Chapter 7: Spread Spectrum Modulation

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Objectives

- *Spreading sequences in the form of pseudo-noise sequences, their properties, and methods of generation.*
- *The basic notion of spread-spectrum modulation.*
- *The two commonly used types of spread-spectrum modulation: direct sequence and frequency hopping*

Definition

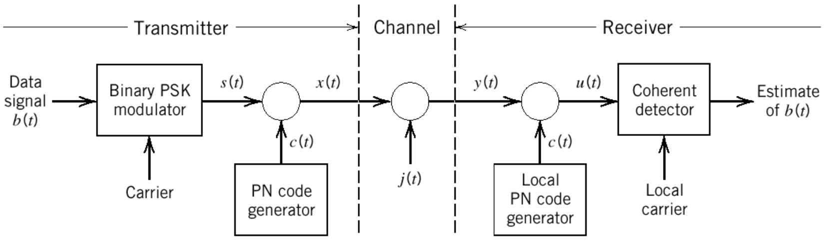
The definition of spread-spectrum modulation may be stated in two parts:

- Spread spectrum is a means of transmission in which the data sequence occupies a bandwidth in excess of the minimum bandwidth necessary to send it.
- The spectrum spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The same code is used in the receiver (operating in synchronism with the transmitter) to despread the received signal so that the original data sequence may be recovered.

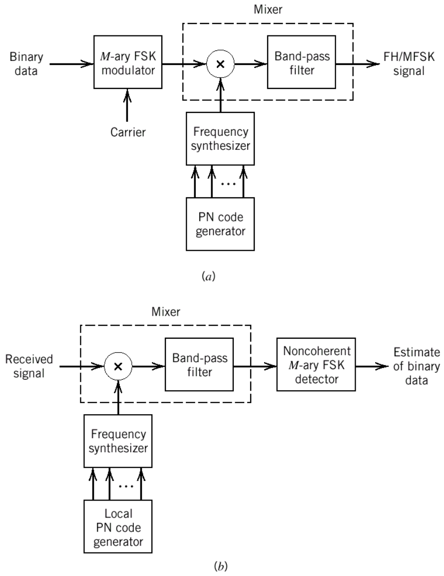
Advantages

- Secure communication
- Low probability of detection
- Resistance to jamming and interference
- Multipath rejection
- Multiple-access communications

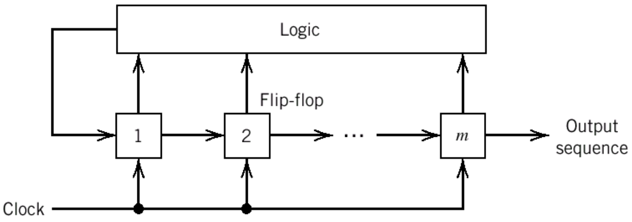
Direct Sequence Block Diagram



Frequency Hoping Block Diagram

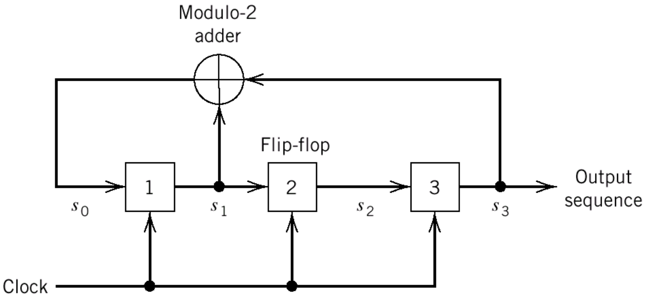


Pseudo-Noise Sequences



Feedback shift register

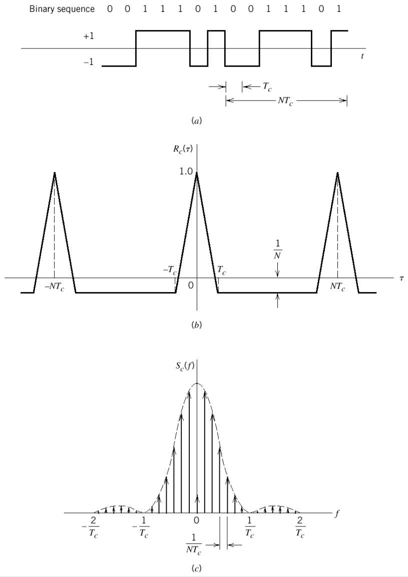
Maximal-length sequence generator for *m* = 3



