A SIMPLIFIED ANALYSIS OF EDGE SETTLEMENT OF A LARGE ABOVEGROUND LIQUID STORAGE TANK

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ABSTARCT

This work is concerned with the analysis of deformation and bending moment distribution along sections of bottom plate of a large aboveground cylindrical liquid storage tank with appreciable out of plane differential edge settlement. The analysis uses approximate simple beam bending theory to model the settled section of the plate and takes into account the effects of foundation compliance, shell and hydrostatic loading and the shell-bottom plate junction stiffness. Results are presented for the effects of edge settlement maximum amplitude, plate thickness, foundation stiffness, and hydrostatic loading on the allowable edge displacement maximum amplitude. Comparison of the results of the present study with the API Standard 653 shows that the API standard 653 which does not take into account the effects foundation stiffness, plate-shell junction stiffness and plate thickness gives in general a conservative estimate of the allowable edge displacement limit and that this limit, depending on system parameters, may be relaxed by as much as 20%..

Keywords: *Edge type settlement, API Standard 653, Allowable maximum displacement amplitude, Localized dispersion, Localized bulge, Soft foundation, Rigid foundation, String action and Yield moment.*

الملخص

يهتم هذا البحث بتحليل النشوة وجهة الانحناء المرافق له علي طول مقاطع من منطقة الاستقرار الجانبي اللاسطحي في الصفيحة السفلية لخزانات السوائل أسطوانية الشكل ذو الحجم الكبير . يستخدم هذا البحث نظرية أنحاء العوارض السهلة لتمثيل الاستقرار الجانبي في الصفيحة السفلية للخزان أخذا بعين الاعتبار تأثير ليونة أرضية وقاعدة الخزان والحمولة الناتجة عن ضغط السائل علي جدار الخزان وليونة منطقة اتصال الصفيحة السفلية بجدار الخزان . يقدم هذا البحث النتائج بشكل بياني يوضح تأثير السعة القصوى للنشوء الجانبي ، ليونة القاعدة ، سمك الصفيحة وضغط السائل علي الحد المسموح به للنشوء الجانبي . ويقدم هذا البحث مقارنة للنتائج مع تلك المحددة بمواصفات معهد البترول الأمريكي ٦٥٣ حيث تبين نتائج هذا البحث بأن هذه المواصفات هي بشكل عام متحفظة بمقدار قد يصل في بعض الأحيان إلى ٢٠%

1. INTRODUCTION

Ground supported liquid storage large steel tanks, depending on their design characteristics, soil conditions and loading history, are prone to different modes of shell and bottom plate settlements. The tank various settlement modes, which may originate from different causes, may be considered to be one or a combination of shell and bottom plate basic settlement patterns: uniform, planar tilt and out of plane settlement patterns [Marr et.al, 1982, and API Standard 653, 1995]. Furthermore the out of plane settlements of a tank bottom plate may be one or a combination of the following three main types:1) dish type;2) localized dispersions and/or bulges and 3) edge settlements [Marr et al. 1982]. Due to structural flexibility, a large tank is more likely to settle into a non-planar mode Furthermore, while uniform and planar rigid body tilt settlements of a tank are not known to cause a threat to its structural integrity, the out of plane settlements can cause tank failures and thus are of main concern to engineers They are the result of localized and usually randomly distributed deformations and thus induce localized overstresses and radial distortions, known as ovality. Beyond permissible displacement limits the induced localized stresses can cause rupture and spillage of a tank content, and an excessive ovality can cause a floating roof malfunction. Evaluation of the maximum allowable settlement amplitude, and consequently the decision on the fitness-to-service and the choice of an appropriate repair procedure for a tank with a given deformation profile requires in general a rigorous stress analysis of the tank structure, specially for the areas of the tank with noticeable deformations. Such a stress analysis is, however, rather involved when carried out numerically, and is, even after introducing significant simplifying assumptions, for the novice complicated analytically. Therefore various researches, design groups and engineers have in many cases used simplified stress -strain relations of a simple beam bending, thin plate or thin shell theories, simplified geometric considerations and boundary conditions, and field data to develop criteria defining the allowable limits for various principal patterns of settlement of aboveground liquid storage cylindrical steel tanks [Rinne, 1963, DeBeer, 1969, Hayashi, 1973, Greenwood, 1974, Guber, 1974, Langeveld,1974, Malik et al., 1977, Bell and Iwakiri, 1980, and Marr et al., 1982]. Each of these criteria is usually concerned with a particular element of the tank structure(e.g the shell or the bottom plate) and with a single failure mechanism for a specified loading and boundary conditions. Marr et al. [1982] analyzed the measured performance of 90 large cylindrical steel tanks used to store liquid at ambient pressure and temperature and used approximate stress analysis of simple beam bending theory and arbitrary factors of safety to study the performance and refine various available criteria for the allowable displacement limits in various patterns of tank settlements. Their study has shown that in many cases the available criteria are over-conservative and are far from being standarized.

