# OPTIMUM TRAJECTORY FUNCTION FOR MINIMUM ENERGY REQUIREMENTS OF A SPHERICAL ROBOT

**S. A. Alshahrani[1], H. Diken[2], A. A. N. Aljawi[2]**

*1: Master student*

*2: Associate Professor, Production and Mechanical Systems Design Department King Abdulaziz University, Jeddah, 21589, Saudi Arabia.*

*E-mail: hdiken@yahoo.com*

## ABSTRACT

*In this study simple harmonic, cycloid and 3-4-5 polynomial time functions, which are used in cam design, are introduced into robotics as new trajectory functions and compared with trajectories of cubic segment and bang bang parabolic blend, which are already used in robotics. For this purpose dynamic equations of a Stanford type spherical robot is developed. Straight line trajectory is chosen for the end effector of the manipulator. On this trajectory robot travels with the above mentioned time functions. Total consumed energy curves are obtained with respect to travel time for each time function. Results show that cubic segment trajectory function spends minimum energy, cycloid and bang-bang parabolic blend trajectories spend maximum energy. When the travel time gets bigger all trajectories approach to an asymptotic energy value. Because inertial loads are becoming small and negligible compared to the gravitational forces and moments. Cycloid and 3-4-5 polynomial trajectories start and end with zero accelerations which will not cause jerk or vibration but a smooth running of the robot. That is why, if travel time is relatively big, cycloid or 3-4-5 polynomial trajectories should be preferred. If the energy consumption is the prime concern and the travel time is short, cubic segment trajectory is the best.*

**Keywords:** *Robotics, Dynamics, Trajectory planning.*

## الملخص

في هذة الدراسة تم تطوير المعادلات الديناميكية لنوع روبوت ستانفورد. فقد تم إختيار نوع مسار الخط المستقيم لتوجية حركة الذراع النهائي المؤثر وذلك بإستخدام معادلات وأوقات مختلفة. . من هذة المعادلات:

cubic segment, Bang-bang parabolic lend, simple harmonic, cycloid, and 3-4-5 polynomial

وبإستخدام المعادلات الثلاث الأخيرة لتوجية المسار فقد تم الحصول على مجموع الطاقة المستخدمة بالنسبة للوقت وذلك لكل معادلة من هذة المعادلات. فقد أوضحت النتائج أن أقل طاقة تستخدم عند إستخدام معادلة من نوع cubic segment وأكثر طاقة عند إستخدام معادلة من نوع cycloid and bang-bang blend . وفي حالة إزدياد وقت المسار نجد أن جميع المسارات تلتقي بنفس قيمة الطاقة المستخدمة وذلك بسبب إزدياد أهمية قوة الجاذبية ومحور الذراع إذا ماقورنت بالقوى الديناميكية. إلا أنة يفضل طريقة cycloid أو 3-4-5 polynomial وذلك في حالة مسار الوقت الطويل. أما إذا كان الهدف هو كمية الطاقة في أقل وقت ممكن فيفضل طريقة cubic segment.

## 1. INTRODUCTION

Efficient use of industrial robots has been the subject of great interest for the past decade as shown by the great volume of work reported in the literature. Because of the non-linearity and highly coupled nature of the manipulator dynamics, the conventional optimal control approach has proven to be complicated and too time consuming for an on-line implementation. Instead a two-stage optimization approach has been commonly used to tackle the problem. The first stage involves an off-line trajectory-planning model, which yields a time history of joint angle and joint velocities (and joint acceleration or joint torques in some cases), to be followed by the robotic arm in the actual task. The trajectory is usually planned with the objective of achieving minimum cost or minimum time. The second stage is the on-line path tracking problem which is concerned with making the robot's actual Cartesian position and velocity follow the derived values as closely as possible. [Bhattacharya and Agrawal, 2000] described a prototype and analytical studies of a spherical rolling robot. Methods are developed for planning feasible, minimum time energy trajectories for the robot. [Diken, 1994] assumed a sinusoidal path in Cartesian coordinates. It is assumed that the end point of the manipulator travels on a sinusoidal path trajectory with simple harmonic time function. Considering the amplitude of the sinusoidal motion as a variable, he searched for the amplitude of sinusoidal path that makes the energy consumption minimum. His computations showed that sinusoidal paths, outward and downward from the body of the manipulator, complete the task with minimum energy.

