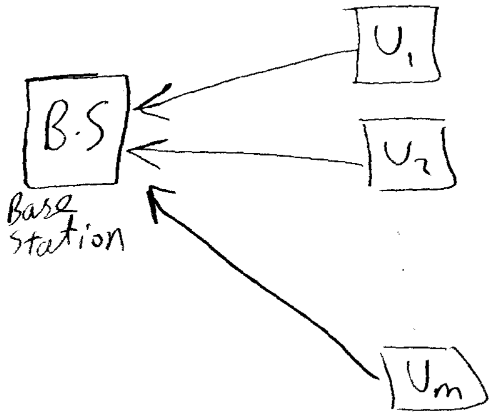
Chapter 14 [Cover]

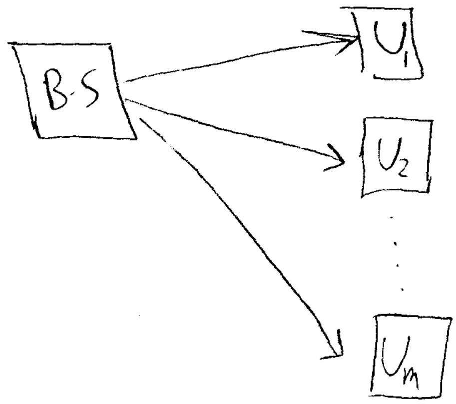
Network Information Theory

– Multiple Access Channels



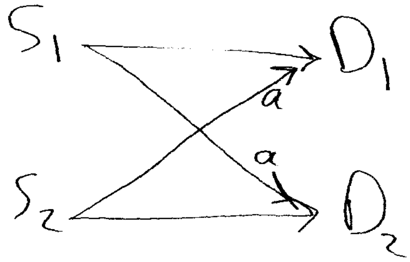
- How do the various users cooperate with each other to send information?
- What rates of communication are simultaneously achievable?
- What limitations does interference among the senders put on the total rate of communication?

– Broadcast Channels



- How does the Base station encode information meant for different receivers in a common signal?
- What are the rates at which information can be sent to the different receivers?

– Interference Channel



– Relay Channel

$$S \to R \to D$$

– Two-Way Channel

$$S_1 \longrightarrow D_2$$
$$D_1 \longleftarrow S_2$$

14.1 Gaussian Multiple User Channels

A discrete time AWGN channel with input power P and noise variance N is modeled by:

$$Y_i = X_i + Z_i \quad , \quad i = 1, 2, \cdots$$

where Z_i are i.i.d Gaussian R.V. with mean 0 and variance N.

The signal $\vec{X} = (X_1, X_2, \cdots, X_n)$ has a power constraint

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P$$

The shannon capacity C is

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right)$$ bits per transmission.

14.1.1 Single User Gaussian Channel

$$Y = X + Z$$

- Choose a rate $R < \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right)$
- Fix a good $(2^{nR}, n)$ codebook of power P.
- Choose an index i in the set 2^{nR}
- Send the i^{th} codeword $X(i)$
- The receiver observes $Y = X(i) + Z$ and finds the index $\hat{i}$ of the closest codeword to Y.

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14.1.2 The Gaussian Multiple Access Channel with m Users

- Consider m transmitters, each with a power P.

$$Y = \sum_{i=1}^{m} X_i + Z$$

and Let

$$C\left(\frac{P}{N}\right) = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right)$$

to be the capacity of a single user with $SNR = \frac{P}{N}$

- The achievable rate region for the Gaussian Channel takes on the simple form given in the following equations:

$$R_i < C\left(\frac{P}{N}\right)$$

$$R_i + R_j < C\left(\frac{2P}{N}\right)$$

$$R_i + R_j + R_k < C\left(\frac{3P}{N}\right)$$

$$\vdots$$

$$\sum_{i=1}^{m} R_i < C\left(\frac{mP}{N}\right)$$

- Note that when all the rates are the same, the last inequality dominates the others.