This work focus on the analysis of the bottom plate out of plane edge type settlement which is of major interest to engineers as it is frequently found in large storage tanks and can lead to tank failure or costly unnecessary repair if not evaluated properly. The edge type settlement develops when the tank shell settles sharply around the periphery leading usually to excessive and localized bottom plate deformations near the plate-shell junction as shown for example in figure (1). Studies dealing with this type of bottom plate out of plane settlements, which may take different patterns [API Standard 653, 1995], are not readily available in the open literature. The API Standard 635 provides guidelines for measurement procedure of the localized depression type edge settlement and recommends using the following criterion to evaluate the allowable edge deflection limit:

$$\frac{S}{L} \leq 0{,}03083 \quad (\ S \text{ and } L \text{ have same units})$$

where, see figure (1), S is plate edge maximum deflection and L is the radial length of the plate settled area . The API Standard 635 also provides a set of curves for evaluating S for different values of tank diameters in cases where the area of the localized edge includes floor lap-welds approximately parallel to the shell and another for edge settled areas with no floor welds, or only floor butt-welds, or lap welds in the floor that are approximately perpendicular to the shell. The API Standard 653 indicates that these curves which were developed for a plate of ¼ inch in thickness may be used with reasonable accuracy for plate thickness in the range of 5/16 to 3/8 inches and it also provides an interpolation formula for evaluating S for the cases for which the area of the localized edge settlement has welds at an arbitrary angle to the shell. The API Standard 653, however, does not indicate the deformation analysis procedure and the failure mode used in developing these curves, nor do theses curves show the effects of plate thickness, and foundation and shell flexibilities where these flexibilities tend to relax part of the stresses induced in the bottom plate by local settlements adjacent to the shell. [Guber, 1974] developed a set of curves for the allowable settlement limits for partial ring type depressions of the bottom plate adjacent to the shell which may be as long as $d < D/4$ and $\bar{d} > 2d$, where (see figure (2)) d is the diameter of the largest horizontal circle that can be inscribed inside the depression, D is the tank diameter and $\bar{d}$ is the length of the partial ring depression. These curves show that at failure the allowable settlement limits for local edge depressions range from $d/17$ to $d/33$ for depressions with single pass welds at failure and from $d/13$ to $d/26$ for depressions with multiple pass welds. [Marr et al, 1982] indicated that the limits in these curves may be expressed by the approximate relation:

$$S \leq d\left(\frac{2.25.\sigma_f .D}{d^{0.75} E.FS.h}\right)^{0.5}$$

where S is the allowable maximum settlement, E is Young's modulus, FS is a factor of safety, h is the tank height, σ_f is the ultimate stress of the welds in the bottom plate, and d and D are as defined before; (S , d, D, and h are in meters). They concluded that, based on their evaluation of performance study of large steel cylindrical liquid storage tanks at various facilities, the above relation provides a "rational" limit on the maximum allowable bottom plate settlement adjacent to the shell and recommended that the factor of safety FS should be

≤ 4 in cases where localized yielding is possible and ≤ 2 in cases where severe overstress and rupture may occur.

The present work is an extension of parts of the author's contribution to a SAUDI ARAMCO funded project concerned with an analytical evaluation of localized edge depressions in large, aboveground liquid storage tanks. It uses linear beam bending theory to analysis a localized edge settlement depression in a tank bottom plate which is assumed to be resting on an elastic foundation as shown in figure (1). A stress-strain analysis of the settlement deformation which takes into account the deformation history is very complicated, therefore the present analysis will be carried out assuming zero initial stress and strain in the depression profile. This approach, despite its inherent limitations, may provide a valuable insight into such e frequently encountered edge settlement deformations of a large liquid storage tank. In this connection, it is noted that a similar approximate approach (e.g use of a beam element model to analyze bottom plate deformations) has been employed by [Malhotra and Veletsos,1994] in their study of uplifting resistance of the base plate of a large cylindrical liquid storage tanks. They presented results which showed that the approximate beam model yields reasonably accurate predictions of the behavior of the uplifted bottom plate.

2. MODEL AND ASSUMPTIONS

The localized edge dispersion of a uniform bottom plate resting on elastic foundation of stiffness K_f per unit area with settlement extending over a plate section of radial length L and having a maximum displacement amplitude S at the plate edge is analyzed by considering a unit width radial strip of length L, and end displacement S as shown in figure (1). The uniform beam model representing the deformed strip is assumed to be of thickness t, cross-sectional area flexural rigidity EI, unit width, resting on elastic foundation of stiffness K_f and subjected to a uniform liquid pressure P. At the interior (breakover) end point (e.g, at $x=0$) the beam vertical deflection y and bending moment M are assumed, as was done by [Malhotra and Veletsos,1994], to be zero. And at the connecting end to the shell, the beam is assumed to be elastically constrained against both rotation and axial displacement by a torsional and a translational linear springs of stiffnesses K_r and K_t, respectively. These end springs are assumed to be induced by a linearly elastic and infinitely long cylindrical shell subjected at its base, due to hydrostatic loading, to an axisymmetric bending moment M_a and transverse shearing force N_a where M_a, N_a and the spring coefficients K_r and K_t, are given by, [Timoshinko and Woinowsky-Krieger, 1984],

