The Impact of Pulse Shaping on Non-coherent DLL Tracking in DS/SS Systems

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***Abstract*— This paper analyses the impact of pulse shaping on the performance of the non-coherent delay lock loop used in code tracking of direct sequence spread spectrum systems. The effect of non-rectangular despreading is demonstrated in the presence of multiple access interference. Several families of pulses are considered. A method is highlighted that demonstrates the relative performance of a chosen chip waveform in terms of minimum tracking error variance, compared to others. The results show that some non-rectangular pulse shapes outperform that of the rectangular. The method can be extended to any given chip waveform used in current or future proposed spread spectrum systems.**

***Index Terms*—Chip waveforms, delay-locked loop, direct sequence spread spectrum, PN code tracking, synchronization.**

I. INTRODUCTION

EFFICIENCY of any communication system is highly dependent on the capability of the receiver to maintain synchronization with the incoming signal. For direct sequence spread spectrum systems (DS/SS), in addition to carrier phase and frequency, the receiver must continuously maintain synchronization between the received and the locally generated pseudo-noise (PN) codes. PN code synchronization of the receiver to the received spread spectrum signal is typically achieved in two distinct phases: code acquisition and code tracking [2]. During acquisition the receiver obtains the relative delay between the received signal and the locally generated code to within a code chip interval. The delay error between the input and local sequence is reduced to less than one chip interval. Further adjustment to remove the slight maladjustment left and fine tune the delay to as close to zero as possible is done during tracking.

Tracking is typically implemented by means of the classical delay lock loop (DLL) [1]-[2]. The structure of the DLL is derived from the principle of finding the zero of the first derivative of the correlation between the received signal and a locally generated reference. The performance of the DLL depends on the slope of its S-curve [2] evaluated at the origin, when the delay error is zero. The S-curve on the other hand is dependent on the locally generated spreading codes since it is obtained as the difference of the early and late correlation functions. In addition to the correlation properties of these spreading codes, an important factor that has an impact on the performance of the tracking loop is the actual chip waveform.

Most of earlier research on the DLL focus on the structure of the DLL [3]-[5] which change the S-curve, while the effect of non-rectangular and unmatched despreading chip waveforms in the transmitter and receiver of the coherent delay lock loop has been demonstrated in [6]-[7] to reduce the tracking error variance. The effect of chip waveform on the synchronization has also been undertaken in [8]. This paper outlines a method for determining the effect of non-rectangular despreading on the non-coherent delay lock loop (NCDLL), illustrated in Fig. 1, in terms of its S-curve and the interference properties of the chip waveform. The NCDLL is more common in practical systems as it is relatively insensitive to data modulation and does not presume reliable carrier tracking prior to PN code acquisition as do the coherent delay lock loops.

The rest of the paper is organized as follows. In Section II, the system model used is presented. In Section III, the performance analysis of the NCDLL is discussed using the frequency domain formulation for the slope of the S-curve and the interference properties of the chip waveform. Section IV discusses the comparative results with regards the variance. The final conclusions are presented in Section V.

II. SYSTEM MODEL

The received baseband CDMA is expressed as [2]:

$$r(t) = \sqrt{2P}c(t-\tau_o)d(t-\tau_o)\cos\left(\omega_o t + \phi(t)\right) + n(t) \quad (1)$$

where P is the signal power, τ_o is the propagation delay, $\phi(t)$ is the random phase of the carrier, $d(t)$ is the binary data sequence, $n(t)$ is additive white noise processes with two-sided power spectral density equal to $N_o/2$ and $c(t)$ is the spreading waveform given by:

$$c(t) = \sum_{n=-\infty}^{\infty} c_n h(t-nT_c) \quad (2)$$

with c_n representing a binary PN code sequence of ± 1 and

$h(t)$ is the chip waveform defined over a chip interval T_c.

The non-coherent delay lock loop (Fig. 1) is based on the early-late scheme where the received signal is correlated with two locally generated, mutually delayed replicas of the PN code, the early $c(t-\bar{\tau}_o+\Delta)$ and late $c(t-\bar{\tau}_o-\Delta)$ local codes. The output waveforms are envelope-detected, squared and the filtered difference is used to drive the voltage controlled oscillator (VCO). The VCO is controlled by the timing error measurement after filtering by the loop filter. The sum of the frequency changes is used by the VCO to modify the timing of each chip. Considering the output of the discriminator in a noiseless environment, and assuming the ideal case of no arm gain imbalance, the difference of the early and late quadrature detector output is [2]:

