Performance Evaluation of Multi-Layered Space Frequency Time Codes for MIMO-OFDM Systems

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Abstract: **In this paper, we study multi-layered concatenated coding for MIMO-OFDM systems. The proposed system combines spatial multiplexing with transmit diversity at each layer. In addition, each layer concatenates a powerful class of bandwidth efficient coding called IQ-TCM with STBC. This allows the code to achieve full spatial and frequency diversity for each layer at much lower complexity in terms of number of states without any bandwidth expansion. This paper evaluates the performance of serial and parallel interference nulling and cancellation algorithms with several ordering criteria. Furthermore, we demonstrate the advantage of multi-layered approach compared to V-BLAST and we show that the multi-layered system is much more power efficient over MIMO-OFDM channels.**

Keywords: MIMO systems, Space Frequency Time Codes, Concatenated TCM-STBC, IQ-TCM

I. INTRODUCTION

Space frequency time (SFT) coding for OFDM applies spatial coding across multiple antennas, frequency coding across OFDM subcarriers and temporal coding across successive OFDM symbols. The first space frequency coding study was done by [1] where they adapted Tarokh's space time codes [2] to OFDM with multiple transmit antennas. However, these codes are designed for quasi-static fading channels. Thus, they are not optimized for OFDM channels and couldn't benefit from the available frequency diversity. In [3], it was shown that the maximum achievable diversity for a MIMO-OFDM system is M_TLM_R, where L is the frequency selective channel length and M_T and M_R are the number of transmit and receive antennas, respectively. In order to achieve this diversity, the minimum effective length of the SFT code should be equal to M_TL, which requires a large number of states for practical cases. Furthermore, since the OFDM channel in the frequency domain is highly correlated and slowly varying, interleaving across frequency tones is a vital requirement that allows the code to exploit the available frequency diversity.

To achieve full spatial-frequency diversity, trellis code design needs large number of states. In order to simplify the design and reduce the complexity of the code, [4] proposed to concatenate trellis coded modulation (TCM) with space time block coding (STBC). The spatial diversity is guaranteed by STBC while the frequency diversity is achieved by TCM. This separation allows for a less complex, lower number of states, TCM design. However, further reduction in number of states is possible and this is the main motivation of this work.

In [9], we proposed a SFT code that achieved full spatial and frequency diversity at much lower number of states compared to previous designed. The proposed code concatenated a powerful class of trellis codes, known as inphase-quadrature trellis coded modulation (IQ-TCM) [5], with STBC. IQ-TCM provides larger effective lengths compared to conventional TCM. At the same rate and number of states, it could at most doubles the minimum effective length.

Our main contribution in this paper is proposing and evaluating the performance of a high data rate architecture for MIMO-OFDM systems. This architecture consists of coded multi-layered scheme that combines spatial multiplexing and transmit-diversity. The information at the transmitter is divided into layers and each layer is applied to IQ-TCM code followed by STBC. After that, each layer is OFDM modulated and transmitted over the MIMO frequency selective channels. At the receiver, serial and parallel nulling and interference cancellation algorithms with several ordering criteria are compared. In addition, the advantage of this proposed architecture compared to full spatial multiplexing systems, such as V-BLAST, is demonstrated in this work.

II. MIMO-OFDM CHANNEL MODEL

A MIMO frequency selective channel (FSC) consists of M_TM_R FSC. Assume that each FSC between the n^{th} transmit antenna and the m^{th} receive antennas be of length L, and denote it by $\mathbf{h}_{mn} = [h_0 \quad h_1 \quad \cdots \quad h_{L-1}]^T$. OFDM transforms MIMO-FSC into N_c parallel MIMO flat fading channels, where N_c is the number of subcarriers. The OFDM channel in the frequency domain between the n^{th} transmit antenna and the m^{th} receive antenna is:

$$\mathbf{h}^f_{mn} = \mathbf{F}\mathbf{h}_{mn} \tag{1}$$

where $\mathbf{F}$ is a partition of the FFT matrix and it is defined as:

$$\mathbf{F}_{k,l} = \frac{1}{\sqrt{N_c}}\exp\left[-i\frac{2\pi}{N_c}(k-1)(l-1)\right]; \quad k = 0,1,\ldots,N_c-1 \quad l = 0,1,\ldots,L-1 \tag{2}$$