$$K_r = \frac{E.t_s^2.(t_s/R)^{1/2}}{2[3(1-\mu^2)]^{3/4}} \tag{1-a}$$

$$K_t = \frac{E(t_s/R)^{3/2}}{[3(1-\mu^2)]^{3/2}} \tag{1-b}$$

$$M_a = (1-\frac{1}{\beta.h})\frac{\gamma.R,h.t_s}{\sqrt{12(1-\mu^2)}} \tag{1-c}$$

$$N_a = \frac{\gamma.R.h.t_s}{\sqrt{12(1-\mu^2)}}(2\beta-\frac{1}{R}) \tag{1-d}$$

where $\beta = (\frac{3(1-\mu^2)}{R^2.t_s^2})^{1/4}$, E is Young's modulus, t_s is the shell wall thickness, R is the tank radius, h is the tank(liquid) height, γ is the stored liquid specific weight, and μ is Poisson's ratio. Using Euler-Bernoulli beam bending theory the deflection v of the above described beam in the presence of a constant axial force N may be described by the following linear ordinary differential equation:

$$EI\frac{d^4v}{dx^4} - N\frac{d^2v}{dx^2} + K_f v = P$$

which for convenience is rewritten in the following form:

$$\frac{d^4v}{d\xi^4} - K_1\frac{d^2v}{d\xi^2} + K_2 v = q \tag{2}$$

where $\xi = x/L$, $K_1 = \frac{NL^2}{EI}$, $K_2 = \frac{K_f L^4}{EI}$ and $q = \frac{PL^4}{EI}$. Based on the above assumptions, the four boundary conditions associated with the above equation may be specified as follows:

$$\text{At } x=0: \quad v=0 \text{ and } \frac{d^2v}{d\xi^2}=0 \tag{3-a,b}$$

$$\text{At } \xi=1: \quad v=S \text{ and} \tag{3-c}$$

$$M = EI\frac{d^2v}{d\xi^2} = \begin{cases} -\frac{K_r}{L}\frac{dv}{d\xi} + M_a & for \;\; M(1) < M_y \\ -M_y & for \;\; M(1) \geq M_y \end{cases} \tag{3-d}$$

where M_y is the yielding moment at the beam -shell junction, i.e. at $\xi = 1$. In this work, the shell thickness is assumed, as in a typical tank, to be greater than (i.e., about twice) the plate thickness so that that yielding at the plate-junction, if occurs, is initiated in beam and not in shell. Therefor M_y in equation (3-d) will be taken to be the beam yield moment. In addition to the above four boundary conditions, the following relation, obtained by assuming the beam to be inextensible and has zero horizontal displacement at $\xi = 0$, will be used later on to determine the unknown axial force N in equation (2):

$$N = -K_t U + N_a = -\frac{K_r}{2}\int_0^l \left(\frac{dv}{dx} \right)^2 dx + N_a \tag{4}$$

where $U = \dfrac{1}{2l}\displaystyle\int_0^1 (\frac{dv}{d\xi})^2 d\xi$ is the axial shortening of the bent inextensible beam. model in equations (2)-(4).

3. **ANALYSIS**

The total solution $v(\xi)$ of the beam deflection model in equation (2) which is a non-homogeneous linear differential equation with constant coefficients is given by:

$$v(\xi) = v_p(\xi) + v_H(\xi) \tag{5}$$

where $v_p(\xi) = \dfrac{q}{K_2} = \dfrac{P}{K_f}$ =constant is the particular solution, and $v_H(\xi)$ is the homogenous solution which, since equation (2) has constant coefficients may be assumed in the form;

$$v_H(\xi) = Ae^{r\xi} \tag{6}$$

where r in an unknown constant. Substituting equation (6) into equation (2), after setting $q = 0$, yields the following fourth order characteristic equation for the parameter r:

$$r^4 - K_1 r^2 + K_2 = 0 . \tag{7}$$

The four roots r_i ,i=1,…,4 of equation (7) are given by:

$$r_i^2 = \frac{K_1}{2} \pm \frac{1}{2}\sqrt{K_1^2 - 4K_2} \quad , \quad i = 1,...,4 . \tag{8}$$

It is noted that the behavior of the solution $v_H(\xi)$, and thus that of $v(\xi)$, will depend on whether the characteristic roots r_i of equation (8) are real, complex, pure, imaginary, and being repeated or non- repeated. The analysis in the present work will be limited to the case for which the relation $K_1^2 < 4K_2$, which may be satisfied in many practical designs, holds.