It is assumed that the initial state of the shift register is 100 (reading the contents of the three flip-flops from left to right). Then, the succession of states will be as follows:
100, 110, 111, 011, 101, 010, 001, 100

The output sequence (the last position of each state of the shift register) is therefore
00111010 ...
which repeats itself with period 2^3 — 1 = 7.

Maximal-length sequence generator for m = 3

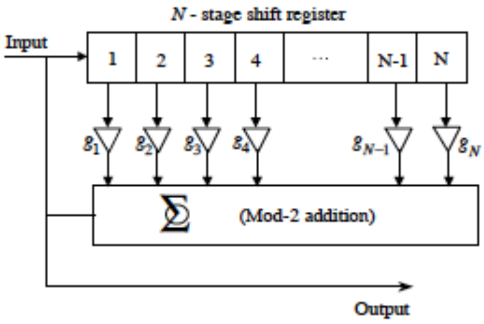


(a) Waveform of maximal-length sequence for length m = 3 or period N = 7.

(*b*) Autocorrelation function.

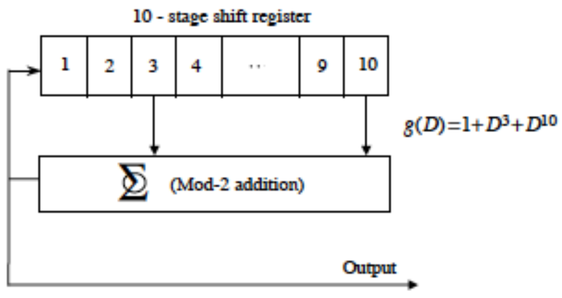
(*c*) Power spectral density.

PN Sequence Generator



$$g(D)=g_0+g_1D+g_2D^2+\cdots+g_{N-1}D^{-1}+g_ND^N$$

PN Sequence Generator (N=10)



Note: $g(D)$ is a primitive polynomial.

Primitive Polynomials

Where did **g**(**D**) come from?

- A PN sequence generator will have maximum period if **g**(**D**)is primitive.
- Fortunately we have tables of primitive polynomials.
  - See for example: R. E. Ziemer and R. L. Peterson, **Digital Communications and Spread Spectrum Systems,** Macmillan, 1985, pp. 390-391.

g(D)= 2 0 1 1 (octal)

g(D)= 0 1 0 0 0 0 0 0 1 0 0 1 (binary) → 1**+D^3+D^{10}**

Primitive Polynomials / 2

Definition of a primitive polynomial:

- The polynomial **g**(**D**) of degree **N** is a primitive polynomial if the smallest integer **k** for which **g**(**D**) divides $\mathbf{D^k+}1$ is $\mathbf{k=}2^{\mathbf{N}}-1$.

Note that testing a polynomial of large degree is a time consuming task.

Proof Octal [1,3] is Primitive

- Claim: Octal [1,3] => 001 101 is primitive

Proof: $g(D)=1+D^2+D^3$

$$m = 7$$

$$\begin{array}{r l}
 & 1+D^2+D^3+D^4 \\ \cline{2-2}
1+D^2+D^3 \big) & 1+D^7 \\
 & 1+D^2+D^3 \\
 & \quad D^2+D^3+D^7 \\
 & \quad D^2+D^4+D^5 \\
 & \qquad D^3+D^4+D^5+D^7 \\
 & \qquad D^3+D^5+D^6 \\
 & \qquad\quad D^4+D^6+D^7 \\
 & \qquad\quad \underline{\underline{D^4+D^6+D^7}}
\end{array}$$