In this study, a Stanford type of a spherical robot is chosen. Dynamic equations are derived using Lagrange equations. Torques are computed at each servomotor for a straight-line work space trajectory. The end point of the robot travels on this trajectory with different time functions. These functions are cubic segment, bang-bang parabolic blend, simple harmonic, 3-4-5 polynomial and cycloidal trajectories. For each trajectory Total energy consumptions are calculated with respect to travel times.

## 2. ANALYSIS

The Stanford type spherical robot is shown in Figure 1. Equations of motion are derived considering only the body, back arm and the forearm. The second set of three motions for the manipulator hand, which gives orientation to the payload, are ignored. The Denavit-Hartenberg method is used to find forward and inverse kinematic relations [Wolovich, 1987]. Total kinetic energy $K$ of the robot is then obtained by using kinematic relations and dynamic parameters.

$$K = \sum_{q=1}^{3} K_q \qquad (1)$$

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Kinetic energy of any link can be obtained by the following equation

$$K_q = \frac{1}{2} V_q^T m_q V_q + \frac{1}{2} \omega_q^T I_q \omega_q \tag{2}$$

Here $m_q$ is the $q$th link mass, $V_q$ is the linear velocity, $\omega_q$ is the angular velocity of the $q$th link, and $I_q$ is the mass moment of inertia about the mass center of the $q$th link. Since robot motion is in three dimensions, $V_q$ and $\omega_q$ are vectors with three components. Dynamic equation of each link can be obtained by using the Lagrange equation.

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_q}\right) - \left(\frac{\partial K}{\partial \theta_q}\right) = Q_q \text{, for } q = 1,2,3. \tag{3}$$

Here $K$ is the total kinetic energy of the robot, $\theta_q$ is the $q$th link rotation, $Q_q$ is the $q$th generalized moment coming from gravity forces and motor torques. When Lagrange equations are applied, the following equations of motion, in matrix form are obtained.

$$D(\theta)\ddot{\theta} + N(\dot{\theta}^2) + C(\dot{\theta}_i \dot{\theta}_j) + G(\theta) = \tau \tag{4}$$

Here $D$ is the inertia matrix, $N$ is the normal force matrix, $C$ is the Coriolis force matrix, $G$ is the gravitational force matrix and $\tau$ is the torque vector, respectively.

## 3. TRAJECTORIES

It is assumed that the end point of the manipulator travels from one point to another in Cartesian space on a straight line, which can be given as,

$$P = P_0 + \lambda(t)\left\lfloor P_f - P_0 \right\rfloor \tag{5}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \lambda(t)\begin{bmatrix} x_f - x_0 \\ y_f - y_0 \\ z_f - z_0 \end{bmatrix} \tag{6}$$

Here $x, y, z$ are Cartesian coordinates, $x_0, y_0, z_0$ and $x_f, y_f, z_f$ are the initial and final positions, respectively. $\lambda(t)$ is the time function, such that $0 \le \lambda \le 1$ for $0 \le t \le t_f$. Here $t_f$ is the travel time. In closed form, position, velocity and acceleration for a straight-line trajectory can be given as,

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$$
\begin{aligned}
P &= P_0 + \lambda(t)\left[P_f - P_0\right] \\
\dot{P} &= \dot{\lambda}(t)\left[P_f - P_0\right] \\
\ddot{P} &= \ddot{\lambda}(t)\left[P_f - P_0\right]
\end{aligned}
\tag{7}
$$

Trajectory functions that will be used for total energy calculations are cubic segment, bang-bang parabolic blend, simple harmonic, 3-4-5 polynomial and cycloid.

