A Threshold-Based Generalized Selection Combining Scheme in Nakagami Fading Channels

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ABSTRACT - The severity of fading on mobile communication channels calls for the combining of multiple diversity sources to achieve acceptable error rate performance. Traditional approaches perform the combining of the different diversity sources using either: the Conventional Selective diversity combining (CSC), Equal-Gain combining (EGC), or Maximal-Ratio combining (MRC) Schemes. CSC and MRC are the two extremes of compromise between performance quality and complexity. Some researches have proposed a generalized selection combining scheme (GSC) that combines the best M branches out of the L available diversity resources (M \leq L). In this paper we analyze a generalized selection combining scheme based on a threshold criterion rather than a fixed-size subset of the best channels. In this scheme, only those diversity branches whose energy levels are above a specified threshold are combined. Closed-form analytical solutions for the BER performances of this scheme over Nakagami Fading Channels are derived. We also discuss the merits of this scheme over GSC.

Index Terms — Selection Diversity, Maximal ratio, Threshold, Nakagami,

I. INTRODUCTION

Diversity techniques are based on the notion that errors occur in reception when the channel is in deep fade -a phenomenon more pronounced in mobile communication channels. Therefore, if the receiver is supplied with several replicas, say L, of the same information signal transmitted over independently fading channels, the probability that all the L independently fading replicas fade below a critical value is p^L (where p is the probability that any one signal will fade below the critical value). The bit error rate (BER) of the system is thus improved without increasing the transmitted power.

A crucial issue in diversity system however, is how to combine the available diversity branches to

achieve optimum performance within acceptable The three traditional combiners are: complexity. Conventional Selective combiner (CSC) which selects the signal from that diversity branch with the largest instantaneous SNR; Equal-Gain combiner (EGC) which coherently combines all L diversity branches weighting each with equal gain; and Maximal-Ratio combiner (MRC) which coherently combines all L diversity branches but weighs each with the respective gain of the branch. CSC gives the most inferior BER performance, MRC gives the best and the optimum performance, and EGC has a performance quality in between these two [1].

CSC and MRC are the two extremes of complexityquality trade off. CSC on one end is extremely simple, but the contributions from the other branches are wasted, irrespective of their strength. MRC on the other end combines the outcome of all branches regardless of how poor some of them may be, resulting in the best possible combining performance gain. The cost for this performance is the heavy processing complexity and extremely complicated circuitry required for phase coherence and amplitude estimation on each branch. It should be noted that the lower the received SNR the less efficient the phase and amplitude estimation circuit will be; therefore presence of accurate Channel State Information, often presumed in analytical procedures, will not be valid for such branches. Also, processing power and other resources dissipated into combining very weak branches are more costly for wireless and high order diversity systems than the marginal contribution such branches make to the total combined SNR. MRC is known to be optimal in the BER performance sense. However, when both the BER performance and complexity should be considered, as is the case in mobile systems, then a scheme that has good balance between BER performance and complexity is required. Mobile units using high order receiver diversity can rarely afford MRC because of power limitations. In addressing this problem, [2] proposes a sub-optimal

scheme that retains most of the advantages of the MRC scheme, and has been widely studied [3-5].

The scheme proposed in [2] combines a fixed number of branches, say M, that have the largest instantaneous SNR out of the L available branches. As $1 \le M \le L$, the scheme was called a generalized diversity selection combining (GSC) scheme; M=1 corresponds to CSC, while M=L corresponds to MRC. Here we refer to that scheme as M-GSC (i.e. M-based GSC).

Combining a fixed number of branches, however, has obvious shortcomings. At times of deep fade, some of the *M* selected branches will still have marginal contribution to the total combined energy and they could be discarded to simplify processing. At other times when the channels are good, some of the *L-M* discarded branches, although inferior to the *M* selected branches, have significant contribution, and combining them will then be advantageous. An M-GSC scheme cannot make any advantage of such improvements in channel conditions since *M* is fixed, and the remaining *L-M* branches must be discarded regardless of their energy levels. Furthermore, we show later that M-GSC incurs a major processing complexity increase in ordering the branches' SNRs.