Proof Octal [1,3] is Primitive

$$m = 6 \qquad \begin{array}{r|l} & 1+D^2+D^3+D^4 \\ \hline 1+D^2+D^3 & 1+D^6 \\ & 1+D^2+D^3 \\ & D^2+D^4+D^5 \\ & D^3+D^4+D^5+D^6 \\ & D^3+D^5+D^6 \\ & D^4 \\ & D^4+D^6+D^7\,XXXXX \end{array}$$

$$m = 5 \qquad \begin{array}{r|l} & 1+D^2+D^3 \\ \hline 1+D^2+D^3 & 1+D^5 \\ & 1+D^2+D^3 \\ & D^2+D^3+D^5 \\ & D^2+D^4+D^5 \\ & D^3+D^4 \\ & D^3+D^5+D^6\,XXXXX \end{array}$$

$$m = 4 \qquad \begin{array}{r|l} & 1+D^2 \\ \hline 1+D^2+D^3 & 1+D^4 \\ & 1+D^2+D^3 \\ & D^2+D^3+D^4 \\ & D^2+D^4+D^5\,XXXXX \end{array}$$

$$m = 3 \qquad \begin{array}{r|l} & 1+D^2 \\ \hline 1+D^2+D^3 & 1+D^3 \\ & 1+D^2+D^3 \\ & D^2 \\ & D^2+D^4+D^5\,XXXXX \end{array}$$

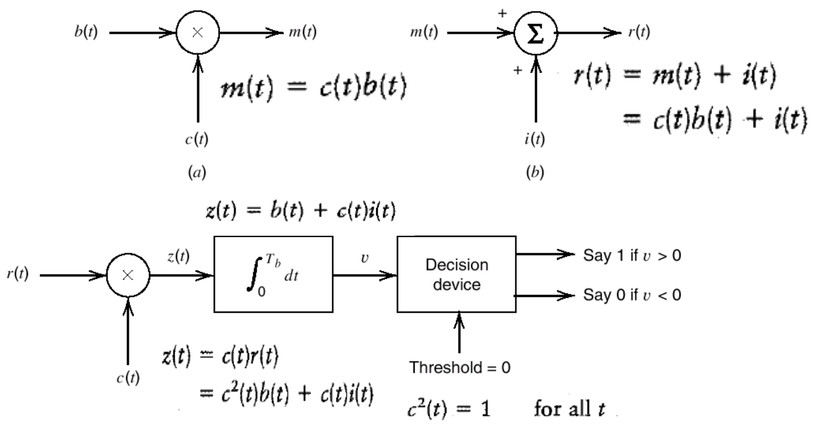
Table for Primitive Polynomials

Table 7.1 Short Table of Primitive Polynomials

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}
3	1	0	1											
4	1	0	0	1										
5	0	1	0	0	1									
6	1	0	0	0	0	1								
7	0	0	1	0	0	0	1							
8	0	1	1	1	0	0	0	1						
9	0	0	0	1	0	0	0	0	1					
10	0	0	1	0	0	0	0	0	0	1				
11	0	1	0	0	0	0	0	0	0	0	1			
12	1	0	0	1	0	1	0	0	0	0	0	1		
13	1	0	1	1	0	0	0	0	0	0	0	0	1	
14	1	0	0	0	0	1	0	0	0	1	0	0	0	1

