Transfer-Function Matrix From State Space

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

$sx(s)-x(0)=Ax(s)+Bu(s)$

$y(s)=Cx(s)+Du(s)$

Or $x(s) =(sI-A)^{-1}x(0)+(sI-A)^{-1}Bu(s)$

$Y(s) = C(sI-A)^{-1}x(0)+C(sI-A)^{-1}Bu(s)+Du(s)$

If $x(0)=0$

Then

$y(s)=((sI-A)^{-1}B+D)u(s)$

But transfer model Function model

$y(s)=G(s)u(s)$

$G(s)=C(sI-A)^{-1}B+D$

Interconnections of Linear Time Invariant Systems

i. Series Connection

ii. Parallel Connection

iii. Feedback Connection

iv. General Connection

Suppose we are given two systems S_1 and S_2. Their state space models are

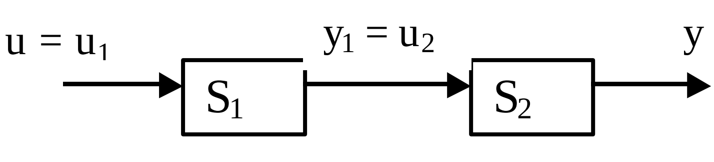
$$\dot{x}_1(t) = A_1 x_1(t) + B_1 u_1(t) \qquad \dot{x}_2(t) = A_2 x_2(t) + B_2 u_2(t)$$

$$y_1(t) = C_1 x_1(t) + D_1 u_1(t) \qquad y_2(t) = C_2 x_2(t) + D_2 u_2(t)$$

Their transfer functions are

$G_1(s)$ and $G_2(s)$

a) Series Connection



Find the transfer function and the state space models of the overall system

Transfer Function

$y(s) = G_2(s)u_2(s) = G_2(s)G_1(s)\, u(s)$

$\Longrightarrow \quad G(s) = G_2(s)G_1(s)$

State Space

For the interconnected system, the vector $x = [x_1 \;\; x_2]^T$ qualifies as a state.

We therefore want to express the state equations in terms of x

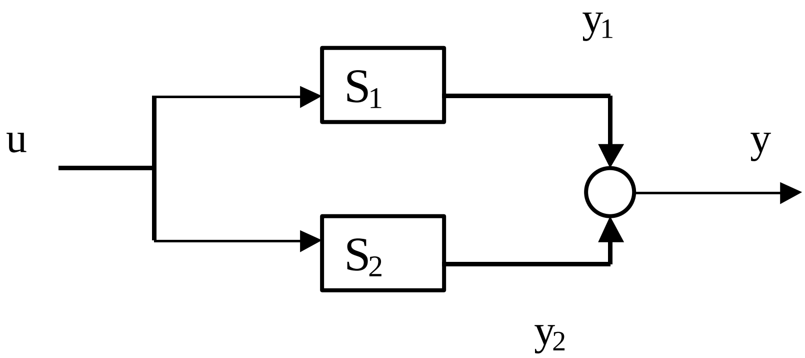
$$\dot{x}_1 = A_1 x_1 + B_1 u_1 = A_1 x_1 + B_1 u$$
$$\dot{x}_2 = A_2 x_2 + B_2 u_2 = A_2 x_2 + B_2 (C_1 x_1 + D_1 u)$$
$$\dot{x}_2 = B_2 C_1 x_1 + A_2 x_2 + B_2 D_1 u$$

In matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} u$$

$$y = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_2 D_1 u$$

ii) Parallel Connection



Transfer Function Model

$$
\begin{aligned}
y(s) &= y_1(s) + y_2(s) \\
&= G_1(s)u(s) + G_2(s)u(s) \\
&= (G_1(s) + G_2(s))u(s)
\end{aligned}
$$

$$\Longrightarrow \quad G(s) = G_1(s) + G_2(s)$$

State Space Model

$$\dot{x}_1 = A_1 x_1 + B_1 u$$

$$\dot{x}_2 = A_2 x_2 + B_2 u$$

$$
\begin{aligned}
y = y_1 + y_2 &= C_1x_1 + D_1u + C_2x_2 + D_2u \\
&= C_1x_1 + C_2x_2 + (D_1 + + D_2)u
\end{aligned}
$$

In matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [D_1 + D_2]u$$