- We need m codebooks, the ith codebook having 2^{nR_i} codewords of Power P.

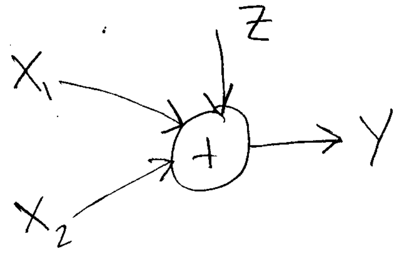
- Each of the independent transmitters chooses an arbitrary codeword from its own codebook. The users simultaneously send these vectors.

- The receiver sees these codewords added together with the Gaussian noise Z.

- Optimal decoding consists of looking for m codewords, one from each codebook, such that the vector sum is closest to $\underline{Y}$ in Euclidean distance.

14.1.2 Continue

Example Two-users MAC



Define the channel capacity function

$$C(x) = \frac{1}{2}\log_2(1+x)$$

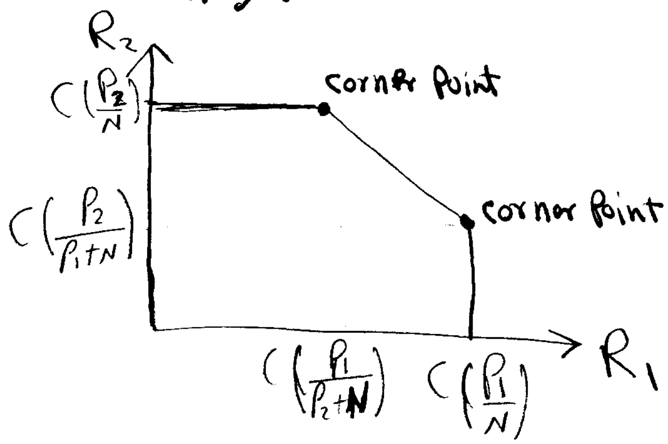
Then, the achievable rates are:

$$R_1 \leq C\left(\frac{P_1}{N}\right)$$

$$R_2 \leq C\left(\frac{P_2}{N}\right)$$

$$R_1 + R_2 \leq C\left(\frac{P_1+P_2}{N}\right)$$

- These upper bounds are achieved when $X_1 \sim \mathcal{N}(0,P_1)$ and $X_2 \sim \mathcal{N}(0,P_2)$



- Notice that the sum of the rates can be as large as $C\left(\frac{P_1+P_2}{N}\right)$ which is the rate acheived by a single transmitter sending with a power equal to the sum of the powers.

- **Corner Points**

To achieve the corner points, the decoding is done in two-stage process:

1- The receiver considers X_1 as noise and decodes X_2. This is possible if $R_2 < C\left(\frac{P_2}{P_1+N}\right)$

$\hat{X}_2 = D(Y)$

2- The receiver subtracts the decoded codeword $\hat{X}_2$ from the received signal and decode X_1

$$\hat{X}_1 = D(Y - \hat{X}_2)$$

- This is possible if $R_1 < C\left(\frac{P_1}{N}\right)$

Capacity Region

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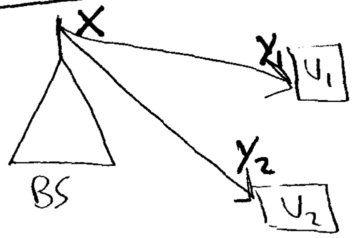
14.1.3 The Gaussian Broadcast Channel



$$Y_1 = X + Z_1$$

$$Y_2 = X + Z_2$$

- The sender has a power P.
- Z_1 and Z_2 are arbitrarily correlated Gaussian R.V. with variances N_1 and N_2.
- The sender wishes to send independent messages at rates R_1 and R_2 to receivers Y_1 and Y_2.
- The capacity region of the Gaussian broadcast channel is:

$$R_1 < C\left(\frac{\alpha P}{N_1}\right)$$

$$R_2 < C\left(\frac{(1-\alpha)P}{\alpha P + N_2}\right)$$

where α may be arbitrarily chosen $(0 \le \alpha \le 1)$ to trade ~~off~~ rate R_1 for rate R_2 as the transmitter wishes.