7.3 Spread Spectrum

baseband spread-spectrum system



$$m(t) = c(t)b(t)$$

$$r(t) = m(t) + i(t) = c(t)b(t) + i(t)$$

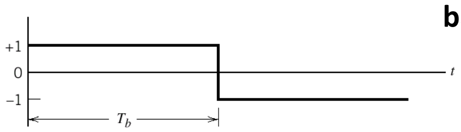
$$z(t) = b(t) + c(t)i(t)$$

$$z(t) = c(t)r(t) = c^2(t)b(t) + c(t)i(t)$$

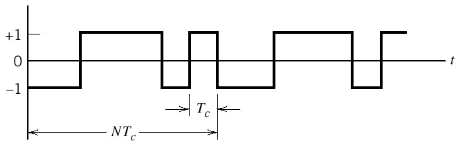
$$c^2(t) = 1 \quad \text{for all } t$$

7.3 Spread Spectrum

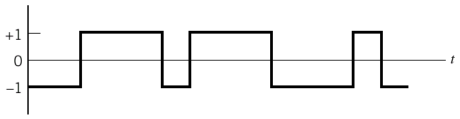
baseband spread-spectrum system



(a) Data signal $b(t)$

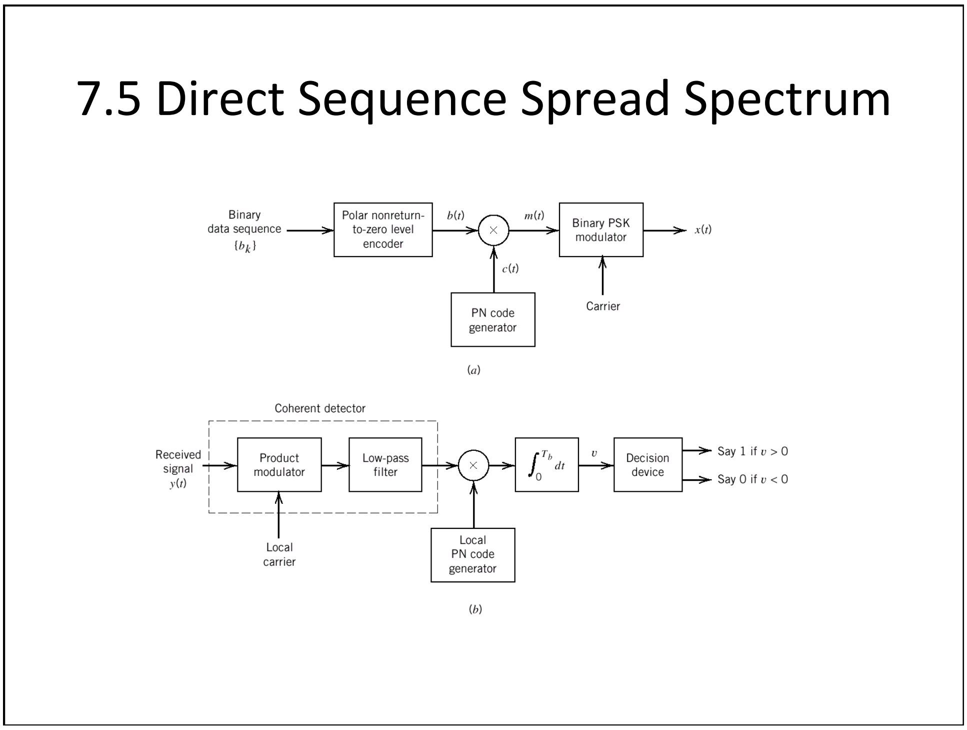


(b) Spreading code $c(t)$

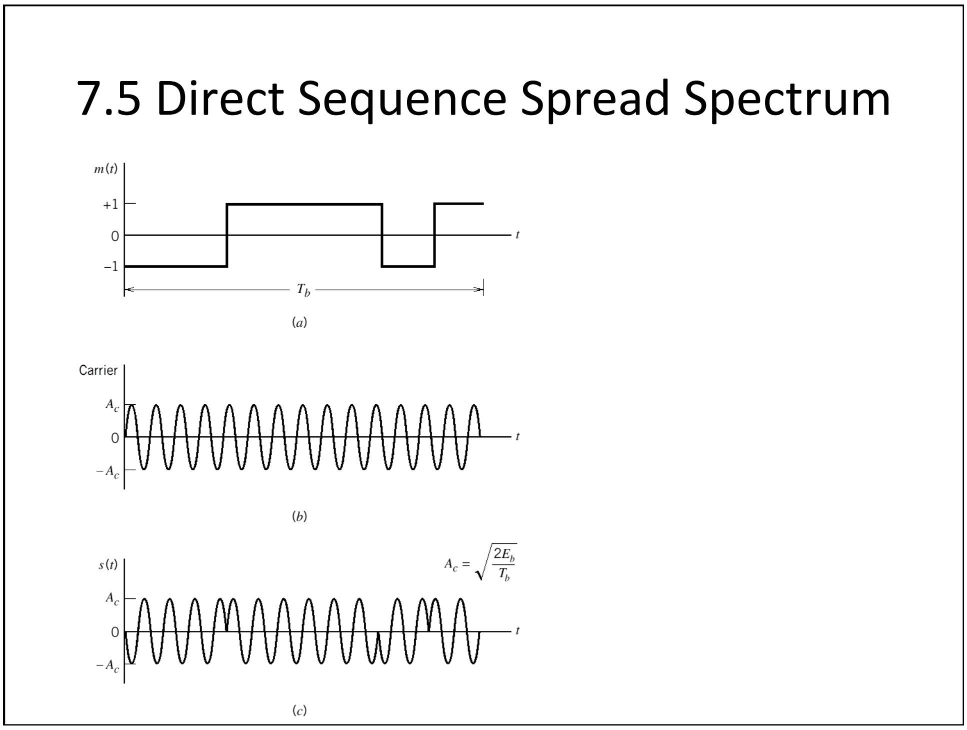


(c) Product signal $m(t)$

the product (modulated), signal m(t) will have a spectrum that is nearly the same as the wideband PN signal.



7.5 Direct Sequence Spread Spectrum



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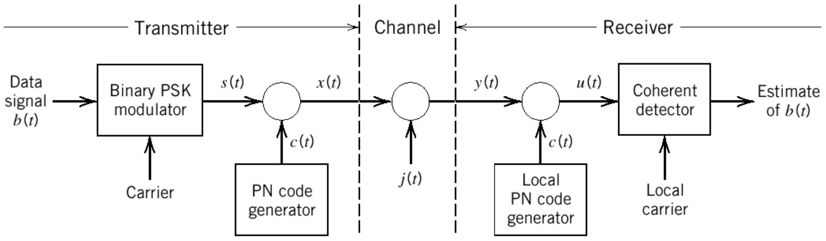


$$\begin{aligned} y(t) &= x(t) + j(t) \\ &= c(t)s(t) + j(t) \end{aligned}$$

$$\begin{aligned} u(t) &= c(t)y(t) \\ &= c^2(t)s(t) + c(t)j(t) \\ &= s(t) + c(t)j(t) \end{aligned}$$