The authors have proposed a Threshold-Based Generalized Selection combining (T-GSC) scheme that overcomes the aforementioned shortcomings [6]. The T-GSC scheme combines all the strong diversity branches available at any time instant, discarding only the weak ones. The proposed scheme is more suitable for mobile channels, which frequently and intermittently improve and degrade during usage, and where power resource savings are critically important must be made without compromising performance quality. The BER performance of T-GSC was simulated over Nakagami fading compared with environment, and M-GSC. Apparently, the system in [6] has attracted other researchers [7, 8]. In [8], Simon and Alouini analyzed the system for Rayleigh fading channels with a slight modification to the threshold definition.

In this work, we extend our work in [6] by providing a detailed analysis of the BER performance of T-GSC over Nakagami fading channels. The rest of the paper is organized as follows. In Section 2 we review the combining rules of T-GSC. Detailed analysis of the BER performance of the system is furnished in Section 3. Some results are presented and discussed in Section 4. A comparison between T-GSC and M-GSC is provided in Section 5. Main conclusions of this work are finally summarized in Section 6.

2. PROPOSED T-GSC SCHEME

The proposed scheme combines diversity branches based on a criterion which we call "branch relative strength" (BRS). The BRS is the ratio of the SNR of each branch to the SNR of the best branch at the same instant of time [6].

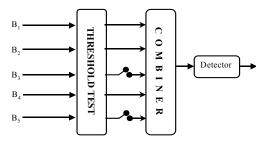
$$BRS_i = \frac{\gamma_i}{\gamma_{\text{max}}} \qquad i = 1, 2 ... L \tag{1}$$

where $\gamma_{\text{max}} = \max \{ \gamma_1, \gamma_2, ..., \gamma_L \}$ is the maximum SNR

received at each time instant, and γ is the SNR in the i^{th} branch, i = I, 2...L. The combining rule is then stated as follows: If the BRS_i is larger than or equal to a specified threshold T (where $0 \le T \le 1$), the branch is combined; otherwise it is discarded. Equivalently, one could compare each γ to γ_{th} where $\gamma_{th} = T \cdot \gamma_{max}$.

The T-GSC scheme thus combines only the significant branches at any time, discarding the weak ones whose energy are below the threshold value. **Processing** resources, notably power, are therefore not dissipated in combining very weak branches that have no appreciable contribution to the total combined SNR -extending battery life for mobile units. Significant branches for different mobile situations can be selected by proper choice of T suitable for the fading environment and mobile scenario concerned. A novel advantage here is that if all the branches' SNRs meet the specified threshold (i.e. they are all strong), they are all combined and no useful information is 'thrown off'. It is then obvious that M, the number of branches combined at each time instant, will not be fixed but varies in correspondence to the channel fading level. Performance gains due to improvements in channel conditions will thus be reflected in the system performance all the time. The scheme is as illustrated in Fig. 1 for L = 5. In the figure, only branches 1, 2 and 4 are above threshold, and are therefore combined.

Next we derive the bit error rate performance for the above scheme. Nakagami m-fading [9] is assumed for the channel fading model. The m-distribution proposed by Nakagami [10] is a general fading statistics from which other fading statistics approximating the mobile communication environments can be modeled by setting the Nakagami parameter m to an appropriate value. We recall that m = 1 corresponds to Rayleigh, and as m is increased, the fading becomes less severe. Binary PSK signal is used throughout the analysis.



B₁, B₂ and B₄ above threshold. B₃ and B₅ below threshold

Figure 1: Block Diagram of the T-GSC scheme

3. BER PERFORMANCE – ANALYTICAL DERIVATION

Given L available diversity branches at the receiver, each branch having instantaneous SNR per bit, $\chi = \alpha^2 E_b$

$$\frac{\alpha^2 E_b}{N_0}$$
, $l = 1,...,L$, where α is the fading coefficient,

and $\frac{E_b}{N_0}$ is the transmitted bit energy-to-Gaussian

noise spectral density ratio. The T-GSC receiver searches for the branch with the maximum SNR, $\gamma_{\rm max}$, and chooses a threshold based on it.