### 3.1. Cubic Segment Trajectory

One of the most frequently used trajectory functions in robotics is a cubic polynomial function, which is given as,

$$x(t) = a + bt + ct^2 + dt^3 \tag{8}$$

The four constant *a, b, c* and *d* can be calculated from the initial conditions, which usually are

$$x(0) = x_0, \quad x(t_f) = x_f, \quad \dot{x}(0) = 0, \quad \dot{x}(t_f) = 0 \tag{9}$$

Here $x_0$ and $x_f$ are initial and final coordinates, and $t_f$ is the travel time. After finding constants, the trajectory is

$$P(t) = P_0 + \left(\frac{3t^2}{t_f^2} - \frac{2t^3}{t_f^3}\right)(P_f - P_0) \tag{10}$$

### 3.2. Bang-Bang Parabolic Blend Trajectory

The bang-bang parabolic blend trajectory is also used in robotics applications, which consists of two parabolas. One is for $0 \le t \le \frac{t_f}{2}$ and the second one is for $\frac{t_f}{2} < t \le t_f$. For the first part of the trajectory function

$$P(t) = P_0 + \frac{2t^2}{t_f^2}(P_f - P_0), \qquad 0 \le t \le \frac{t_f}{2} \tag{11}$$

For the second part

$$P(t) = P_f + \left(\frac{4t}{t_f} - \frac{2t^2}{t_f^2} - 2\right)(P_f - P_0), \qquad \frac{t_f}{2} < t \le t_f \tag{12}$$

### 3.3. Simple Harmonic Trajectory

Simple harmonic trajectory function is a trigonometric function, this and the following functions are mostly used in cam design [Shigley, 1980]. The simple harmonic trajectory function is given as,

$$P(t) = P_0 + \frac{1}{2}(1 - \cos\frac{\pi t}{t_f})(P_f - P_0), \quad 0 \le t \le t_f \tag{13}$$

### 3.4. Cycloidal Trajectory

The equation of the cycloidal trajectory function is given as,

$$P(t) = P_0 + (\frac{t}{t_f} - \frac{1}{2\pi}\sin\frac{2\pi t}{t_f})(P_f - P_0), \quad 0 \le t \le t_f \tag{14}$$

### 3.5. 3-4-5 Polynomial Trajectory

The 3-4-5 polynomial trajectory function is given as,

$$P(t) = P_0 + (10\frac{t^3}{t_f^3} - 15\frac{t^4}{t_f^4} + 6\frac{t^5}{t_f^5})(P_f - p_0) \quad 0 \le t \le t_f \tag{15}$$

Figure 2 shows the cycloidal trajectory, along with its velocity and acceleration as a sample. It may be noted that the cubic segment, bang-bang parabolic blend, and the simple harmonic functions have finite acceleration values at the beginning and at the end. These finite values of acceleration will cause jerk in the motion, which are undesirable in robotics. Cycloidal motion and 3-4-5 polynomial functions, on the other hand, starts and ends with zero acceleration, which will result in a smooth motion.

## 4. ENERGY CALCULATIONS

When two points in the workspace are chosen, then $P_0(x_0, y_0, z_0)$ and $P_f(x_f, y_f, z_f)$ are known. For a chosen trajectory and travel time $t_f$, $\lambda(t)$ is also known. By using equation (7) any number of trajectory points $P_i$ $(i=1,2,\ldots,N)$, velocity $\dot{P}_i$ and acceleration $\ddot{P}_i$ can be calculated.

When the Cartesian position, velocity and acceleration values are computed in this manner, then the link space values, i.e., servomotor rotations, velocities and accelerations can be calculated by using the following inverse kinematic relations

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$$Q = T^{-1} P$$
$$\dot{Q} = J_p^{-1} \dot{P} \qquad (16)$$
$$\ddot{Q} = J_p^{-1} \ddot{P} - J_p^{-1} \dot{J}_p J_p^{-1} \dot{P}$$

Here $Q^T = [\theta_1, \theta_2, \theta_3]$ is the link rotation vector, $P^T = [x, y, z]$ is the Cartesian coordinate vector, $T^{-1}$ is the inverse of the transformation matrix, $J_p^{-1}$ is the inverse of the Jacobian matrix and $\dot{J}_P$ is the time derivative of the Jacobian matrix. Using the link angles, angular velocities and angular accelerations, it is possible to compute servomotor torques by the use of equation (4). Once torques and angular velocities are known, the total power requirement of the robot becomes

$$W(t) = \sum_{q=1}^{3} \left| \tau_q \, \dot{\theta}_q \right| \qquad (17)$$

The total energy spent during the travel time $t_f$ is

$$E = \int_0^{t_f} W(t) \, dt \qquad (18)$$

To serve as a practical application of the above, a series of computations were carried out and total energy consumption curves are obtained for five trajectories [Shahrani, 2001]. Geometrical values and dynamic parameters that are used for simulation are tabulated in Table 1.

