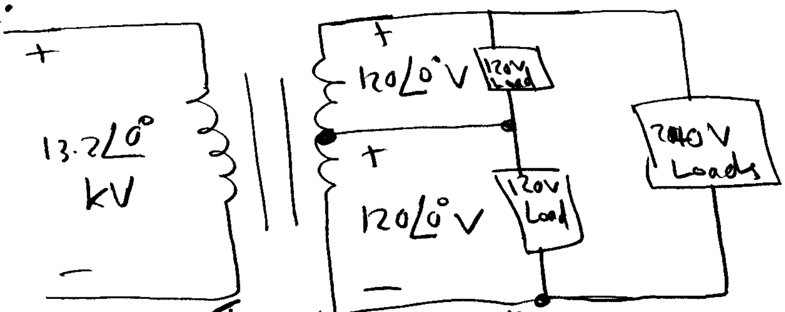
Mutual Inductance and Transformers Ch. 6

Textbook Sections 6.4 and 6.5
and 9.10 and 9.11

What is a Transformer?

- A transformer transfer electrical energy between two circuits which are physically seperated. For example, in power distribution, step up and step down tranformers are very common. The step down transformer are used to reduce the utility distribution voltage from 13.2 kV to 240 V.

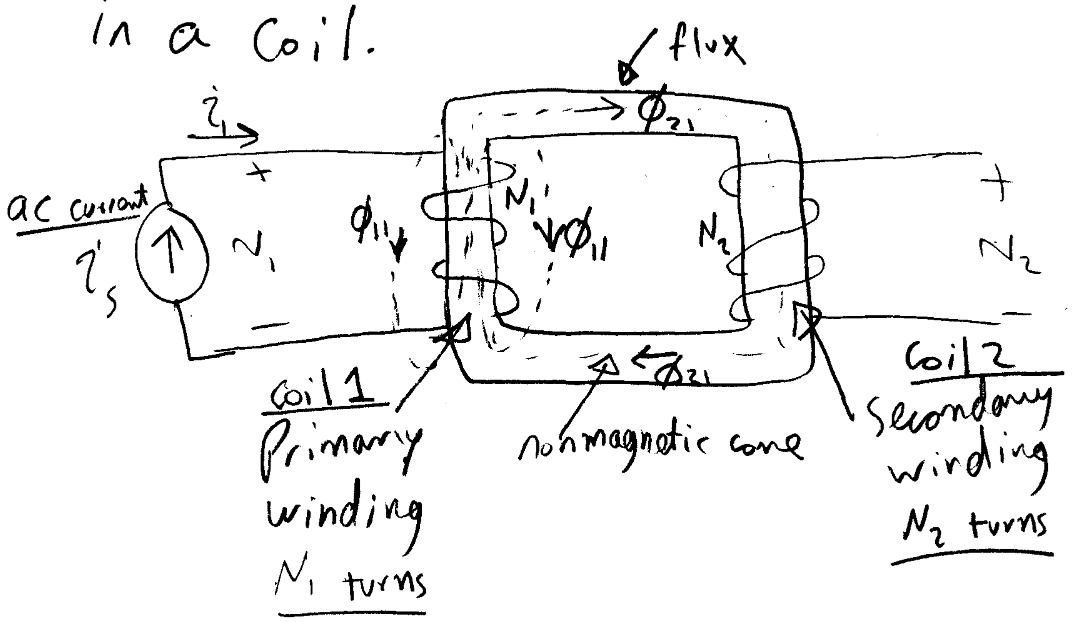


Step down transformer

- In addition, transformers are used in circuits and electronic designs to match impedances and eliminate dc signals from portions of the systems.

How does it work?

It is based on the electromagnetic theory of mutual inductance where a time varying magnetic field will introduce a time varying current in a coil.



What is happening?

- Coil 1 is energized by a time-varying current source that establishes the current i_1 in the N_1 turns.
- Coil 2 is not energized and is open.
- The coils are wound on nonmagnetic core.
- The time varying current in Coil 1 introduce a flux ϕ_1.
- Flux $\phi_1 = \phi_{11} + \phi_{21}$

ϕ_{11}: flux linking Coil 1

ϕ_{21}: flux linking Coil 2 caused by Coil 1

Mutual Inductance and Transformers

— Definitions:

— Flux ~~Em~~ strength of magnetic field (webers) (Wb)

— $\phi = \mathcal{P} N i$

- $\mathcal{P}$ → Permeance "Quantity that describes the magnetic properties of a space."
- N → # of Turns
- i → current

— Flux Linkage $\lambda = N\phi$

— So, $\phi_1 = \mathcal{P}_1 N_1 i_1$; $\phi_{11} = \mathcal{P}_{11} N_1 i_1$; $\phi_{21} = \mathcal{P}_{21} N_1 i_1$