In contrast to M-GSC in which a fixed number of diversity branches M is combined, the number of diversity branches to be combined in the T-GSC scheme is a random variable l, $l \in \{1,L\}$. Using the theorem on total probability [11], the average BER for T-GSC can be derived as a weighted sum of the average BER for the M-GSC corresponding to M=1,2,...,L. Hence,

$$P_{b,T}(E) = \sum_{l=1}^{L} \Pr(M = l). \ P_{b,M}(E \mid M = l)$$
 (2)

where $P_{b,M}(E \mid M = l)$ is the average BER for the M-GSC given that the number of branches combined, M, is equal to the variable l.

Pr(M=l) denotes the probability of the event that l branches have their SNRs equal to or exceed γ_{th} and are combined, while L-l branches have their SNRs lower than γ_{th} and are thus discarded. The probability of this event is given by [11]:

$$Pr(M = l) = \frac{\binom{L-1}{l-1}}{\left[\int_{0}^{\gamma_{\text{max}}} p_{\gamma}(\gamma) d\gamma\right]^{L}}$$

$$.T \int_{0}^{\gamma_{\text{max}}} p_{\gamma}(\gamma) d\gamma \left[\int_{\gamma_{th}}^{\gamma_{\text{max}}} p_{\gamma}(\gamma) d\gamma\right]^{l-1}$$

$$.\left[\int_{0}^{\gamma_{th}} p_{\gamma}(\gamma) d\gamma\right]^{L-l}$$
(3)

For Nakagami-m branch fading coefficients, each branch's SNR, γ , is a gamma random variable with pdf given as [1]:

$$p_{\gamma}(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^{m} \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left\{-\left\{\frac{m}{\bar{\gamma}}\gamma\right\}\right\} \quad (4)$$

where the lower case letter m refers to the Nakagami

parameter, and
$$\bar{\gamma}=E[\alpha^2]\frac{E_b}{N_0}$$
 . Substitution of (4) in (3)

and making use of the reduction formula [12] in evaluating the integrals in the resulting expression, we arrive at:

$$Pr(M=l) =$$

$$\frac{\binom{L-1}{l-1}}{\left[-\exp\{(-m\beta_{\max})\}\right]\sum_{n=0}^{m-1}\frac{(m-1)!}{(m-1-n)!}(m\beta_{\max})^{m-1-n}+(m-1)!}$$

$$\cdot \left[-\exp\left\{ \left(-m\beta_{\max} \right) \right\} \sum_{k=0}^{m-1} \frac{(m-1)!}{(m-1-k)!} (m\beta_{\max})^{m-1-k} + \exp\left\{ \left(-m\beta_{th} \right) \right\} \sum_{k=0}^{m-1} \frac{(m-1)!}{(m-1-k)!} (m\beta_{th})^{m-1-k} \right]^{l-1} \times \left[-\exp\left(-m\beta_{th} \right) \right\} \sum_{k=0}^{m-1} \frac{(m-1)!}{(m-1-q)!} (m\beta_{th})^{m-1-q} + (m-1)! \right]^{l-1}$$

where
$$\beta_{\max} = \frac{\gamma_{\max}}{\overline{\gamma}}$$
 and $\beta_{th} = \frac{\gamma_{th}}{\overline{\gamma}}$. Note that the

solution in Eq. (5) for Nakagami fading is valid only for integer values of the Nakagami parameter m.

Substitution of (5) into (2) above gives the desired result for the average BER of T-GSC, $P_{b,T}(E)$, over Nakagami-m fading channels, in terms of the average BER of M-GSC, $P_{b,M}(E \mid M)$. Expressions for $P_{b,M}(E \mid M)$ over Rayleigh fading and Nakagami fading channels can be obtained from works in [13] and [14] respectively.