**Table 1**. Geometric and dynamic properties of the spherical robot.

| | | |
|---|---|---|
| $h=1.5\ m$ | $m_2=27\ kg$ | $I^3_{yy} = 0.0291 kgm^2$ |
| $r=0.5\ m$ | $m_3=63\ kg$ | $I^3_{xx}=0.0291\ kgm^2$ |
| $f_{min}=0.15\ m$ | $I^1_{zz} = 0.8 kgm^2$ | $I^3_{yy}=0.0134\ kgm^2$ |
| $f_{max}=0.6 m$ | $I^2_{xx} = 0.0134 kgm^2$ | $I^3_{zz}=0.0582\ kgm^2$ |
| $a_2=0.3m$ | $I^2_{yy}=0.0537\ kgm^2$ | $m_p=5\ kg$ |
| $a_3=0.75m$ | $I^2_{zz}=0.0134\ kgm^2$ | |

The end point of the manipulator will travel from $P_0$ (0.6, -0.6, 0.5) m to $P_f$ (0.5,0.1,0.6) m during a travel time $t_f$=3.5s. Figures 3, 4 and 5 show angular positions, angular accelerations and angular velocities of the links of the manipulator for cycloidal trajectory, respectively. Figure 6 shows the torque requirements of servomotors during the travel. Figure 7 shows the power needed for each servomotor. Figure 8 is the plot of the total power requirement of the robot.The integral of the curve, which is given in Figure 7 will yield the total energy required

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for the manipulator, which is 66.63 Joule. If these calculations are repeated for different travel times $t_f$, the plot of the total consumed energy per travel time can be obtained. The result of a such set of computations for the cycloidal trajectory function is summarized in Figure 9.

Total energy consumption curves can be obtained for the other four trajectories as well in a similar manner.The plot of the total consumed energy curves for five trajectories are shown in Figure 10. Cyloid and bang-bang parabolic blend trajectories are spending almost same amount of energy. For longer travel times, curves are approaching to an asymptotic value. This is because, the effect of inertial forces are decreasing and becoming negligible for longer travel times, servomotors are overcoming only gravitational forces and moments. In terms of energy consumption cubic segment is the best choise. For example, for the travel time 1.4 second, simple harmonic trajectory spends 3.7% more, 3-4-5 polynomial trajectory spends 22% more, bang-bang parabolic blend spends 29% more and cubic segment trajectory spends 31% more energy than the cubic segment trajectory. In terms of minimum energy consumption, cubic segment trajectory is the best than comes simple harmonic. Since 3-4-5 polynomial trajectory and cycloidal trajectory functions are starting and ending with zero accelerations, they can be preffered to eliminate vibrations and for smooth running of the robot. In many practical applications, the maximum velocity of the end effector is assumed as 1 m/s, corresponding travel times to this velocity are also calculated, which is 1.5 s for cycloid, 1.44 s for bang-bang parabolic blend, 1.35 s for 3-4-5 polynomial, 1.12 s for simple harmonic and 1.07 s for cubic segment.

## 5. CONCLUSION

In this study simple harmonic, cycloid and 3-4-5 polynomial time functions, which are used in cam design, are introduced into robotics as new trajectory functions and compared with trajectories of cubic segment and bang bang parabolic blend, which are already used in robotics. For this purpose dynamic equations of a Stanford type spherical robot is developed. Straight line trajectory is chosen for the end effector of the manipulator. On this trajectory robot travels with the above mentioned time functions. Total consumed energy curves are obtained with respect to travel time for each time function. Results show that cubic segment trajectory function spends minimum energy, cycloid and bang-bang parabolic blend trajectories spend maximum energy. When the travel time gets bigger all trajectories approach to an asymptotic energy value. Because inertial loads are becoming small and negligible compared to the gravitational forces and moments. Cycloid and 3-4-5 polynomial trajectories start and end with zero accelerations which will not cause jerk or vibration but a smooth running of the robot. That is why, if travel time is relatively big, cycloid or 3-4-5 polynomial trajectories should be preferred. If the energy consumption is the prime concern and the travel time is short, cubic segment trajectory is the best.