— Note that $\mathcal{P}_1 = \mathcal{P}_{11} + \mathcal{P}_{21}$

— By using Faraday's Law,

$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d}{dt}(\phi_{11} + \phi_{21})$$

$$= N_1^2 (\mathcal{P}_{11} + \mathcal{P}_{21}) \frac{di_1}{dt} = N_1^2 \mathcal{P}_1 \frac{di_1}{dt}$$

$$= L_1 \frac{di_1}{dt}$$

(L_1: self-inductance of coil 1)

Also, $v_2 = \frac{d\lambda_2}{dt} = \frac{d(N_2 \phi_{21})}{dt} = N_2 \frac{d}{dt}(\mathcal{P}_{21} N_1 i_1)$

$$= N_2 N_1 \mathcal{P}_{21} \frac{di_1}{dt}$$

($N_2 N_1 \mathcal{P}_{21}$: Mutual inductance between coil 1 and 2)

Define: mutual inductance $M_{21} = N_2 N_1 \mathcal{P}_{21}$

similarly $M_{12} = N_1 N_2 \mathcal{P}_{12}$

For nonmagneti...

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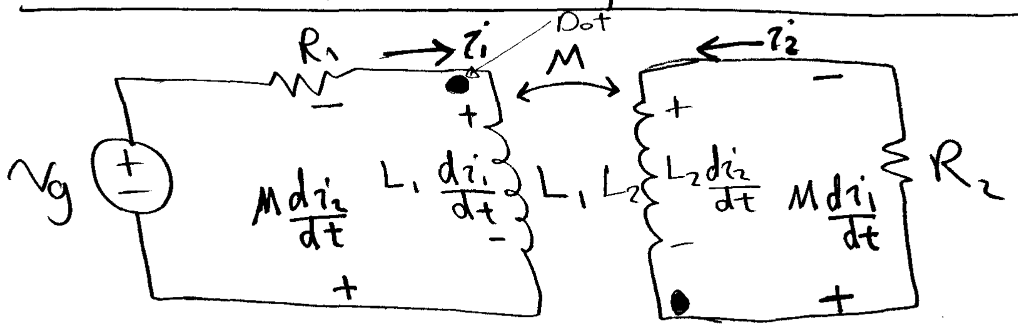
What is the relation between Mutual inductance and Self inductance?

$$M = k\sqrt{L_1 L_2} \quad ; \quad 0 \leq k \leq 1$$

k ← coefficient of Coupling

Induced Voltage Polarity and Dot Convension:

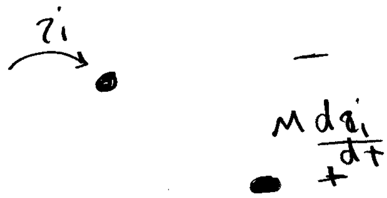


There are Four voltages

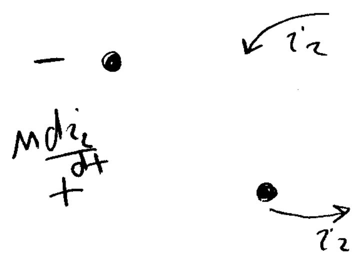
- $L_1\frac{di_1}{dt}$, $L_2\frac{di_2}{dt}$ — Self inductance
- $M\frac{di_1}{dt}$, $M\frac{di_2}{dt}$ — Mutual Inductance

● Dot Convention:

- When the reference direction of the current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminals.



- When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative

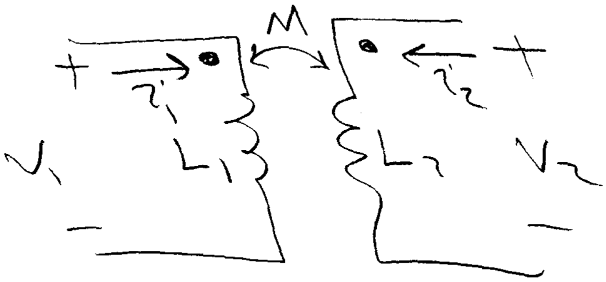


Thus, the sum of voltages at each loop are:

$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0$$

$$i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

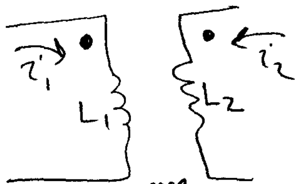
Energy Calculations?



What is the total Energy stored in the magnetically coupled coils?

$$w(t) = \underbrace{\frac{1}{2} L_1 i_1^2}_{\text{coil 1}} + \underbrace{\frac{1}{2} L_2 i_2^2}_{\text{coil 2}} \pm \underbrace{M i_1 i_2}_{\text{Mutual Energy}}$$

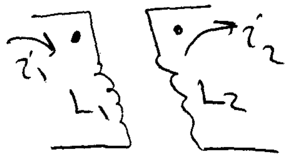
How to decide on the sign, "+" or "−"?



i_1 and i_2 are same
Both enter the Dot.

⇒ "+"

$$\Rightarrow w(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$



i_1 and i_2 are opposite
one current enters while the other leaves.

⇒ "−"

$$\Rightarrow w(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2$$