As an illustration of the evaluation of $P_{h,T}(E)$ using (2) and (5), we consider the case of Nakagami-m branch fading with m=1 (which is equivalent to Rayleigh fading). For this example, $P_{bM}(E \mid M = l)$ is obtained from Equation (40) in [14] after substituting $l=L_c$ as:

$$P_{b,M}(E|M=l) = \binom{L}{l} \sum_{k=0}^{L-l} \frac{(-1)^k \binom{L-l}{k}}{1+\frac{k}{l}} I_{l-l} \left(\frac{\pi}{2}; g\overline{\gamma}, \frac{g\overline{\gamma}}{1+\frac{k}{l}}\right), (6)$$

where g=1 for Binary PSK signals, $I_n(\theta;c_1,c_2)$ is

$$\frac{1}{\pi} \int_0^{\theta} \left(\frac{\sin^2 \phi}{\sin^2 \varphi + c_1} \right)^n \left(\frac{\sin^2 \phi}{\sin^2 \varphi + c_2} \right) d\varphi. \quad \text{Closed}$$

form result for this integral has been obtained in [13]. Setting m=1 in (5) and expanding the result in

binomial series leads to:
$$A = \frac{1}{2} \arctan\left(\frac{N}{D}\right) + \frac{\pi}{2} \left[1 - \operatorname{sgn} N\left(\frac{1 + \operatorname{sgn} D}{2}\right)\right]$$
(10)
$$\Pr(M = l) = \frac{\binom{L-1}{l-1} \sum_{k=0}^{l-1} \binom{l-1}{k} (-1)^{l-1-k} \exp\left\{\left(-\beta_{\max}[l-1-k(1-T)]\right)\right\} - \frac{l-1}{2} \sqrt{c(1+c)} \sin(2\theta) \text{ and } \sum_{n=0}^{L-1} \binom{L-1}{n} (-1)^{L-1-n} \exp\left\{\left(-\beta_{\max}[L-1-n]\right)\right\} - D = (1+2c)\cos(2\theta) - 1. \text{ For } c_1 \neq c_2, \text{ we have:}$$

Note from Eq. (7) that T=0, corresponding to MRC, yields

Pr(M=l)=0, l=1,2,...,L-1,

Pr(M=L)=1.

Similarly, for T=1, corresponding to CSC,

Pr(M=l)=0, l=2,3,...,L

Pr(M=1)=1

Thus verifying the upper and lower bounds on the BER for the T-GSC scheme.

Substituting (6) and (7) into (2) yields the following expression for the average BER of T-GSC:

$$\cdot \binom{L}{l} \sum_{p=0}^{L-l} \frac{(-1)^{p} \binom{L-l}{p}}{1 + \frac{p}{l}} I_{l-l} \left(\frac{\pi}{2}; \frac{-\gamma}{\gamma}, \frac{-\gamma}{1 + \frac{p}{l}} \right) \cdot (8)$$

where $I_{l-1}(\theta; c_1, c_2) = I_l(\theta; c)$ for $c_1 = c_2 = c$

given by [13]:

$$I_{l}(\theta;c) = \frac{\theta}{\pi} - \left(\frac{1 + \operatorname{sgn}(\theta - \pi)}{2} + \frac{A}{\pi}\right)$$

$$\cdot \sqrt{\frac{c}{1+c}} \sum_{i=0}^{l-1} {2i \choose i} \frac{1}{\left[4(1+c)\right]^{i}}$$

$$-\frac{2}{\pi} \sqrt{\frac{c}{1+c}} \sum_{i=0}^{l-1} \sum_{j=0}^{i-1} {2i \choose j} \frac{(-1)^{i+j}}{\left[4(1+c)\right]^{i}} \cdot \frac{\sin[(2i-2j)A]}{2i-2j},$$

$$0 \le \theta \le 2\pi \tag{9}$$

where

$$A = \frac{1}{2}\arctan\left(\frac{N}{D}\right) + \frac{\pi}{2}\left[1 - \operatorname{sgn}N\left(\frac{1 + \operatorname{sgn}D}{2}\right)\right]$$
(10)
$$-\operatorname{wit}\sum_{a} \left(\sum_{c=1}^{L-l} \left(\sum_{c=1}^{L-l} \sqrt{c(1+c)}\right) \sin(2\theta)$$
 and