Assume that $\mathbf{h}_{mn} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{C}_{\mathbf{h}_{mn}})$, then the covariance matrix of $\mathbf{h}^f_{mn}$ will be:

$$\mathbf{C}_{\mathbf{h}^f_{mn}} = \mathbf{F}\mathbf{C}_{\mathbf{h}_{mn}}\mathbf{F}^H \tag{3}$$

Thus the OFDM channel in the frequency domain is highly correlated even when the paths of FSC are independent, i.e

$\mathbf{C_h}$=**I**. The fade rate is slower at low number of paths and it is faster at higher number of paths.

III. IQ-SPACE FREQUENCY TIME CODES

An important advantage of the concatenated TCM-STBC is the design separation between temporal and spatial diversity. The spatial diversity is guaranteed by STBC which allows the designer to focus on TCM design to get more frequency diversity. There are number of trellis code designs that increase the minimum effective length (l_{min}) over fast fading channels. The minimum effect length is known as the diversity of the code. An interesting design that increases the minimum effective length of TCM is to code the inphase and quadrature components separately by two parallel TCM encoders [5]. This code is called IQ-TCM. It shows superior performance improvements over conventional TCM and it is easily implemented from off the shelf codes.

The minimum effective length of TCM is upper bounded by $\left(l_{\min} \leq \lfloor v/k \rfloor + 1 \right)$ [5], where v is the number of memory elements in the encoder and k is the number of inputs. Thus, at the same number of states, reducing k increases l_{min}. When k is reduced by a half, l_{min} at most doubles and this is the reason behind the diversity increase of IQ-TCM. For example, for a 2 bps/Hz efficiency system, the IQ-TCM encoder consists of two half rate 4-AM trellis codes. Each input is trellis coded and mapped to a 4-AM signal set. Then the output is combined to form a 16-QAM signal. At 8-states, the IQ-16QAM trellis code provides a frequency diversity of order four while it is only two for a conventional 8PSK TCM. Table 1 demonstrates the advantage of IQ-SFT codes, in terms of complexity needed to achieve full spatial-frequency diversity at each layer, compared to Tarokh's STTC-OFDM and 8PSK-STBC-OFDM. Detailed explanation and performance comparisons of these codes were presented in [9].

In this paper, each layer is an IQ-SFT code. The encoder and the decoder are shown in Figures 1 and 2, respectively. IQ-TCM output two codewords each is an OFDM symbol of length N_c. After interleaving, STBC encodes the two OFDM symbols using Alamouti code [6] at each subcarrier.

For illustration, assume that the receiver is equipped with one receive antenna as shown in Figure 2. Extension to more than one antenna is straight forward. After FFT, the received signals over two periods in the frequency domain are:

$$\begin{bmatrix} \mathbf{Y}^{t_1} & \mathbf{Y}^{t_2} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 & -\mathbf{s}_2^* \\ \mathbf{s}_2 & \mathbf{s}_1^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}^{t_1} & \boldsymbol{\eta}^{t_2} \end{bmatrix} \tag{4}$$

where $\mathbf{Y}^{t_1} = \begin{bmatrix} y_1^{t_1} & y_2^{t_1} & \cdots & y_{N_c}^{t_1} \end{bmatrix}^T$ is the OFDM received symbol at time t_1. Similarly, $\boldsymbol{\eta}^{t_1}$ is the complex AWGN vector of all subcarriers of zero mean and variance $N_0/2$ per dimension. Furthermore, the OFDM channel matrix in the frequency domain between transmit antennas n and receive antenna m is:

$$\mathbf{H}_{mn} = \begin{bmatrix} h_{mn,1} & 0 & \cdots & 0 \\ 0 & h_{mn,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{mn,N_c} \end{bmatrix} \tag{5}$$

where $h_{mn,l}$ is a complex Gaussian channel coefficient for the l^{th} subcarrier of zero mean and variance 0.5 per dimension. The $diag\left(\mathbf{H}_{mn}\right)$ is as defined in (1).

