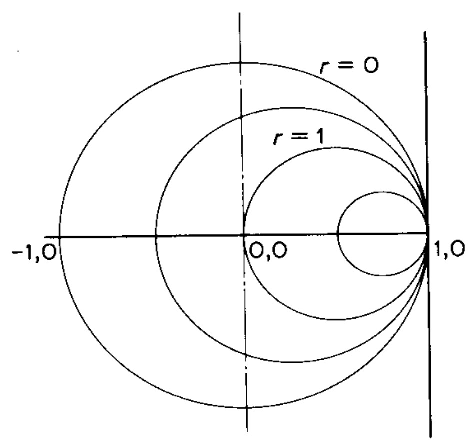
{concentric at (U_0,V_0) & radius=R}

(Note: ↑ 'r' ⇔ ↓ 'circle size')

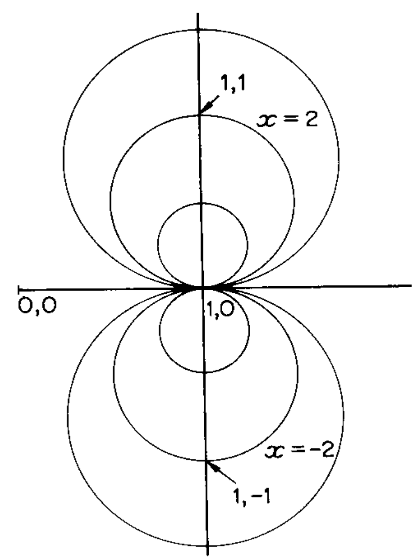
- Eq.2 ⇒ Reactance circles

{concentric at (U'_0,V'_0) & radius=R'}

(Note: ↑ 'x' ⇔ ↓ 'circle size')