Processing Gain $\mathrm{PG} = \dfrac{T_b}{T_c}$

Input SNR $(\mathrm{SNR})_I = \dfrac{E_b/T_b}{J}$

Output SNR $(\mathrm{SNR})_O = \dfrac{2T_b}{T_c}(\mathrm{SNR})_I$

$$10 \log_{10}(\mathrm{SNR})_O = 10 \log_{10}(\mathrm{SNR})_I + 3 + 10 \log_{10}(\mathrm{PG}) \text{ dB}$$

7.6 Average Probability of Error for DS/BPSK

For Coherent DS / BPSK

$$P_e \simeq \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

Recall that for Coherent BPSK without spreading

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Thus for direct sequence spreading, the wideband noise power spectral density is

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

Since the signal energy per bit is E_b=PT_b, where P is the average signal power and T_b is the bit duration.

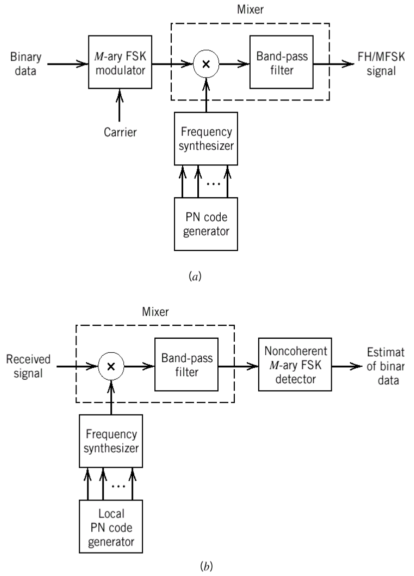
$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right) \longrightarrow \frac{J}{P} = \frac{\mathrm{PG}}{E_b/N_0}$$