Since the OFDM system transforms FSC into N_c parallel flat fading channel as apparent in (5), the STBC combiner operates on each subcarrier separately. The STBC received signal at the i^{th} subcarrier is:

$$\begin{bmatrix} y_i^{t_1} & y_i^{t_2} \end{bmatrix} = \begin{bmatrix} h_{11,i} & h_{12,i} \end{bmatrix} \begin{bmatrix} s_{1,i} & -s_{2,i}^* \\ s_{2,i} & s_{1,i}^* \end{bmatrix} + \begin{bmatrix} \eta_i^{t_1} & \eta_i^{t_2} \end{bmatrix} \tag{6}$$

it is rearranged into:

$$\begin{bmatrix} y_i^{t_1} \\ y_i^{t_2 *} \end{bmatrix} = \begin{bmatrix} h_{11,i} & h_{12,i} \\ h_{12,i}^* & h_{12,i}^* \end{bmatrix} \cdot \begin{bmatrix} s_{1,i} \\ s_{2,i} \end{bmatrix} + \begin{bmatrix} \eta_i^{t_1} \\ \eta_i^{t_2 *} \end{bmatrix} \tag{7}$$

After that, STBC combiner is used to estimate the IQ coded symbols. The uncoupled estimates are then deinterleaved and passed to the IQ-trellis decoders.

IV. MULTI-LAYERED IQSFT CODES

The MLIQSFT coded system consists of K parallel IQ-SFT codes. Each code is described in details in . Each encoder consists of N_G antennas and is called a group. Thus, the total number of transmit antennas is M_T=K·N_G. The receiver has M_R receive antennas. The information bits encoded by each encoder are called a layer. A bock diagram of the multi-layered system is shown in Figure 3. At the receiver and after OFDM demodulation, the multi-layered received signal at the i^{th} subcarrier over T time slots, where T is the STBC length, is:

$$\begin{aligned} \mathbf{y}_i &= \mathbf{H}_i \mathbf{x}_i + \boldsymbol{\eta}_i \\ &= \begin{bmatrix} \mathbf{H}_{1,i} & \mathbf{H}_{2,i} & \cdots & \mathbf{H}_{K,i} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1,i} \\ \mathbf{x}_{2,i} \\ \vdots \\ \mathbf{x}_{K,i} \end{bmatrix} + \boldsymbol{\eta}_i \end{aligned} \tag{8}$$

where $\mathbf{y}_i$ is an $M_R \cdot T \times 1$ received signal, $\mathbf{H}_{k,i}$ is an $M_R \cdot T \times N_G$ orthogonal channel matrix for the k^{th} group, $\mathbf{x}_{k,i}$ is the $N_G \times 1$ transmitted symbols from the k^{th} group, which are the IQ-trellis coded symbols, and $\boldsymbol{\eta}_i$ is an $M_R \cdot T \times 1$ AWGN vector. To get to the above model, we used the property of the short code length to rearrange the received matrix to a vector.

In the next section, different multi-layered detection techniques will be compared. They have one common operation which is group interference nulling. It needs to be done at each subcarrier since the MIMO channel at each frequency tone is different. Based on an ordering criterion, assume that the first detected group is the k^{th} group. Then, the algorithm calculates the orthonormal bases of the null space of $\mathcal{H}_{k,i}$, which is the channel matrix of all interfering groups at the i^{th} subcarrier:

$$\mathcal{H}_{k,i} = \left[\mathbf{H}_{1,i} \quad \cdots \quad \mathbf{H}_{k-1,i} \quad \mathbf{H}_{k+1,i} \quad \cdots \quad \mathbf{H}_{K,i} \right] \qquad (9)$$