- Assume $N_1 < N_2$
  $\Rightarrow$ channel 1 is better than channel 2.
- To encode the messages, the transmitter generates two codebooks, one with power αP ~~at~~ rate R_1 and another with power $\bar{\alpha} P$ at rate R_2
- The transmitter takes $X(i)$ from the first codebook and $X(j)$ from the second codebook, where $i \in \{1, 2, \dots, 2^{nR_1}\}$
  $j \in \{1, 2, \dots, 2^{nR_2}\}$
- The transmitter sends
  $$X = X(i) + X(j)$$
- At Y_2, we have
  $$Y_2 = X(j) + X(i) + Z_1$$

since $N_2 > N_1$, Y_2 considered to ~~be~~ have a bad channel.

Decoding: consider $X(i)$ to be noise and look at the second codebook to find the closest codeword to the received vector Y_2.

the effective SNR in this case is

$$\frac{\bar{\alpha} P}{(\alpha P + N_2)}$$

14.1.3 Continue

- ~~Y_2~~ The good receiver Y_1 first decodes Y_2's codeword.

$$\hat{X}_2 = D(Y_1) = D(X(i) + X(j) + Z_2)$$

- Subtract $\hat{X}_2$ from Y_1 and then decode the first codeword.

$$\hat{X}_1 = D(Y_1 - \hat{X}_2)$$

14.1.4 The Gaussian Relay Channel

$$\begin{array}{ccc} & Y_1 : X_1 & \\ \nearrow & & \searrow \\ X & \longrightarrow & Y \end{array}$$

$$Y_1 = X + Z_1$$

$$Y = X + Z_1 + X_1 + Z_2$$

- where Z_1 and Z_2 are independent zero mean Gaussian R.V. with variance N_1 and N_2
- Sender X has power P and sender X_1 has power P_1.

Case 1 no cooperation

$$X \xrightarrow[SNR_1 = \frac{P}{N_1}]{} Y_1 : X_1 \xrightarrow[SNR_2 = \frac{P_1}{N_2}]{} Y$$

$$C = \min\left\{ C\left(\frac{P}{N_1}\right), C\left(\frac{P_1}{N_2}\right) \right\}$$

- Assume that the relay to the receiver channel is better than the sender X to the relay

$$\Rightarrow \frac{P_1}{N_2} \geq \frac{P}{N_1}$$

$$\Rightarrow C = C\left(\frac{P}{N_1}\right)$$

- Notice that the total noise in the system is $N = N_1 + N_2$
- By using a relay, the rate $C\left(\frac{P}{N_1+N_2}\right)$ is increased to $C\left(\frac{P}{N_1}\right)$

Case 2 Sender and Relay Cooperation.

First transmission

$$X \longrightarrow Y_1$$

$$Y_1 = \sqrt{\alpha}\, X + Z_1 \quad ; \text{where } \alpha \in [0,1]$$

$$\Rightarrow R_1 < C\left(\alpha \frac{P}{N_1}\right)$$

Second transmission

Relay and sender collaborate and partition the codeword

$$\begin{array}{ccc} & Y_1 : X_1 & \\ & & \searrow \\ X & \longrightarrow & Y \end{array}$$

$$Y = \sqrt{\bar{\alpha}}\, X + X_1 + Z_1 + Z_2$$

where $\bar{\alpha} = 1 - \alpha$

- In the second transmission, the relay and the sender cooperate in sendending the codeword to the receiver.
- The power of the sum at the receiver is $(\sqrt{\bar{\alpha}P} + \sqrt{P_1})^2$

$$\cancel{\Rightarrow R_2 < C(P} \quad \cancel{\Rightarrow R_2 < C(\bar{\alpha}P +}$$