$$(-1)^{L-l-q} \exp\{(-T\beta_{\max}(L-l-q))\} \cdot (I_l)_1(\theta;c1,c2) = I_{l-1}(\theta;c1) - \left(\frac{1+\operatorname{sgn}(\theta-\pi)}{2} + (\frac{A_2}{\pi})\right)$$

$$\cdot \sqrt{\frac{c_2}{1+c_2}} \left(\frac{c_2}{c_2-c_1}\right)^{1-\epsilon} + \left(\frac{1+\operatorname{sgn}(\theta-\pi)}{2} + \frac{A_1}{\pi}\right) \sqrt{\frac{c_1}{1+c_1}}$$

$$\cdot \sum_{i=0}^{l-2} \left(\frac{c_2}{c_2 - c_1} \right)^{l-1-i} {2i \choose i} \frac{1}{\left[4(1+c_1) \right]^i}$$

$$P_{b,T}(E) = \sum_{l=1}^{L} \frac{\binom{L-1}{l-1} \sum_{k=0}^{l-1} \binom{l-1}{k} (-1)^{l-1-k} \exp\{(-\beta_{\max}[l-1] \frac{2}{\pi^{k}} (\sqrt[l]{\frac{c_{1}}{1+1}}) \sum_{i=0}^{l-2} \sum_{j=0}^{i-1} \left(\frac{c_{2}}{c_{2}-c_{1}}\right)^{l-1-i}}{\sum_{n=0}^{L-1} \binom{L-1}{n} (-1)^{l-1-n} \exp\{(-\beta_{\max}[L-\binom{2i}{n}]) \frac{(-1)^{i+j}}{j} \frac{\sin[(2i-2j)A_{1}]}{2i-2j}, \ 0 \le \theta \le 2\pi \quad (11)$$

where A_1 and A_2 correspond to A of (10) when c is $\cdot \sum_{l=0}^{L-l} \binom{L-l}{q} (-1)^{L-l-q} \exp \left\{ \left(-T\beta_{\max} \left(L - l - q \right) \right) \right\} \text{ laced by } c_1 \text{ and } c_2 \text{ respectively}[13].$

4. RESULTS AND DISCUSSION

The T-GSC system was evaluated over Nakagami-m channels for the Nakagami parameters m=1 (Rayleigh), m=2, and m=4. BER curves obtained for Nakagami m=1,2, and 4 are shown in Figures 2,3, and 4 respectively. In those figures, the curves for T=0 and T=1 correspond to MRC and SC respectively. The following observations are evident:

- 1. For any particular fading channel, the performance of the T-GSC improves as the threshold level is varied from *T*=1 to *T*=0. The Figures also indicate that at the threshold value *T*=0.25, most useful diversity branches that can appreciably contribute to the combined SNR would have been selected and combined. This value of *T* is valid for all the types of channels studied -ranging from the (severe) Rayleigh fading to the less severe Ricean fading channels.
- 2. For any particular threshold level considered, the BER performance improves as the fading becomes less severe.
- 3. It is interesting to note that as the channel fading becomes less severe, the performance of the system at low threshold values becomes indistinguishable from that of MRC. Note the closeness of the curves at T=0.25 and T=0 in both figures 3 and 4. This can be explained as follows. As all diversity channels are not that bad for these values of *m*, they will be most of the time above threshold, and will be combined as in MRC. This is a significant merit of T-GSC over M-GSC that will be illustrated further in the next section.