Denote the orthonormal bases of $\mathcal{H}_{k,i}$ by $\mathcal{N}_{k,i}$, the received signal for the k^{th} group after nulling is:

$$\tilde{\mathbf{y}}_{k,i} = \mathcal{N}_{k,i}\mathbf{y}_i = \tilde{\mathbf{H}}_{k,i}\mathbf{x}_{k,i} + \tilde{\boldsymbol{\eta}}_{k,i} \qquad (10)$$

where $\tilde{\mathbf{H}}_{k,i}$ is the resultant channel matrix after nulling. Then, the STBC combiner is used to estimate the IQ-TCM symbols at the i^{th} subcarrier. The Frobenius norm (FN) of the k^{th} group at the i^{th} subcarrier is defined as:

$$\text{FN}_{k,i} = \frac{1}{T}\left\|\mathbf{H}_{k,i}\right\|_F^2, \text{ where } \left\|\mathbf{A}\right\|_F^2 = trace\left(\mathbf{A}^H\mathbf{A}\right)$$

V. ADVANTAGE OF MLSTBC-OFDM COMPARED TO V-BLAST-OFDM

In order to show the advantage of the multi-layered STBC, we compared its OFDM symbol error rate (SER) with V-BLAST via simulation. An error is counted when at least one of the bits in the whole OFDM symbol is in error. Each OFDM symbol has 64 subcarriers. We use an independent equal path power FSC model of length four. Perfect channel state information is assumed. Also, each layer uses Alamouti full rate code with QPSK modulation. Thus, the rate of each layer is 2bps/Hz and each layer transmits through two antennas and has a transmit diversity of order two.

Figure 5 compares the OFDM SER performance of MLSTBC, V-BLAST and STBC over 4x4 MIMO-OFDM channels. The FSC is of length 4. The results show that the MLSTBC is more power efficient than V-BLAST. That is a result of the higher diversity order obtained by the MLSTBC. At 4x4 MIMO channels, the MLSTBC has two layers and each layer has a transmit diversity of order two. At the receiver, one antenna is used to null out interference and three antennas are used for diversity and detection. On the other hand, V-BLAST has four layers and no transmit diversity while the receiver uses three antennas to null out three interfering layers and one antenna is used for the detection of the first layer. Recall that the performance is dominated by the weakest layer. For the case of 4 bps/Hz, the gain is 27dB and it is 19dB for 16 bps/Hz transmission. The reason for this high gain in performance is that the OFDM channel in the frequency domain is highly correlated and a diversity order of 2x3 greatly improves the performance.

VI. MULTI-LAYERED DETECTION ALGORITHMS

Serial and parallel detection algorithms are described in this section. We also present and compare different ordering criteria for the serial algorithm.

Serial interference nulling/ decoding and cancellation

This algorithm operates on the MIMO received signal in the frequency domain after FFT. This algorithm serially nulls out interfering OFDM symbols without ordering. Ordering is not a straight forward operation because each subcarrier has a different channel and the ordering needed to be done at the OFDM symbol level since each OFDM symbol is a codeword. Different ordering criteria are discussed next section.

At each subcarrier, the algorithm calculates the orthonormal bases of the null space of the interfering groups. After that, a STBC combiner is used to uncouple the space time symbols at each subcarrier. These soft outputs and the channel information are passed to the IQ- trellis decoders. Then, the symbols of the first layer are regenerated and their contributions are subtracted from the received signal. This process is repeated serially to detect each IQ coded layer.

Ordering criteria

Ordering is very essential in serial processing since detecting strong layers first result in less error propagations to successive layers and hence improves the performance of the system. In this paper, we evaluate the performance of several ordering criteria. They are described as follows:

- Post-nulling ordering with hard detection: This algorithm performs ordering and detection based on maximum post-processing Frobenius norm of each layer at each subcarrier.
- *MaxMin FN*: this approach orders whole OFDM symbols instead of per carrier bases. The serial nulling/ decoding and cancellation algorithm is similar to the unordered case which has the advantage of soft symbol decoding. The algorithm first calculates FNs of the MIMO channels at each subcarrier for all groups then the group that has the maximum minimum FN is detected first.
- *MaxAverage FN*: this ordering criterion is similar to the MaxMin FN but it orders based on the average FN.
- *Blind power allocation*: this method doesn't do ordering at the receiver. Instead, the power at the transmitter is distributed unequally among the groups such that the first detected group has the highest power. The power is distributed so that there is a 3dB difference between consecutive groups. Meanwhile, the sum is constant and equal to a single transmit antenna power.