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For such a case the four characteristic roots r_i in equation (8) become the set of two complex conjugate non-repeated pairs:

$$r_{1,2} = (a_1 \pm jb_1)^{1/2} \quad , \quad r_{3,4} = -(a_1 \pm jb_1)^{1/2} \tag{9}$$

where $a_1 = \dfrac{K_1}{2}$, and $b_1 = \dfrac{1}{2}\sqrt{4K_2 - K_1^2}$; a_1 and b_1 are real. In order to express $v_H(\xi)$ in a convenient form, the characteristic complex roots r_i in equation (9) are represented in simple rectangular form as follows: for r_1, one has;

$$r_1 = (R_1 e^{-j\theta})^{1/2} = R_1^{1/2} e^{-j\theta/2} = R_1^{1/2}(\cos(\theta/2) - j\sin(\theta/2)) = a - jb \tag{10}$$

where

$$R_1 = \sqrt{a_1^2 + b_1^2} = \left[\left(\frac{K_1}{2}\right)^2 + \frac{1}{4}\left(4K_2 - K_1\right)\right]^{1/2} = \sqrt{K_2} \tag{11-a}$$

$$\theta = \arctan\left(\frac{b_1}{a_1}\right) = \arctan\left(\frac{\sqrt{4K_2 - K_1}}{K_1}\right) \tag{11-b}$$

$$a = R_1^{1/2}\cos(\theta/2) = \left(K_2\right)^{1/4}\cos(\theta/2), \tag{11-c}$$

$$b = R_1^{1/2}\sin(\theta/2) = \left(K_2\right)^{1/4}\sin(\theta/2). \tag{11-d}$$

Following a similar procedure as above one obtains

$$r_2 = a - jb, \quad r_3 = -a + jb \text{ and } r_4 = -a - jb, \tag{12-a,b,c}$$

where a and b are real constants defined as in equations(11). Substituting for $r_i, i = 1,..,4$ from equations (11) and (12) into equation (6), using Euler's identity $e^{\pm j\phi} = \cos(\phi) \pm j\sin(\phi)$ and equation (5), one obtains the following form of the general solution for the beam deflection $v(\xi)$;

$$v(\xi) = \frac{P}{K_f} + e^{-a\xi}\left[A_1\cos(b\xi) + A_2\sin(b\xi)\right] + e^{a\xi}\left[A_3\cos(b\xi) + A_4\sin(b\xi)\right] \tag{13}$$

where $A_i, i = 1,..,4$ are constants to be determined using four boundary conditions as specified in equations (3). Substituting equation (13) and its derivatives into equations (3) and noting that the beam bending moment is given by $M(\xi) = \frac{EI}{L^2}\frac{d^2 v}{d\xi^2}$, leads to the following expressions for the constants A_i , $i = 1,..4$, :

$$A_1 = \frac{-P}{K_f} - A_3, \qquad A_2 = A_4 + \frac{b^2 - a^2}{2ab}\frac{P}{K_f}, \tag{14-a,b}$$

$$A_3 = \frac{F_1 C_4 - F_2 C_2}{\Delta}, \quad A_4 = \frac{F_2 C_1 - F_1 C_3}{\Delta} \tag{14-c,d}$$

where

$$\Delta = C_1 C_4 - C_2 C_3, \quad C_1 = (e^{2a} - 1)\cos(b), \quad C_2 = (e^{2a} + 1)\sin(b), \tag{15-a,b,c}$$

$$F_1 = e^a (S - P / K_f) + \frac{P}{K_f}\left(Cos(b) + \frac{a^2 - b^2}{2ab}\sin(b) \right), \tag{15-d}$$

and the remaining constant are, for $M(1) <$ beam end yielding moment M_y , given by:

$$C_3 = (a^2 - b^2)(e^{2a} - 1)\cos(b) - 2ab(1 + e^{2a})\sin(b) + K_e a(e^{2a} + 1)\cos(b) +$$

$$K_e b(1 - e^{2a})\sin(b), \tag{16-a}$$

$$C_4 = (a^2 - b^2)(1 + e^{2a})\sin(b) + 2ab(e^{2a} - 1)\cos(b) + K_e a(e^{2a} - 1)\sin(b) -$$

$$K_e b(e^{2a} + 1)\cos(b), \text{ and} \tag{16-b}$$

$$F_2 = \frac{P}{K_f}\left\{\left[\frac{(a^2 + b^2)^2}{2a} - \frac{a^2 + b^2}{2b}K_e\right]\sin(b) - K_e \frac{3a^2 - b^2}{2a}\cos(b)\right\} + \frac{M_s L^2}{EI}e^{2a} \tag{16-c}$$

where $K_e = \frac{K_r L}{EI}$; while for $M(1) \geq M_y$, these constants become:

$$C_3 = (a^2 - b^2)(e^{2a} - 1)\cos(b) - 2ab(1 + e^{2a})\sin(b) \tag{17-a}$$

$$C_4 = (a^2 - b^2)(e^{2a} + 1)\sin(b) + 2ab(e^{2a} - 1)\cos(b), \text{ and} \tag{17-b}$$

$$F_2 = \frac{P}{K_f}\left[\frac{(a^2 + b^2)^2}{2ab}\sin(b)\right] - \frac{L^2 M_y e^a}{EI} . \tag{17-c}$$