$$z_\Delta(\tau) = R^2(\tau-\Delta) - R^2(\tau+\Delta) \qquad (3)$$

where $R(\tau)$ is the sequence autocorrelation given by:

$$R(\tau) = \frac{1}{NT_c}\int_0^{NT_c} c(t)c(t+\tau)dt \qquad (4)$$

It is seen that the output of the discriminator $z_\Delta(\tau)$ is dependent on $R(\tau)$, the sequence autocorrelation. We assume random codes to remove the effect of the correlation properties of the spreading codes and focus on the effect of the actual chip waveform $h(t)$. The discriminator S-curve of the NCDLL as a normalized function of the timing error is written from (3) as:

$$G(\tau/T_c) = R^2\left(\frac{\tau-\Delta}{T_c}\right) - R^2\left(\frac{\tau+\Delta}{T_c}\right) \qquad (5)$$

The above (5) can be rewritten as:

$$G(\varepsilon) = R^2(\varepsilon-\delta) - R^2(\varepsilon+\delta) \qquad (6)$$

where $\tau = \tau_o - \bar{\tau}_o$ is the delay error, $\varepsilon = \tau/T_c$ is the normalized delay error and $\delta = \Delta/T_c$ is the normalized time difference between the early and late branches $(0 < \Delta \le T_c)$. For constant-envelope phase modulation of interest, the complex autocorrelation is real [2], $R^2(\varepsilon \pm \delta)$ is therefore replaced by $4|R(\varepsilon \pm \delta)|^2$ and (6) becomes:

$$G(\varepsilon) = 4[|R(\varepsilon-\delta)|^2 - |R(\varepsilon+\delta)|^2] \qquad (7)$$

III. PERFORMANCE ANALYSIS

Performance of the delay lock loop is measured by the root mean square (RMS) tracking error. Linear loop analysis of the DLL is used to obtain the RMS tracking error. At high SNR, the tracking error will usually be small enough such that the error measurement can be taken as a linear function of the relative timing error. The results obtained using linear loop analyses are considered to be good approximations of the actual performance at high SNR. Using linear loop analysis, the tracking error variance of the NCDLL is approximated as [9]:

$$Var(\tau/T_c) \approx \frac{2V_o^2}{N^2E_c^2\kappa^2} \qquad (8)$$

where N is the number of chips per bit, E_c is the energy per chip, κ is the slope of the S-curve evaluated at $\tau = 0$ and V_o is the interference variance given by [10]:

$$V_o = N_o + I_o \qquad (9)$$

where N_o is the variance of the additive white noise processes and I_o is the variance of the interference due to other users. The additive white noise variance N_o is neglected with focus on the interference caused by other users since CDMA is user interference limited. The tracking error variance from (8) becomes:

$$Var(\tau/T_c) \approx \frac{2I_o^2}{N^2E_c^2\kappa^2} \qquad (10)$$

The other user interference variance I_o is given by [10]:

$$I_o = \sum_{j\neq k} E_c(j)\int_{-\infty}^{+\infty}|H(f)|^4\,df\,/T_c \qquad (11)$$

where $H(f)$, the wave shaping filter, is the Fourier transform of the chip waveform $h(t)$. We assume perfect power control and normalize the chip energy to be equal for all the pulse shapes, the variance of the timing error of one pulse shape compared to another can be expressed from (10) as:

$$Var(\tau/T_c) = F\left(\frac{I_o^2}{\kappa^2}\right) \qquad (12)$$

The tracking performance is directly related to the interference variance and inversely related to the slope at $\tau = 0$. The slope κ for the NCDLL is evaluated from (7) as:

$$\kappa = \left.\frac{dG(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = 8[R'(\varepsilon-\delta)R(\varepsilon-\delta) - R'(\varepsilon+\delta)R(\varepsilon+\delta)]$$

$$\kappa = 8[R'(-\delta)R(-\delta) - R'(\delta)R(\delta)] \qquad (13)$$

For the design example of $\delta = 0.5$:

$$\kappa = 8\left[R'\left(-\frac{1}{2}\right)R\left(-\frac{1}{2}\right) - R'\left(\frac{1}{2}\right)R\left(\frac{1}{2}\right)\right] \qquad (14)$$

We resort to frequency-domain formulation to simplify the problem at hand. For long PN codes modeled as random signature sequences, the autocorrelation function reduces to:

$$R(\delta) = \int_{-\infty}^{+\infty} |H(f)|^2 \cos(2\pi f\delta)df \qquad (15)$$