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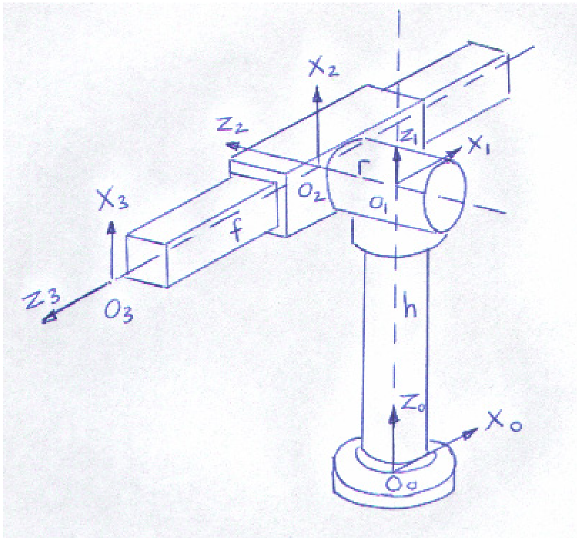


Figure 1. Stanford type spherical manipulator.

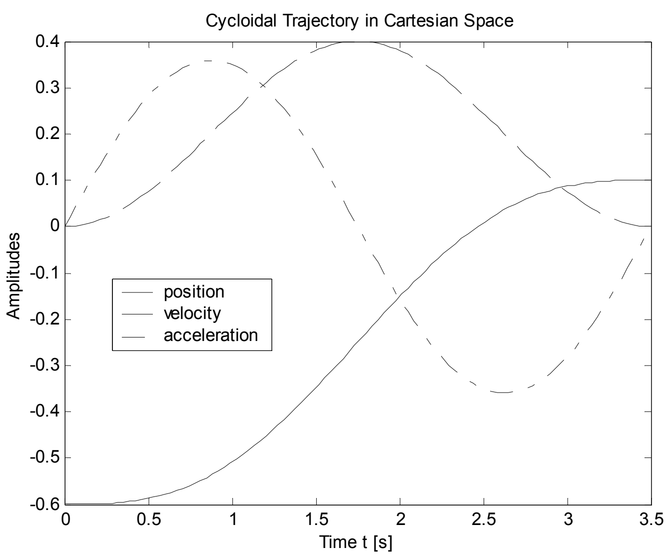


Figure 2. Cartesian position, velocity and acceleration for cycloidal trajectory.

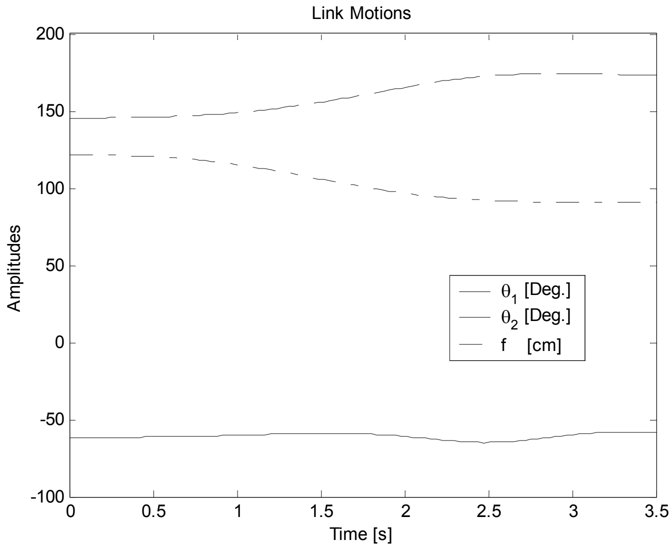


Figure 3. Link angles for cycloidal trajectory.