Note that equations (14) -(17) define a set of two closed form solutions for the coefficients $A_i, i = 1,...,4$ of the beam deflection $v(\xi)$ in equation (13). The first of these solution which corresponds to case where the bending moment $M(1)$ at the beam -shell junction does not reach the yielding value M_y is given by equations (14-16) while the other solution which corresponds to the case where $M(1) \geq M_y$ is defined by equations (14), (15) and (17). The parameters a and b in these equations are however (see equations (2) and (11)) are function of the yet unknown beam axial force N which must satisfy the relation defined by equation (4). However, using equation (13) along with either equations (14)-(16)or (14), (15) and (17) to substitutes for $\frac{dv}{d\xi}$ into equation (4) can be easily seen that it will lead to a complicated transcendental equation which may only be solved numerically for the unknown constant axial force N. To avoid this difficulty, an approximation to the unknown axial force N is obtained in this work by assuming that the profile $v(\xi)$ of edge settlement beam model in figure(1) may be approximated by the following expression:

$$v(\xi) = S \sin(\pi\xi / 2) \ . \tag{18}$$

Substituting for the derivative of equation (18) into equation (4) and carrying out the necessary integration leads to the following approximation to the axial force N:

$$N = \frac{-\pi^2 S^2 K_r}{16L} + N_a \tag{19}$$

Finally, using equation (13) one obtains the following expression for the bending moment distribution $M(\xi)$ along the beam ;:

$$M(\xi) = \frac{EI}{L^2}\frac{d^2v}{d\xi^2} = \frac{EI}{L^2} e^{-a\xi} \Big((a^2 - b^2) \big[(A_1 + A_3 e^{2a\xi}) \cos(b\xi) + (A_2 + A_4 e^{2a\xi}) \sin(b\xi) \big] +$$

$$2ab \big[(A_1 - A_3 e^{2a\xi}) \sin(b\xi) + (A_4 e^{2a\xi} - A_2) \cos(b\xi) \big] \Big\} \ . \tag{20}$$

The results for the effects of various system parameters on the deflection and bending moment distributions along the present beam model, obtained using, respectively, equations (13) and (20) are presented and discussed in the next section.

4-RESULTS AND DISCUSION

The behavior of deflection configurations $v(\xi)$, and associated bending moment $M(\xi)$, given, respectively by equations (13) and (20), of the beam model shown in figure(1), was examined for various selected values of the following system parameters: edge settlement

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radial length L, maximum edge settlement amplitude S , tank (liquid level) height h, plate thickness t and foundation elastic stiffness K_f. Representative examples of the obtained results are shown in figures (3)-(8) . The solutions displayed in all of these figures are for a water filled mild steel tank for which $E = 2.06 \times 10^{11}$ N/m^2, yield stress $\sigma_y = 2.8 \times 10^8\ N/m^2$ N/m^2 Poisson's ratio $\mu = 0.3$, radius $R = 40\,m$, shell thickness $t_s = 15\,mm$, and specific weight of stored liquid $\gamma = 9820 N/m^3$. Since the beam deflection $v(\xi)$, (see figure(1)), is considered to be positive when downward, the displayed deflection variable used in figures (3)-(8) was, for visualization convenience, $w(\xi) = -v(\xi)$ instead of $v(\xi)$. The results presented in these figures also display for each of the considered cases the behavior of the dimensionless bending moment ratio $M(\xi)/M_y$, where M_y is, as indicated in section (1), the beam yield moment, which for the unit width beam under consideration is given by:

$$M_y = \frac{\sigma_y t^2}{12} \tag{21}$$

The procedure used in this work for determining if an edge settlement with a maximum amplitude S is, for given system parameters, below or above the allowable limit is to examine the corresponding plot of the ratio $M(\xi)/M_y\ vs.\xi$. If at any one or more of the beam interior points,(i.e $\xi < 1$), this ratio is found to be $\geq |1|$ then it is assumed that yielding has occurred at the interior point(s) of the beam and the corresponding edge settlement amplitude S_0 is above the allowable limit. On the other hand, the maximum edge settlement amplitude S is considered to be below the allowable limit when $|\ M(\xi\ )/M_y\ | < |1|$ for all $\xi < 1$ even when yielding takes place at the beam-shell junction (at $\xi = 1$), (i.e even when $M(1) = M_y$ .This is justified by the fact that for a typical tank the plate-shell junction is usually designed so that yielding, if it occurs, is initiated within the beam. Note that the above settlement evaluation procedure assumes yielding at a beam section to take place when the maximum bending stress at that section exceeds the beam (e.g. plate) material yield stress σ_y and thus ignores the effect of the normal stress induced by the string action of the axial force N. This is justified by the fact that the normal stress due to N is usually quite small (i.e $\leq 5\%$ σ_y), [Malhotra and Veletsos, 1994], so that disregarding this normal stress is not expected to have a significant effect on the obtained results. As noted in section (1) the API Standard 653 recommends evaluating the allowable edge settlement limit using the general relation:

$$S/L \leq 0.0308, \quad (S \text{ and } L \text{ have same units})$$

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which does not show how the various system parameter may affect this limit. Based on the example results presented in figures (3)-(8) and the above discussion the following remarks may be made:

1. The system parameters, namely the plate thickness and elastic foundation stiffness coefficient, have a significant effect on the edge settlement allowable limit, deflection configuration and associated moment distribution .