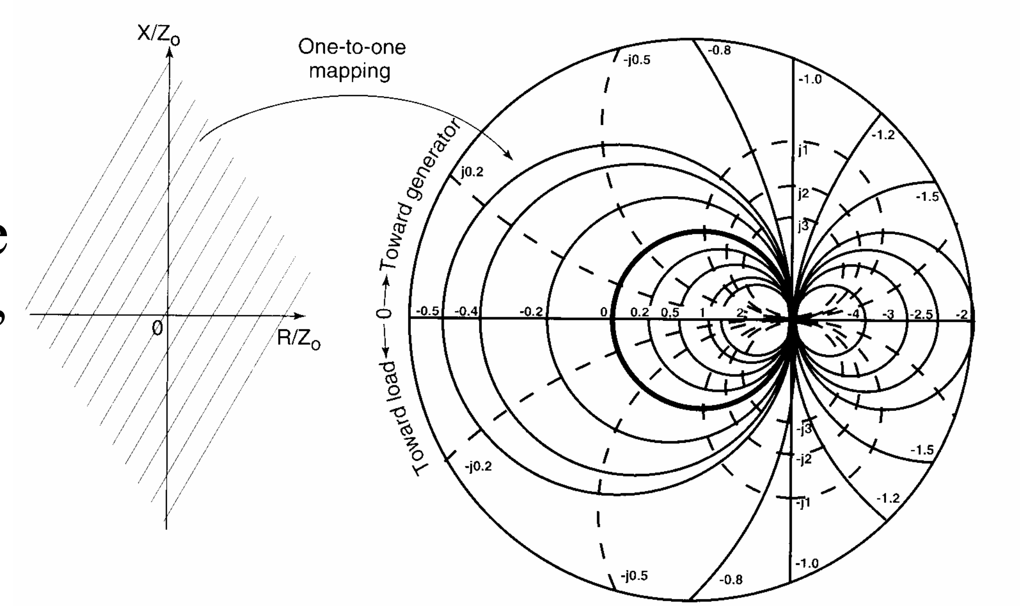
RESISTANCE CIRCLES



REACTANCE CIRCLES

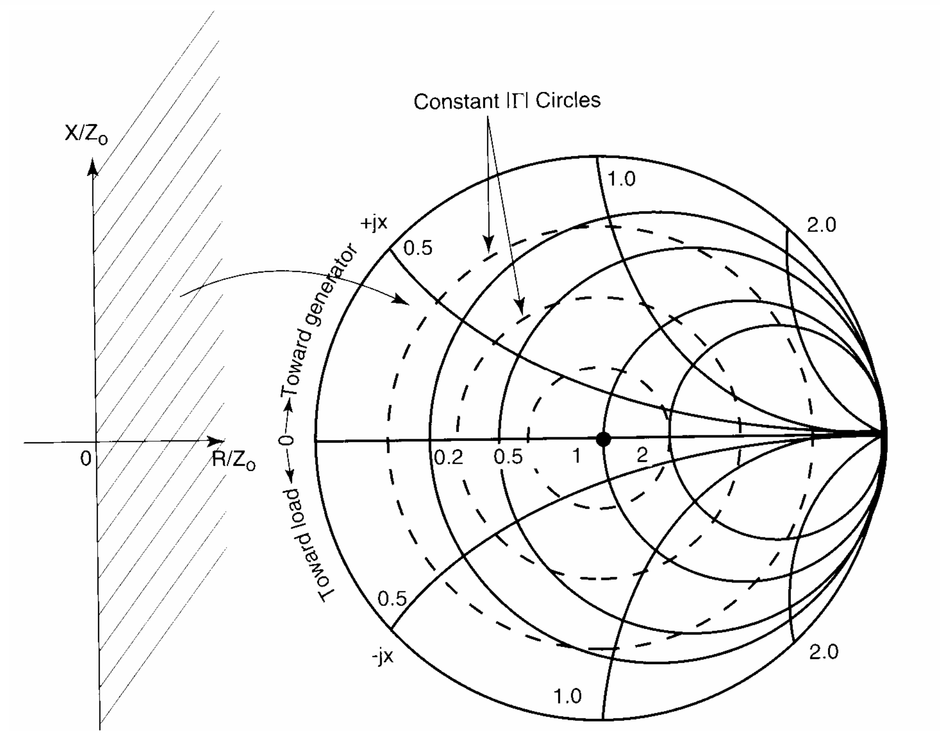
Compressed Smith Chart :

- Plotting two families of circles for all values for (r,x) creates the entire smithchart: “Compressed”
- Applies to active & passive ckts
- Impractical and seldom used



Standard Smith Chart :

- If two families of circles are plotted only for r≥0: “Standard”
- If ‘x ≥ 0’ ⇒ Positive reactance
- If ‘x ≤ 0’ ⇒ Negative reactance
- Heavily used for Passive ckts.
- Ref. Corff. Plane with ‘|Γ| ≤ 1’

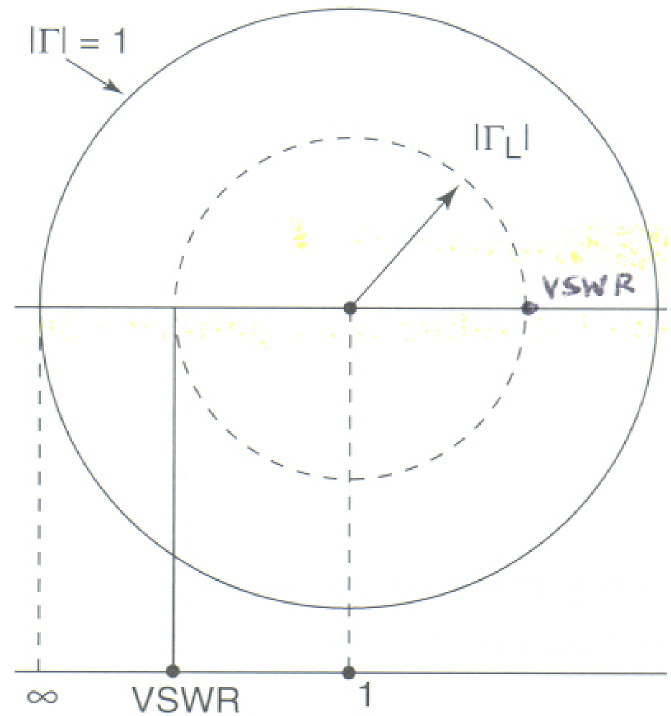


Determine *VSWR* from known Z_L:

1. Plot the normalized impedance $(Z_L)_N$

2. Draw constant *VSWR* circle through $(Z_L)_N$

- From the intersection of circle and left-hand horizontal axis, drop a line on the bottom scale to read *VSWR* value
- Or use intersection of the circle & θ=0 axis