Parallel nulling/ decoding and cancellation

The parallel processing doesn't need ordering. It nulls all groups at the same time. Then, each group space time symbols are uncoupled and estimated by the STBC combiner. After de-interleaving, the output of the combiner is decoded by the IQ trellis decoder. Then the codewords are regenerated and interleaved. After that, parallel cancellation is done at each subcarrier. The output of the cancellation stage is de-interleaved and IQ trellis decoded. This iterative parallel processing is repeated until diminishing returns are observed.

VII. SIMULATION RESULTS

The channel is a MIMO-FSC of length L with equal power paths and each path experience an independent Rayleigh fading. We assume that the channel is constant over two OFDM symbols.

In this section, we evaluate the BER performance of the MLIQSFT detection algorithms. The serial detection algorithms are compared in Figure 4.The hard nulling with ordering performs worse than no ordering because the trellis decoder suffers from hard detection. The result also shows that the different ordering criteria, which are at the receiver, perform very close to each other. Thus, performing ordering based on pre-FN is more computational efficient while its performance is very close to the post-FN ordering. On the other hand, power allocation at the transmitter performs the best with less complexity than ordering since the order is fixed. This shows that the unequal power allocation greatly benefits the serial detection and cancellation algorithm. At BER of 10^{-3}, around 1dB is gained by ordering and around 2dB is gained by the blind power allocation at the transmitter.

Performance with parallel detection and cancellation is shown in Figure 6. The iterative processing gives diminishing returns after four iterations. It gains around 4.5 dB after four iterations compared to no iterations. When it is compared to the ideal performance of no cancellation, it is 6 dB far. Perfect cancellation is a lower bound and it has full receive diversity.

Figure 7 compares the performance of the serial and parallel processing for the MLIQSFT codes. The parallel processing at four iterations gained around 1dB compared to the serial with power allocation. The diversity advantage of the IQ coded system is apparent from the result when compared to the uncoded MLSTBC-OFDM system. The coded plots have sharper slopes as a result of achieving full frequency diversity.

VIII. CONCLUSION

In this paper, we propose bandwidth efficient high data rate architecture for MIMO-OFDM systems. Our proposed system consists of multi-layered transmission where each layer concatenates IQ-TCM with STBC and OFDM. This multi-layered approach combines spatial multiplexing and space time coding. In addition, the concatenated system separates frequency coding from space time coding and it has much lower complexity than a joint design. The IQ-TCM codes provide larger effective lengths at low number of states. In addition, we compared serial and parallel detection algorithms for the multi-layered system with several ordering criteria. The main results of this study show the advantage of the Multi-layered STBC-OFDM compared to V-BLAST-OFDM. In addition, this paper shows the spatial and frequency diversities captured by MLIQSFTC compared to the uncoded MLSTBC.

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REFERENCES

[1] D. Agrawal, V. Tarokh, A. Naguib, and N. Seshadri, "Space-time coded OFDM for high data-rate wireless communication over wideband channels," *IEEE 48th Vehicular Technology Conference*, vol. 3, pp. 2232-2236, May 1998.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-Time Codes for High Data Rates Wireless Communications: Performance Criterion and Code Construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744-765, March 1998.

[3] B. Lu and X. Wang, "Space-time code design in OFDM systems," *IEEE Global Telecommunications Conference*, vol. 2, pp. 1000-1004, 27 Nov.-1 Dec. 2000.