2. The effect of the tank height (i.e liquid pressure) on the edge settlement allowable limit, deflection configuration and moment distribution becomes significant only when the tank foundation is relatively soft or is highly rigid.

3. The evaluation of the permissible edge settlement limit using the above API Standard 653 relation is, depending on system parameters, in many cases fairly conservative. For example, from figure (3) it can be seen that for a plate thickness $t = 7mm$, which is within the range 6.35mm-9.525mm specified in the above API relation, settlement radial length $L = 1m$, and foundation stiffness $K_f = 3 \times (10)^7 N/m^3$, the allowable edge settlement limit is $S \approx 36mm$ which is about 20% more than the limit $S = 30.8mm$ allowed by the above API Standard 653.

4. For a soft foundation, the profile of the settlement deflection, as well as that of the corresponding bending moment diagram, for the loaded tank tends to have only a single maximum located at point within the beam. As the foundation stiffness is decreased, the amplitude of the maximum deflection (as well as that of the maximum bending) tends to increase, and its location tends to move away from the plate -shell junction towards the center of the beam (i.e. center of the edge settlement). For a relatively low foundation stiffness the amplitude of this maximum deflection becomes greater than the allowable limit, i.e. the corresponding bending moment reaches the yield limit due to excessive deformation, even when the amplitude of the settlement at the plate-shell junction is kept well below the allowable limit.

5. For a relatively rigid foundation, i.e. as the foundation stiffness is increased to relatively high values, the deflection profile, as well as the moment profile, depending on system parameters, tends to exhibit local maximum(s) e.g. the edge displacements tends to develop localized bulge(s),. The induced bulge(s) becomes highly localized, its maximum amplitude increases, and tends to move closer to the plate -shell junction as the foundation becomes more rigid. Such bulges, are usually associated with highly localized stresses, which may cause serious problems if their maximum amplitude exceeds a certain limit [Yoshida and Tomiya, 1999]

6. For an intermediate value of foundation stiffness, i.e. for a moderately rigid foundation, the maximum of the edge settlement deflection profile of the loaded tank tends to occur at

the plate-shell junction, i.e. is equal to the unloaded tank edge settlement S , and the deflection profile tends to exhibit a relatively low maximum amplitude internal bulges which tend to disappear (e.g. tend to deform elastically and make full contact with the foundation) as the liquid pressure (i.e. tank height) is increased.. Based on these results, one may conclude that a moderately rigid foundation is the best practical choice for minimizing the possibility of internal yielding in the edge settlement of a large storage tank..

Finally it is noted that the present analysis assumes that the deformation of beam at interior point remains elastic, i.e. the beam does not develop plastic hinges at interior points. That is, the solutions presented in this work are valid provided that $\left| M(\xi)/M_y \right| \leq 1$, for $\xi < 1$. Therefore the curves in figures (3)-(8) for which $\left| M(\xi)/M_y \right| > 1$, for some $\xi < 1$ are not accurate and are used only to indicate that the corresponding maximum settlement amplitude has exceeded the allowable limit. The present solution may however by used with an iterative procedure to find, for given system parameters the allowable edge settlement maximum amplitude by finding the value of S that leads to the initiation of yield at an interior point of the beam.

5. CONCLUSION

The use of simple bending theory of a linear beam to obtain approximate closed form solution for the deformation and bending moment distribution in the bottom plate edge settlement area of a storage tank has advantages in simplicity and significantly reduced computational efforts when compared to approximate analytical and numerical methods using plate theory. Despite its limitations such as, i.e its inability to account for circumferential membrane action, the present simple closed form solution may be used to gain insight into the effects of various system parameters on the allowable edge settlement maximum amplitude. The present solution has considered only the case where the roots of the characteristic equation (8) are two complex conjugate pairs. The consideration of other possible types of these roots can a be useful extension of the present work. The results presented in this work indicate that evaluation of the edge settlement allowable maximum amplitude using the API Standard 653 is in general fairly conservative. They also indicate that the plate thickness and foundation stiffness have a significant effect on the allowable edge settlement maximum amplitude.