Determine Y_N from known Z_N: *(vice versa)*

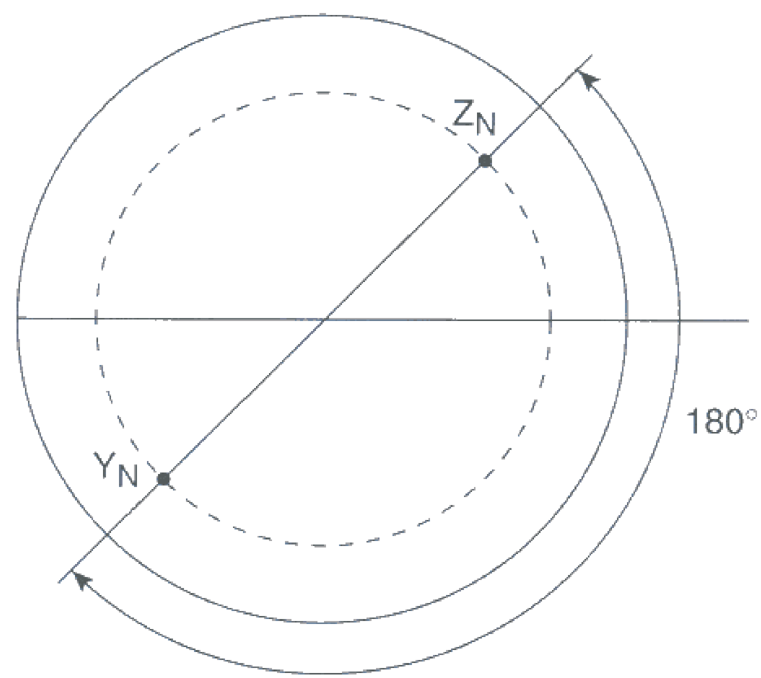
1. Plot the normalized impedance $[(Z)_N = Z/Z_0)]$ on the standard Smith chart.

2. Draw constant *VSWR* circle through $(Z)_N$

3. Draw a line from $(Z)_N$ via the center of the of constant *VSWR* circle

4. $(Y)_N$ is the interaction of the line and circle

* Prove this using relation between $(Z)_N$ & $|\Gamma_N|$



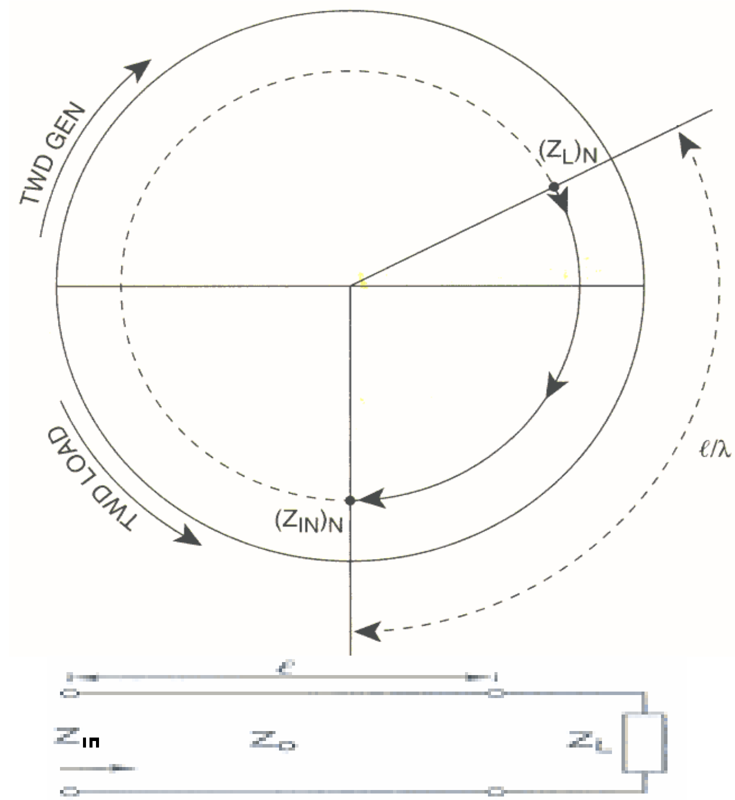
Determine Z_{IN} from known Z_L: *(or vice versa)*