[4] G. Yi and K. B. Letaief, "An efficient space-frequency coded OFDM system for broadband wireless communications," *IEEE Transactions on Communications*, vol. 51, pp. 2019-2029, Dec. 2003.

[5] S. A. Al-Semari and T. E. Fuja, "I-Q TCM: reliable communication over the Rayleigh fading channel close to the cutoff rate," *IEEE Transactions on Information Theory*, vol. 43, pp. 250-262, Jan. 1997.

[6] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1451-1458, Oct. 1998.

[7] H. Jamali and T. Le-Ngoc, *Coded-Modulation Technique for Fading Channels*: Kluwer Academic Publisher 1994.

[8] X. Wang, Y. R. Shayan, and M. Zeg, "On the Code and Interleaver Design of Broadband OFDM Systems," *IEEE Communication Letters*, vol. 8, pp. 653-655, Nov. 2004.

[9] S. Al-Ghadhban, R. M. Buehrer, and B. D. Woerner, "IQ Space Frequency Time Codes for MIMO-OFDM Systems," *Vehicular Technology Conference, 2006. VTC 2006-Spring. IEEE 63rd* , vol.4, no., pp.1817-1820, 7-10 May 2006.

Table 1: Complexity of SFT codes at 2bps/Hz and M_T=2 transmit antennas

FCS Length	Minimum number of states to achieve full diversity ($M_T L M_R$)		
L	Tarokh QPSK	8PSK	IQ-16QAM
2	64	4	2
3	1024	16	4
4	16384	64	8
5	262144	256	16
6	4194304	1024	32
7	67108864	4096	64

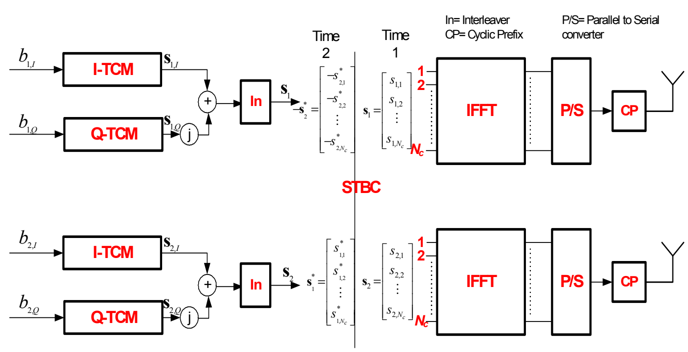

Figure 1: Block Diagram of IQ-SFT Encoder

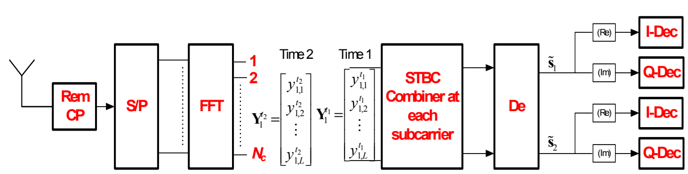

Figure 2: Block Diagram of IQ-SFT Decoder at one receive antenna

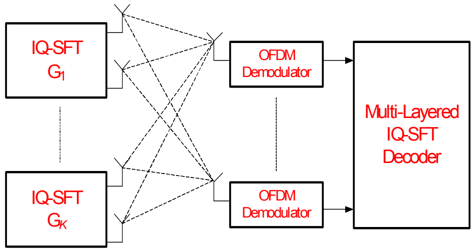

Figure 3: Block diagram of a MLIQSFT coded system

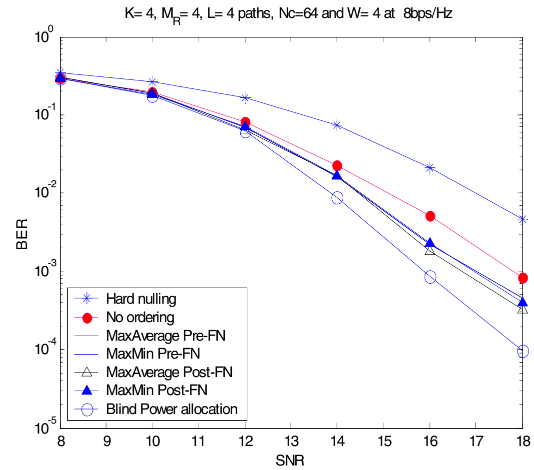

Figure 4: Performance comparison of serial detection algorithms for MLIQSFT codes.