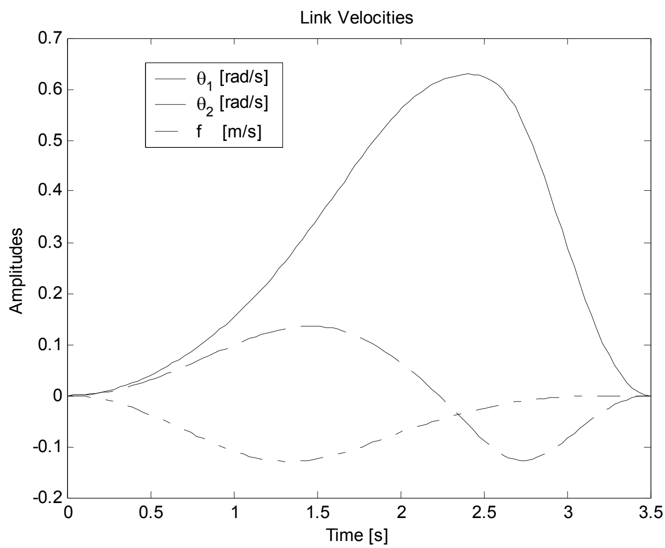


Figure 4. Link angular velocities for cycloidal trajectory.

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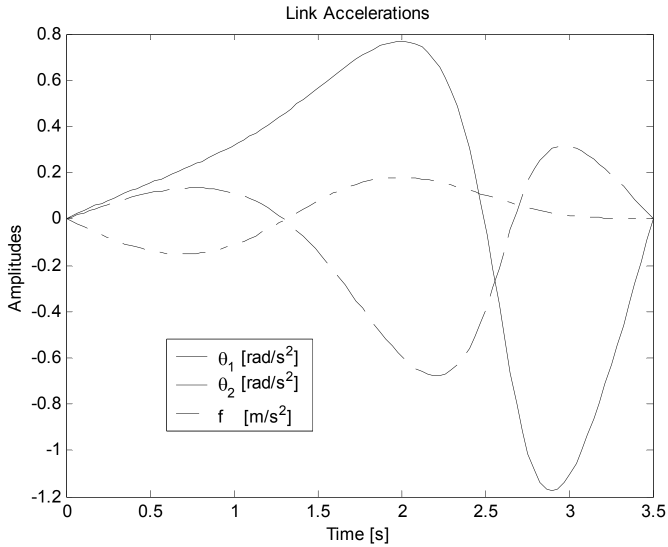


Figure 5. Link angular accelerations for cycloidal trajectory.

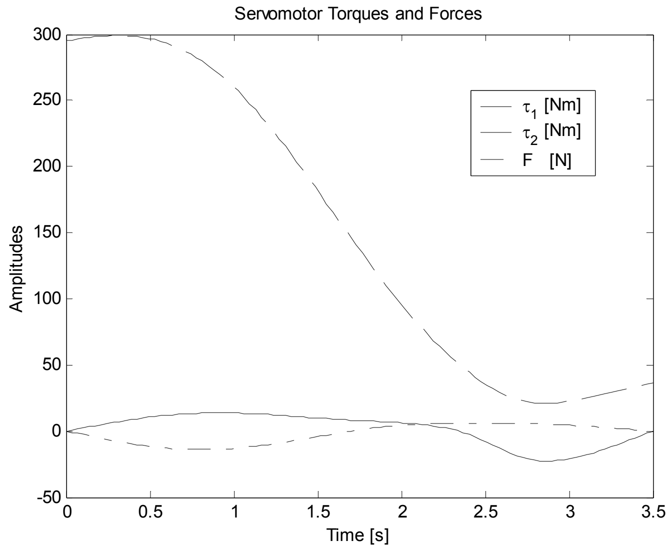


Figure 6. Motor torques of the manipulator.

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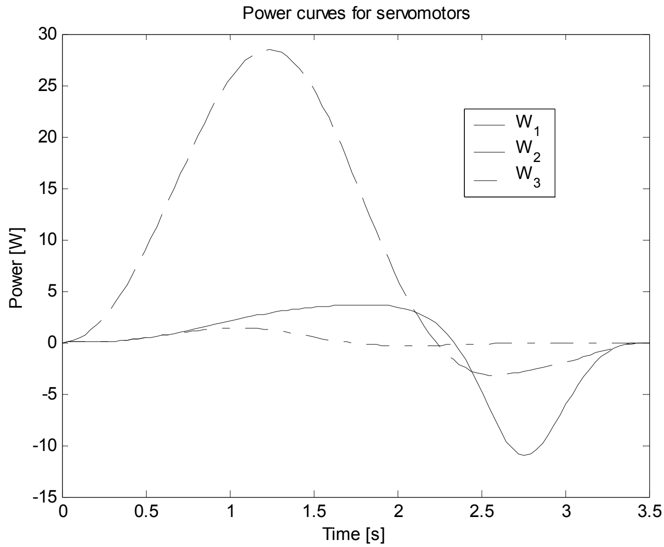


Figure 7. Servomotor power consumption curves.