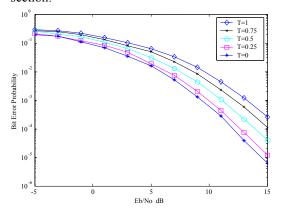


Figure 2: BER Performances of T-GSC in Nakagami channel *m*=1 for different values of T

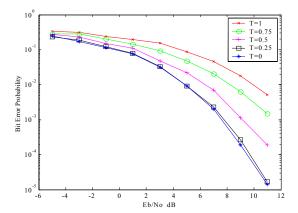


Figure 3: BER Performances of T-GSC in Nakagami channel *m*=2 for different values of T

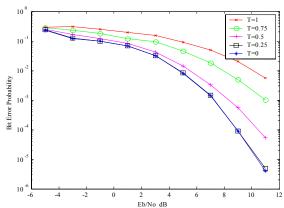


Figure 4: BER Performances of T-GSC in Nakagami channel *m*=4 for different values of T

5. COMPARISON BETWEEN T-GSC AND M-GSC

We have already stated that T-GSC results in power conservation as it does not combine the weak branches, thereby extending battery life for mobile units. In this section, we state other significant differences between the T-GSC and M-GSC schemes.

Figure 5 shows the BER curves of T-GSC for three values of T: 0.25, 0.5 and 0.75 and two values of M: 2 and 3. Again we are assuming L=5. Also shown, as benchmarks, are the BER curves of SC (corresponding to T=1 or M=1) and MRC (corresponding to T=0 or M=5). From the figure, we observe the following:

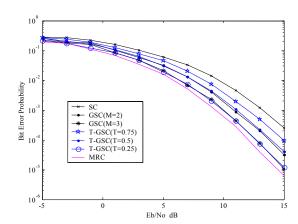


Figure 5: Comparing BER Performances of T-GSC with M-GSC (Nakagami channel *m*=1).

- 1. T-GSC provides a gradual exchange of performance quality and processing intensity. If SC performance is not found to be satisfactory for a certain application, then the next step in M-GSC is to combine two channels all the time, which results in improving the BER by one order of magnitude at $E_b/N_0=15$ dB, for example. However, T-GSC permits any gradual change in BER (and hence processing) by selecting the appropriate threshold T. For example, T=0.75 would provide less improvement in BER over SC as compared to M-GSC with M=2, but will keep the processing intensity lower as it will be combining two channels occasionally. This will obviously has its impact on power consumption.
- 2. We have seen in the previous section that for a particular value of T, most useful diversity branches would be combined for various degrees of fading. This is however not the case with the M-GSC, in which a value of *M* that suits one fading channel can be grossly inadequate for another. Clearly, the T-GSC scheme uses a sound criterion for defining the significant and the insignificant branches that will lead to no loss of appreciable information at any time instant, while operating in any mobile communication channel.
- 3. It is possible to choose a value of T that yields a BER value identical to some M. For example, in Figure 5 T-GSC with T=0.5 has a performance close to M-GSC with M=2. The same observation is true for T=0.25 and M=3. Yet, under these identical performance condition, the M-GSC has slightly higher complexity since it requires the ranking of all diversity branch strengths, where as T-GSC requires only the

knowledge of the branch with the maximum SNR and does not rank the remaining *L*-1 branches after the branch with the maximum SNR is known (i.e., T-GSC does not require full ranking [15]). For L=5, M-GSC requires a pre-combining processing of 10 comparisons and 30 data swaps, while T-GSC requires 8 comparisons and 4 data swaps. The difference in complexities becomes more significant and influential at Large L, as shown in Table 1.

Diversity Order		2	5	10	N
Number Of Comparisons	M-GSC	1	10	45	0.5N(N-1)
	T-GSC	2	8	18	2(N-1)
Number of swaps	M-GSC	3	30	135	1.5N(N-1)
	T-GSC	1	4	9	N-1

Table 1: Pre-combining processing of M-GSC and T-GSC

6. CONCLUSION

This paper analyzes a threshold-based generalized selection combining (T-GSC) scheme, which combines all, and only, the significant diversity branches at any given time instant. The scheme compares the strength of each branch to a predefined threshold, and combines only those branches that pass the threshold test. Compared to the general selective diversity scheme based on combining the best M out of L channels (M-GSC), T-GSC saves power resources that would have been dissipated into combining very weak branches, thereby extending battery life for mobile receivers. Also, T-GSC has less pre-combining operations, and provides a gradual mechanism for exchanging quality with processing intensity.

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7. References

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