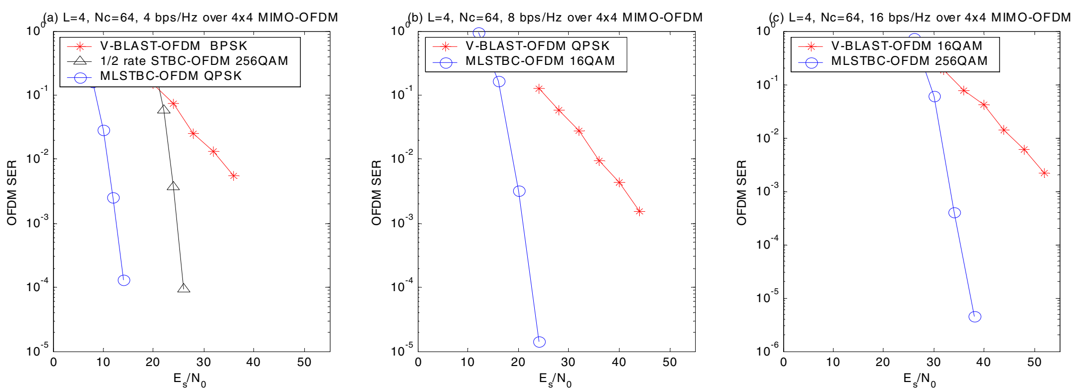

Figure 5: OFDM SER Comparison of MLSTBC, VBLAST and STBC over 4x4 MIMO Channels

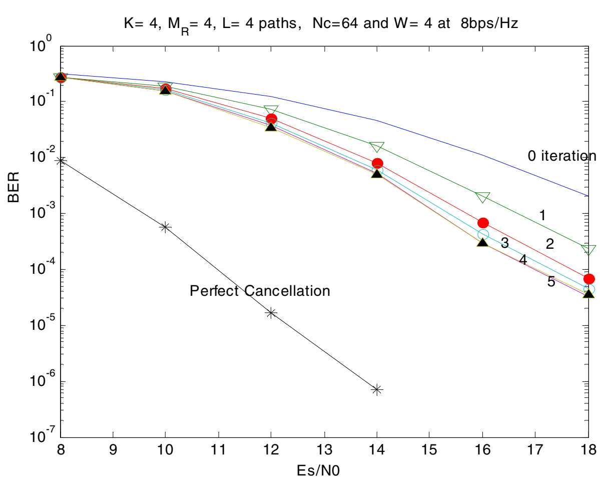

Figure 6: Performance of MLIQSFT codes with parallel iteration detection

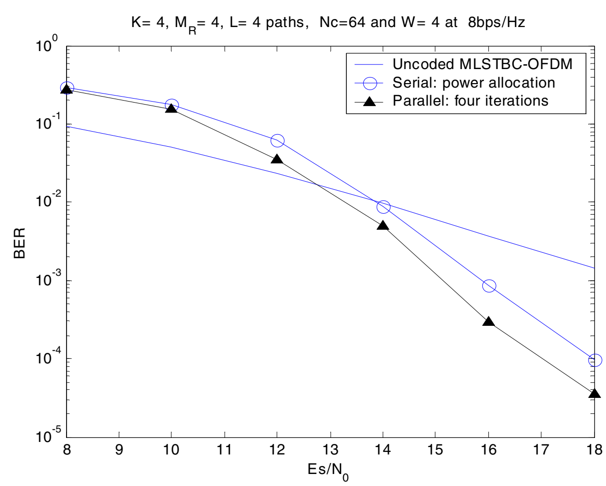

Figure 7: Performance comparison of serial and parallel detection for MLIQSFT codes