1. Plot the normalized load impedance $[(Z_L)_N = Z_L/Z_0)]$ on the standard Smith chart.

2. Draw constant *VSWR* circle through $(Z_L)_N$

3. From $(Z_L)_N$, move to a distance "l/λ" 'toward generator' on constant *VSWR* circle

4. Read the normalized input impedance value $[(Z_{IN})_N = Z_{IN}/Z_0)]$ from the smith chart.



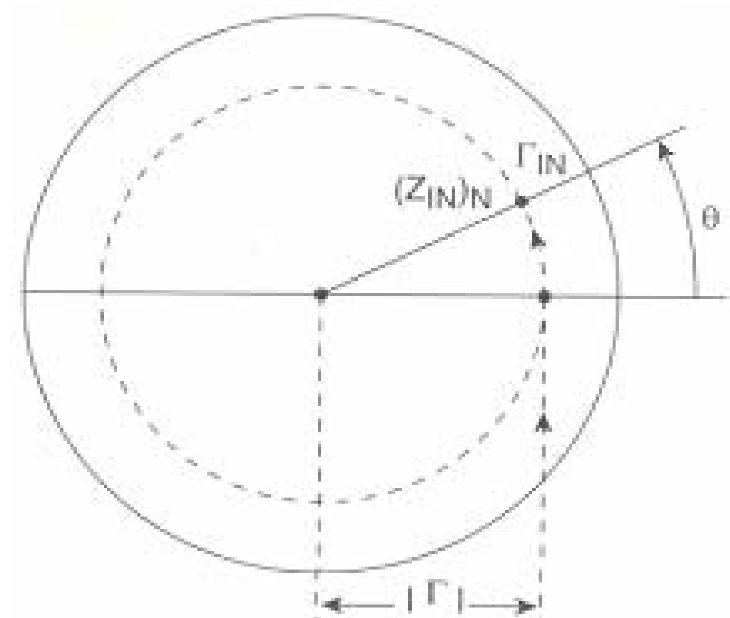
Determine Z_{IN} from known Γ: *(for* $|\Gamma_{IN}| \leq 1$*)*

1. For any point of TL. plot $\Gamma_{IN} = |\Gamma_{IN}|\, e^{j\theta}$,

- Use the bottom scale to plot $|\Gamma_{IN}|$ value
- Use circular scale to pot the angle 'θ'.

2. Read the normalized input impedance value $(Z_{IN})_N$ from the smith chart.

* Conversely find 'Γ' from known 'Z_{IN}'



Examples of how to use the smith chart: *(assume* Z_0=50 Ω *)*

To plot the known impedance of the load: (*say*, Z_0= 50 Ω, Z_L= 50+j100 Ω)

1. Normalizing the load impedance yields $(Z_L)_N$ = 1+j2.0 Ω.
2. Set chart with the central diameter horizontal & point zero on the left. Start from '*r* = 0' on the central diameter & move to point '*r*= 1.0' on the right.
3. Follow (*r*= 1.0) resistance circle upward and locate its intersection with reactance circle 'j2.0'. Point of intersection is $(Z_L)_N$ = 1+j2.0 Ω.

To find the VSWR of the transmission line: *(assume* Z_0=50 Ω, Z_L=100-j50 Ω*)*

1. Plot $(Z_L)_N$. Draw VSWR circle, with center at point (1,0), through the point $(Z_L)_N$. Read point of intersection of the circle with the horizontal diameter (*θ=0 axis*) on the right of the Chart center. This gives the VSWR as 2.6

To find the admittance (Y_L) of the load: *(assume* Z_0=50 Ω, Z_L=150-j75 Ω*)*

1. Plot $(Z_L)_N$. Draw the VSWR circle. From point $(Z_L)_N$, draw a diameter through the center of the chart to intersect with the circle again.
2. Point of intersection is $(Y_L)_N$ = 0.27+j0.14 Ω = $1/(Z_L)_N$. Now $Y_L = (Y_L)_N / Z_0$

To find the load Reflection coefficient (Γ_L): *(assume* **Z_0=50 Ω, Z_L=100+j75 Ω***)*

1. Plot $(Z_L)_N$. Draw the VSWR circle. Also draw a radial line through $(Z_L)_N$.
2. Use the scale below the smith chart (*for Ref. Coeff. E or I*) to read the value of the intersection of the circle and central diameter. Thus, $|\Gamma| = 0.535$
3. The intersection of the radial line & phase angle circle, gives; $\theta = 30°$

To find input impedance (Z_{IN}), if *l*=0.25λ of T.L.: **(*Z_0=50 Ω, Z_L=150-j75 Ω*)**

1. Plot $(Z_L)_N$. Draw VSWR circle. From point $(Z_L)_N$ move along the VSWR circle by a distance of '0.25', using wavelength circle of '*towards generator*
2. From the arrived point, draw a radial line towards the center. Intersection of this radial line and the VSWR circle gives, $(Z_{IN})_N = 0.27+j0.14\ \Omega$?.

Problem: A 600 Ω loss-free transmission line is 105 ft long & used to connect a 200 MHz transmitter, having an output impedance of 600∠0° Ω, to an antenna with a terminal impedance of 700∠-72°. Using smith chart determine;

(a) the VSWR on the antenna feeder.

(b) the voltage reflection coefficient at the antenna terminal.

(c) the impedance presented at the transmitter terminal