$$\Rightarrow R_2 < C(\bar{\alpha}P + P_1 + 2\sqrt{\bar{\alpha}PP_1})$$

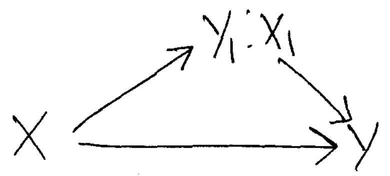
$$\Rightarrow C = \max_{0\le\alpha\le 1} \min\left\{ C\left(\frac{\bar{\alpha}P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{N_1+N_2}\right), C\left(\frac{\alpha P}{N_1}\right) \right\}$$

where $\bar{\alpha} = 1-\alpha$.

- Case 3: Cooperation ~~with~~

  - Notice in case 2 in the second transmition, the sender X only sends $\bar{\alpha}P$ of his power budget.
  - He can send another codeword at αP power which will be used for cooperation in the next cycle.

Thus, in this case.



$$Y = \sqrt{\alpha}X + \sqrt{\bar{\alpha}}X_1 + X_1 + Z_1 + Z_2$$

$$\Rightarrow R_2 < C\left(\frac{P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{N_1+N_2}\right)$$

And

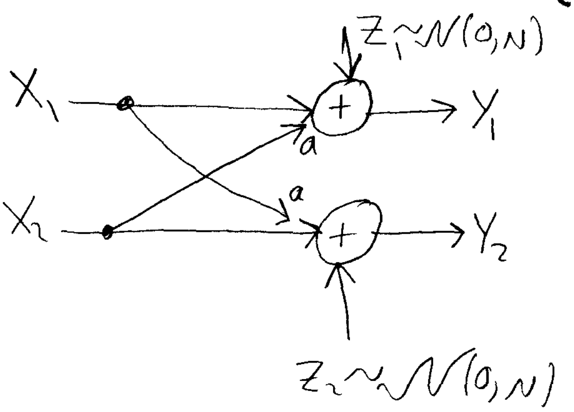
$$C = \max_{0\le\alpha\le 1} \min\left\{ C\left(\frac{P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{N_1+N_2}\right), C\left(\frac{\alpha P}{N_1}\right) \right\}$$

14.1.5 The Gaussian Interference Channel

For symmetric interference,

$$Y_1 = X_1 + \alpha X_2 + Z_1$$

$$Y_2 = X_2 + \alpha X_1 + Z_2$$

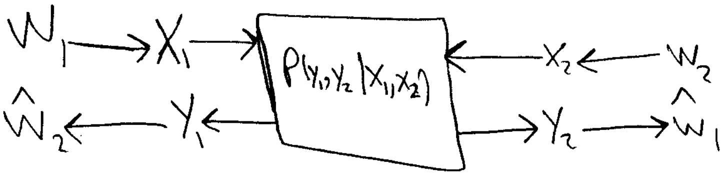


- If the receiver can cancel the interference ~~suessifuly~~ successfully
  $\Rightarrow$ the channel is the same as if there were no interference.
- The condition to ~~have to~~ be able to do this is to have high interference.
- Each sender generates his codebook with power P and rate $C\left(\frac{P}{N}\right)$
- If the interference is high such that $C\left(\frac{a^2 P}{P+N}\right) > C\left(\frac{P}{N}\right)$
  $\Rightarrow$ ~~the~~ each receiver can decode the signal from the other transmitter.

- The interference is subtracted from the received signal.
- The channel is interference free.
- Each can achieve the rate

$$R < C\left(\frac{P}{N}\right)$$

14.1.6 The Gaussian Two-Way channel



- This is similar to interference channel but with possible feedback from the second receiver.
- Similar to the interference channel, ~~to~~ the possible achievable rates are:

$$R_1 < C\left(\frac{P_1}{N_1}\right)$$

$$R_2 < C\left(\frac{P_2}{N_2}\right)$$