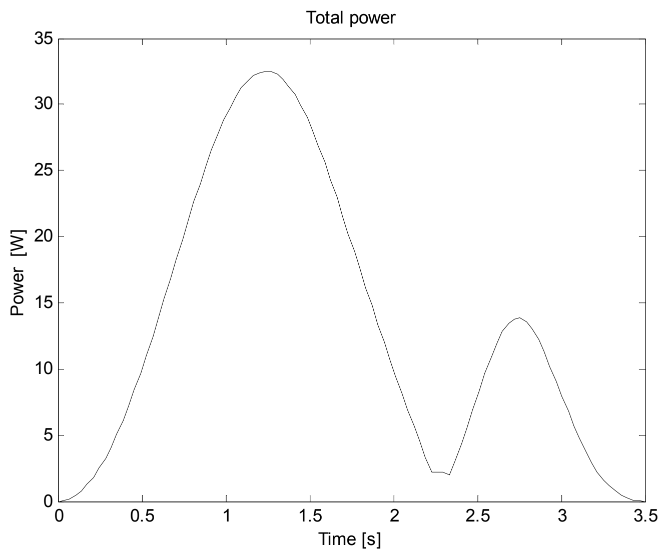


Figure 8. Total power required for the manipulator.

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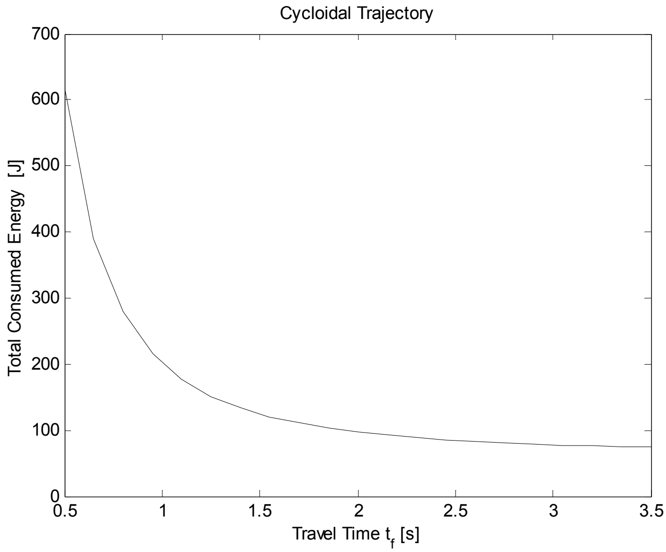


Figure 9. Total consumed energy curve for cycloidal trajectory.

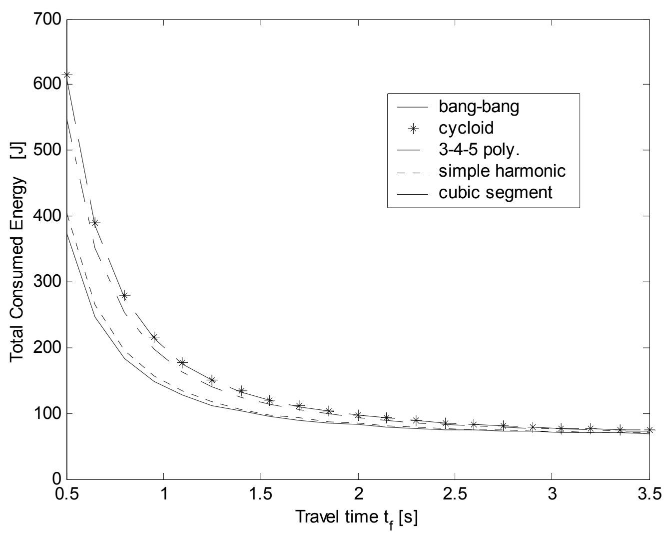


Figure 10. Total consumed energy curves for all trajectories.