$R(\delta)$ is an even function, therefore:

$$R\left(\frac{1}{2}\right) = R\left(-\frac{1}{2}\right) = \int_{-\infty}^{+\infty} |H(f)|^2 \cos(\pi f)df \qquad (16)$$

Similarly $R'(\delta)$ is an odd function, therefore:

$$R'\left(-\frac{1}{2}\right) = -R'\left(\frac{1}{2}\right) = \int_{-\infty}^{+\infty} 2\pi f |H(f)|^2 \sin(\pi f)df \qquad (17)$$

Substituting (16) and (17) into (14) we get:

$$\kappa = 16\left[\int_{-\infty}^{+\infty} |H(f)|^2 \cos(\pi f)df\right]\left[\int_{-\infty}^{+\infty} 2\pi f |H(f)|^2 \sin(\pi f)df\right] \qquad (18)$$

The performance comparison of one pulse shape to the other can directly be estimated by evaluating the interference variance I_o and the slope κ of the S-curve. Table I shows the normalized tracking error variance of several pulse shapes evaluated using this approach. The following pulse shapes are considered:

Rectangular $h(t) = \begin{cases} 1, & 0 \le t < T_c \\ 0, & otherwise \end{cases};$

Triangular $h(t) = \begin{cases} 1 - \left|\frac{2t}{T_c} - 1\right|, & 0 \le t < T_c \\ 0, & otherwise \end{cases};$

Half Sine $h(t) = \begin{cases} \sin\left(\frac{\pi t}{T_c}\right), & 0 \le t < T_c \\ 0, & otherwise \end{cases};$

Raised Cosine $h(t) = \begin{cases} \frac{1}{2}\left(1 - \cos\frac{2\pi t}{T_c}\right), & 0 \le t < T_c \\ 0, & otherwise \end{cases};$

Hamming $h(t) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi t}{T_c}\right), & 0 \le t < T_c \\ 0, & otherwise \end{cases};$

IV. RESULTS AND DISCUSSION

For the design example of $\delta = 0.5$ the S-curve is illustrated in Fig. 2. The slopes at the origin (Fig. 3) of different pulse shapes is accurately evaluated using the derived expression for κ in (18). The respective slopes and the normalized timing error variance are shown in Table I. It is seen that for the design choice of $\delta = 0.5$, the least slope belongs to the Hanning pulse shape with the Rectangular and Half-Sine having equal and largest slopes. The difference in the performance of both arises from the better interference performance of the Half-Sine over the Rectangular pulse shape. The performance in the presence of noise for five users is shown in Fig. 4. As CDMA is interference limited, the tracking error variance can be observed to reach a floor at 5dB. The performance in the presence of multiple users at a fixed noise level $(E_c / N_o = -10dB)$ is illustrated in Fig. 5. In future work, an optimization problem similar to the one undertaken for coherent delay locked loop in [11] can be formulated around the objective function F to minimize the tracking error variance, thereby optimizing the performance of the NCDLL.

TABLE I
NORMALIZED TRACKING ERROR VARIANCE

Pulse Shape	I_o	κ	I_o^2 / κ^2
Raised Cosine	0.24055	0.889	0.07322
Hamming	0.26368	1.357	0.03776
Triangular	0.26964	1.500	0.03231
Rectangular	0.33333	2.000	0.02778
Half-Sine	0.29332	2.000	0.02151

V. CONCLUSION

Tracking performance of several chip waveforms have been presented showing the effect of non-rectangular despreading on the non-coherent delay lock loop. A method was proposed that brings to light the effect of these pulse shapes by evaluating their slopes at the origin along with their interference performances for given early-late DLL spacing. The results show that some non-rectangular pulse shapes outperform that of the rectangular. The method can be extended to other pulses shapes used in current and future DS/SS systems.

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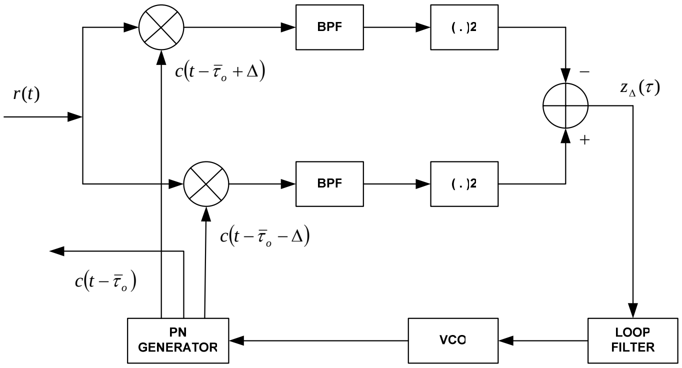