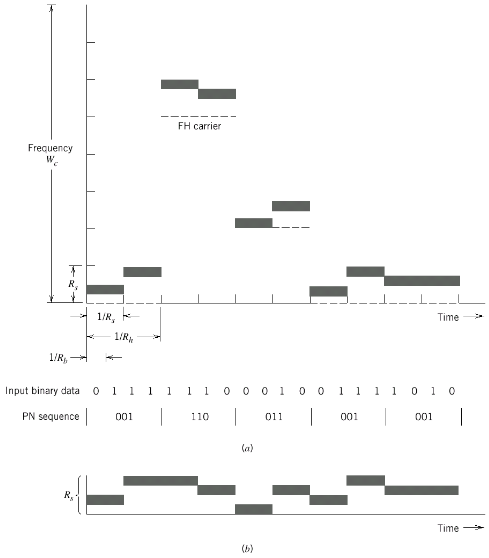
The ration J / P is called the jamming margin

$$(\text{Jamming margin})_{\mathrm{dB}} = (\text{Procesing gain})_{\mathrm{dB}} - 10 \log_{10}\left(\frac{E_b}{N_0}\right)_{\min}$$

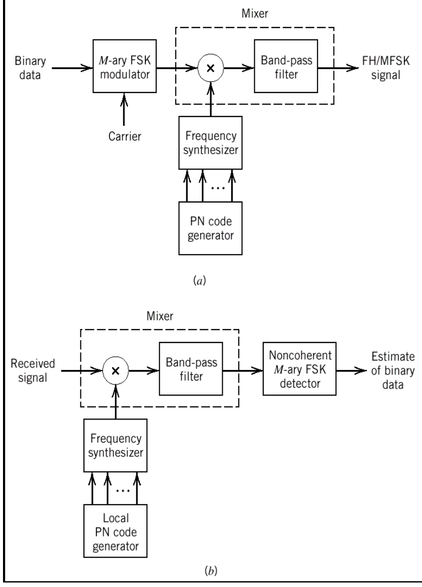
where $(E_b/N_0)_{\min}$ is the minimum value needed to support a prescribed average probability of error.

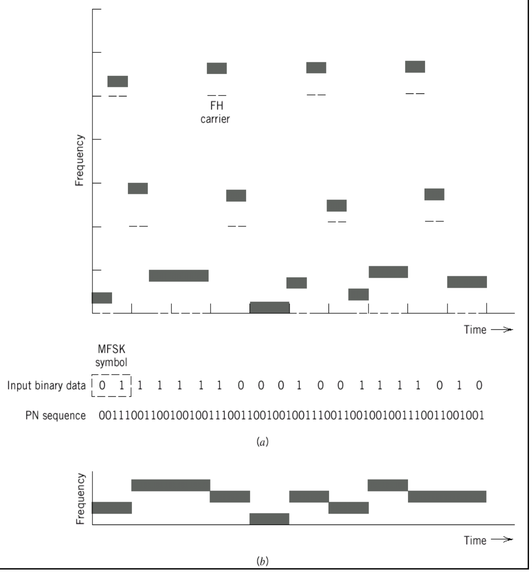
7.7 Frequency Hopping





7.7 Frequency Hopping





Example

A spread-spectrum communication system has the following parameters:

$$\text{Information bit duration, } T_b = 4.095 \text{ ms}$$
$$\text{PN chip duration, } T_c = 1\ \mu s$$

a) Find the processing gain (PG)
b) Find the required PN sequence length N and the shift register length m
c) Find the fading margin if we required that the average probability of error does not exceed 10^{-5} .

Hence, using Equation (7.38) we find that the processing gain is

$$PG = 4095$$

Correspondingly, the required period of the PN sequence is $N = 4095$, and the shift-register length is $m = 12$.

Example

Part c)

For a satisfactory reception, we may assume that the average probability of error is not to exceed 10^{-5}. From the formula for a coherent binary PSK receiver, we find that $E_b/N_0 = 10$ yields an average probability of error equal to 0.387×10^{-5}. Hence, using this value for E_b/N_0, and the value calculated for the processing gain, we find from Equation (7.47) that the jamming margin is

$$\begin{aligned}(\text{Jamming margin})_{\text{dB}} &= 10 \log_{10} 4095 - 10 \log_{10}(10) \\ &= 36.1 - 10 \\ &= 26.1 \text{ dB}\end{aligned}$$