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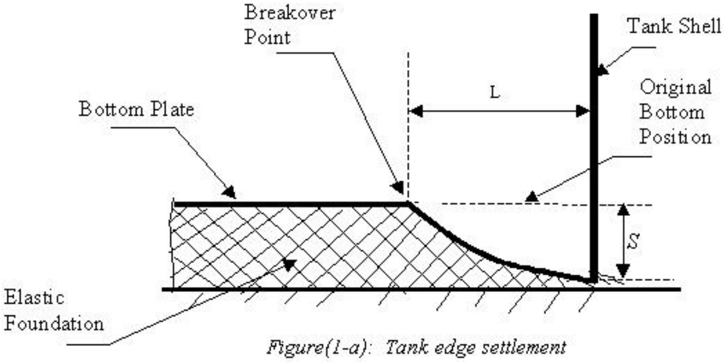

Figure(1-a): Tank edge settlement

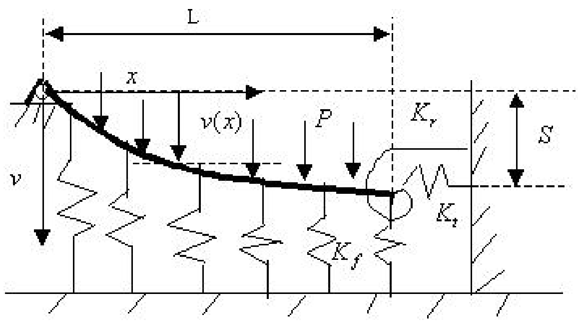

Figure (1-b): Edge settlement beam model.

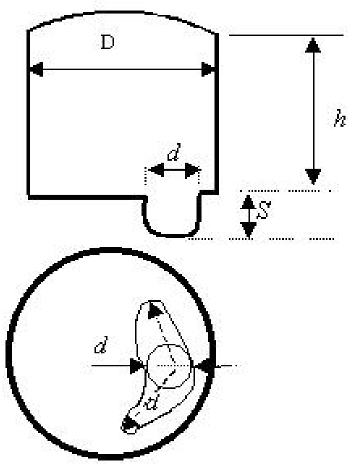

Figure 2. Ring type localized settlement.

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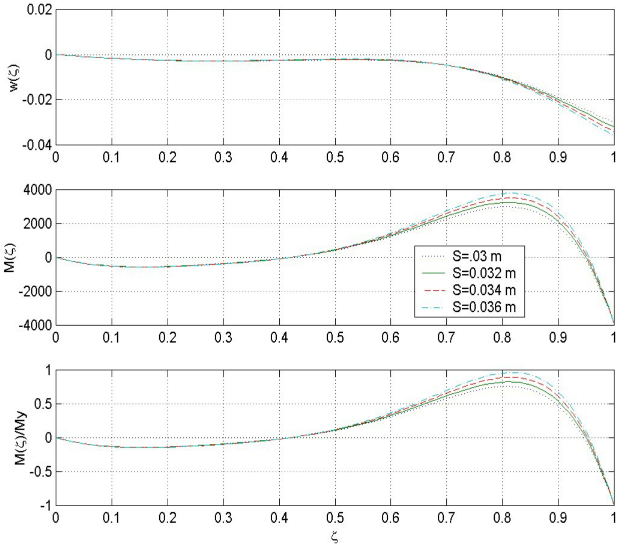

Figure 3**:** Effect of edge settlement S for $K_f = 3 \times 10^7\ N/m^3$, *t=7mm*,and *h=10 m.*

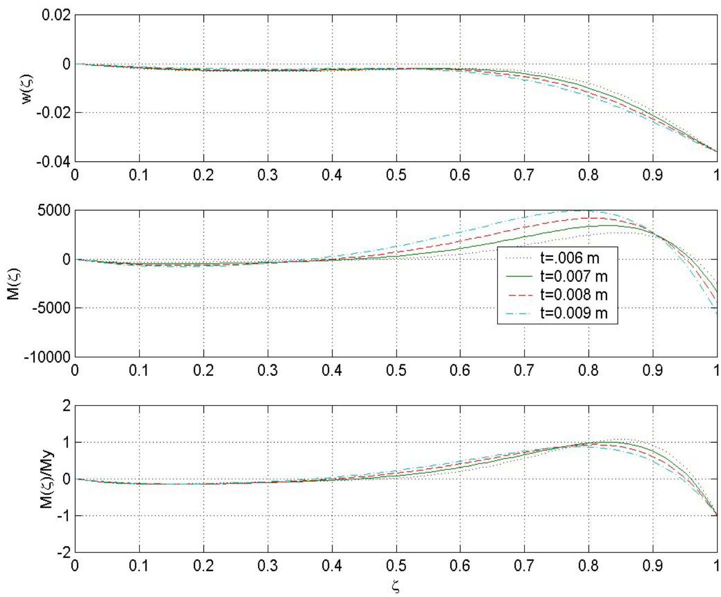

Figure 4**:** Effect of plate thickness t for $K_f = 3 \times 10^7\ N/m^3$, $S = 0.036\,m$ and *h=10 m*

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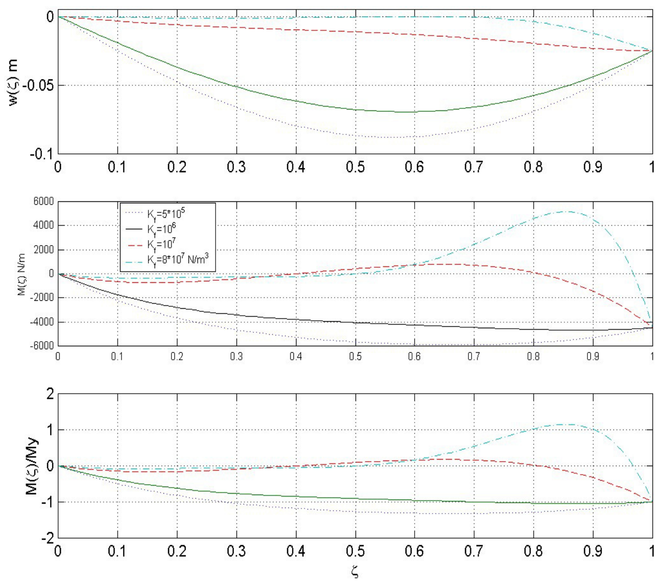