Fig. 1: Noncoherent delay lock loop (NC-DLL)

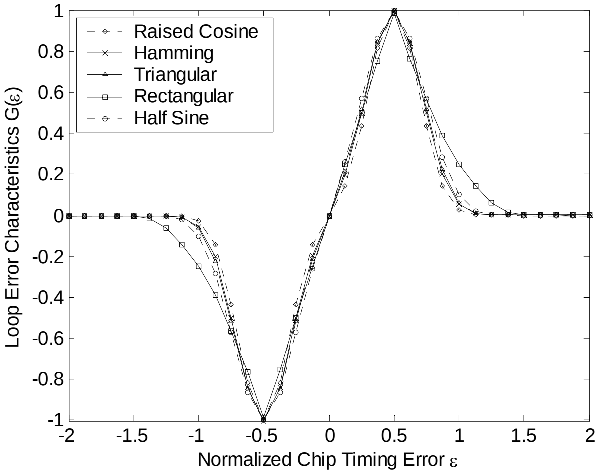

Fig. 2: S-curve for NCDLL for various pulse shapes $(\delta = 0.5)$

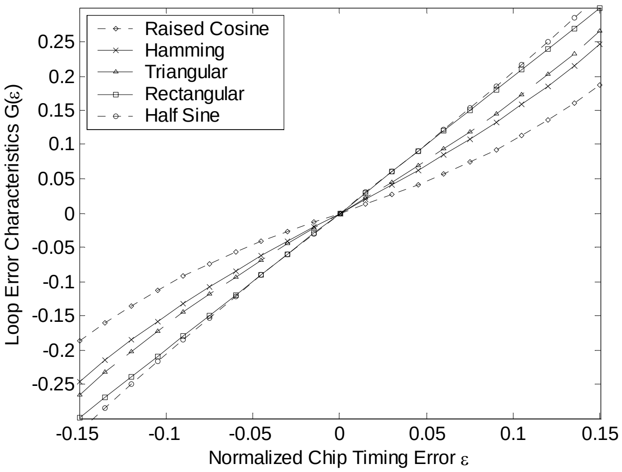

Fig. 3: S-curve for NCDLL for various pulse shapes at the origin $(\delta = 0.5)$

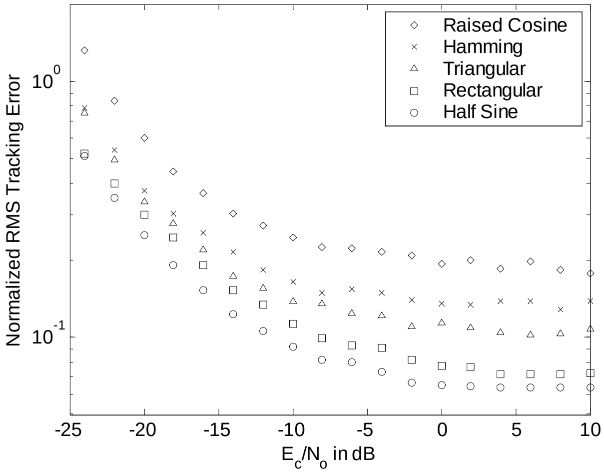

Fig. 4: Normalized RMS Tracking Error for 5 users

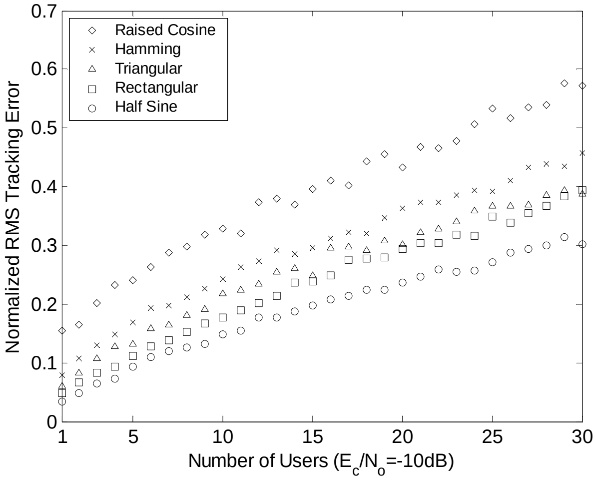

Fig. 5: Normalized RMS Tracking Error for Multiple Users

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