Figure 5**:** Effect of foundation stiffness K_f for $S = 0.025\ m$, *t= 7mm* and *h=10 m*

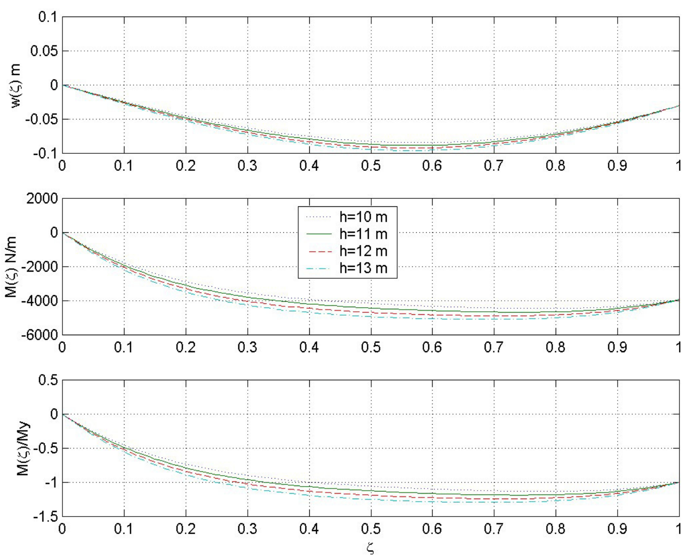

Figure 6**:** Effect of Liquid level *h* for $S = 0.03\,m$, $K_f = 7\times10^5\ N/m^3$ and *t=7.5* mm.

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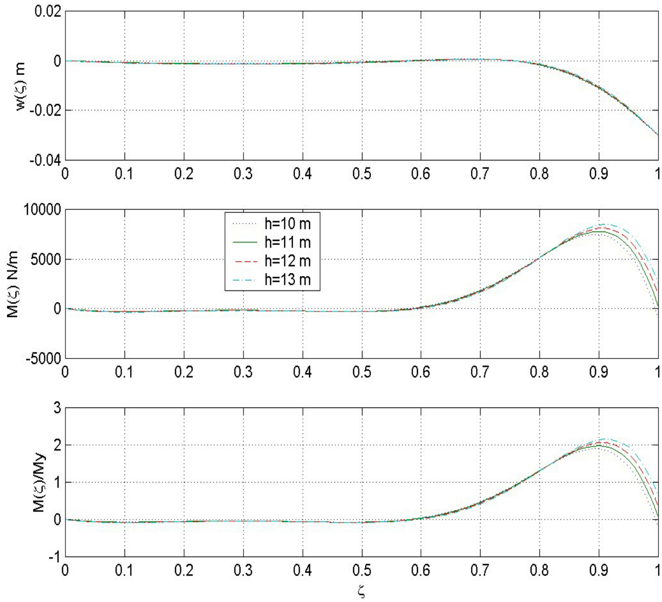

Figure 7**:** Effect of Liquid level *h* for $S = 0.03\,m$, $K_f = 10^8\ N/m^3$ and *t=7.5* mm.

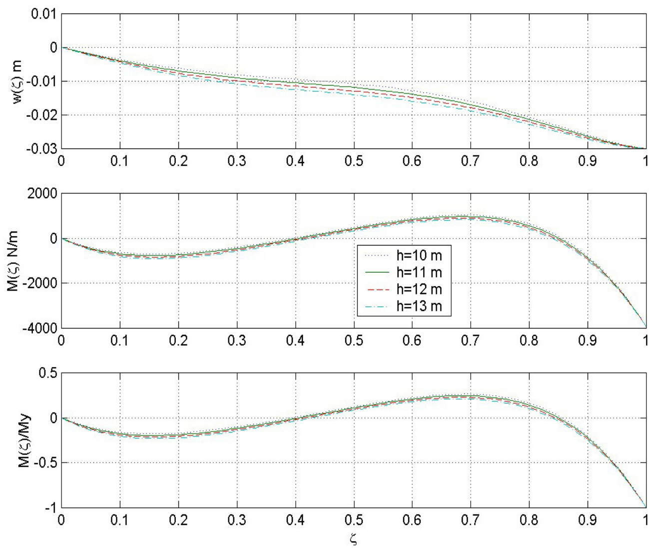

Figure 8**:** Effect of Liquid level *h* for $S = 0.03\,m$, $K_f = 10^7\ N/m^3$ and *t=7.5* m

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