Tables of Transform Pairs

Students and engineers in communications and mathematics are confronted with transformations such as the z-Transform, the Fourier transform, or the Laplace transform. Often it is quite hard to quickly find the appropriate transform in a book or the Internet, much less to have a good overview of transformation pairs and corresponding properties.

In this document I present a handy collection of the most common transform pairs and properties of the

> ▷ **continuous-time frequency Fourier transform** $(2\pi f)$,

> ▷ **continuous-time pulsation Fourier transform** (ω),

> ▷ **z-Transform**,

> ▷ **discrete-time Fourier transform DTFT**, and

> ▷ **Laplace transform**

arranged in a table and ordered by subject. The properties of each transformation are indicated in the first part of each topic whereas specific transform pairs are listed afterwards.

Please note that, before including a transformation pair in the table, I verified their correctness. However, it is still possible that there might be some mistakes due to typos. I'd be grateful to everyone for dropping me a line and indicating me erroneous formulas.

Some useful conventions and formulas

Sinc function	$\mathrm{sinc}\,(x) \equiv \frac{\sin(x)}{x}$
Convolution	$f * g(t) = \int_{-\infty}^{+\infty} f(\tau)g^*(t-\tau)d\tau$
Parseval theorem	$\int_{-\infty}^{+\infty} f(t)g^*(t)dt = \int_{-\infty}^{+\infty} F(f)G^*(f)df$
	$\int_{-\infty}^{+\infty} |f(t)|^2 dt = \int_{-\infty}^{+\infty} |F(f)|^2 df$
Real part	$\Re\mathfrak{e}\{f(t)\} = \frac{1}{2}\left[f(t) + f^*(t)\right]$
Imaginary part	$\Im\mathfrak{m}\{f(t)\} = \frac{1}{2}\left[f(t) - f^*(t)\right]$
Sine / Cosine	$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$ $\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
Geometric sequences	$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ $\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$
	General case : $\sum_{k=m}^{n} x^k = \frac{x^m - x^{n+1}}{1-x}$

Table of Continuous-time Frequency Fourier Transform Pairs

$f(t) = \mathcal{F}^{-1}\left\{F(f)\right\} = \int_{-\infty}^{+\infty} f(t)e^{j2\pi ft}df$	$\xleftrightarrow{\mathcal{F}}$	$F(f) = \mathcal{F}\left\{f(t)\right\} = \int_{-\infty}^{+\infty} f(t)e^{-j2\pi ft}dt$
$f(t)$	$\xleftrightarrow{\mathcal{F}}$	$F(f)$
$f(-t)$	$\xleftrightarrow{\mathcal{F}}$	$F(-f)$
$f^*(t)$	$\xleftrightarrow{\mathcal{F}}$	$F^*(-f)$
$f(t)$ is purely real	$\xleftrightarrow{\mathcal{F}}$	$F(f) = F^*(-f)$ even/symmetry
$f(t)$ is purely imaginary	$\xleftrightarrow{\mathcal{F}}$	$F(f) = -F^*(-f)$ odd/antisymmetry
even/symmetry $f(t) = f^*(-t)$	$\xleftrightarrow{\mathcal{F}}$	$F(f)$ is purely real
odd/antisymmetry $f(t) = -f^*(-t)$	$\xleftrightarrow{\mathcal{F}}$	$F(f)$ is purely imaginary
time shifting $f(t - t_0)$	$\xleftrightarrow{\mathcal{F}}$	$F(f)e^{-j2\pi ft_0}$
$f(t)e^{j2\pi f_0 t}$	$\xleftrightarrow{\mathcal{F}}$	$F(f - f_0)$ frequency shifting
time scaling $f(af)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{|a|}F\left(\frac{f}{a}\right)$
$\frac{1}{|a|}f\left(\frac{f}{a}\right)$	$\xleftrightarrow{\mathcal{F}}$	$F(af)$ frequency scaling
$af(t) + bg(t)$	$\xleftrightarrow{\mathcal{F}}$	$aF(f) + bG(t)$
$f(t)g(t)$	$\xleftrightarrow{\mathcal{F}}$	$F(f) * G(f)$
$f(t) * g(t)$	$\xleftrightarrow{\mathcal{F}}$	$F(f)G(f)$
$\delta(t)$	$\xleftrightarrow{\mathcal{F}}$	1
$\delta(t - t_0)$	$\xleftrightarrow{\mathcal{F}}$	$e^{-j2\pi ft_0}$
1	$\xleftrightarrow{\mathcal{F}}$	$\delta(f)$
$e^{j2\pi f_0 t}$	$\xleftrightarrow{\mathcal{F}}$	$\delta(f - f_0)$
$e^{-a|t|}$ $a > 0$	$\xleftrightarrow{\mathcal{F}}$	$\frac{2a}{a^2 + 4\pi^2 f^2}$
$e^{-\pi t^2}$	$\xleftrightarrow{\mathcal{F}}$	$e^{-\pi f^2}$
$e^{j\pi t^2}$	$\xleftrightarrow{\mathcal{F}}$	$e^{j\pi\left(\frac{1}{4} - f^2\right)}$
$\sin\left(2\pi f_0 t + \phi\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{j}{2}\left[e^{-j\phi}\delta\left(f + f_0\right) - e^{j\phi}\delta\left(f - f_0\right)\right]$
$\cos\left(2\pi f_0 t + \phi\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{2}\left[e^{-j\phi}\delta\left(f + f_0\right) + e^{j\phi}\delta\left(f - f_0\right)\right]$
$f(t)\sin\left(2\pi f_0 t\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{j}{2}\left[F\left(f + f_0\right) - F\left(f - f_0\right)\right]$
$f(t)\cos\left(2\pi f_0 t\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{2}\left[F\left(f + f_0\right) + F\left(f - f_0\right)\right]$
$\sin^2\left(t\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{4}\left[2\delta(f) - \delta\left(f - \frac{1}{\pi}\right) - \delta\left(f + \frac{1}{\pi}\right)\right]$
$\cos^2\left(t\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{4}\left[2\delta(f) + \delta\left(f - \frac{1}{\pi}\right) + \delta\left(f + \frac{1}{\pi}\right)\right]$
$\text{rect}\left(\frac{t}{T}\right) = 1_{\left[-\frac{T}{2}, +\frac{T}{2}\right]}(t) = \begin{cases} 1 & |t| \leqslant \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$	$\xleftrightarrow{\mathcal{F}}$	$T\,\text{sinc}\,Tf$
$\text{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T} & |t| \leqslant T \\ 0 & |t| > T \end{cases}$	$\xleftrightarrow{\mathcal{F}}$	$T\,\text{sinc}^2\,Tf$
$u(t) = 1_{[0, +\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{j2\pi f} + \delta(f)$
$\text{sgn}\left(t\right) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{j\pi f}$
$\text{sinc}\left(Bt\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{B}\,\text{rect}\left(\frac{f}{B}\right) = \frac{1}{B}1_{\left[-\frac{B}{2}, +\frac{B}{2}\right]}(f)$
$\text{sinc}^2\left(Bt\right)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{B}\,\text{triang}\left(\frac{f}{B}\right)$
$\frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{F}}$	$(j2\pi f)^n F(f)$
$t^n f(t)$	$\xleftrightarrow{\mathcal{F}}$	$\frac{1}{(-j2\pi)^n}\frac{d^n}{df^n}F(f)$
$\frac{1}{1 + t^2}$	$\xleftrightarrow{\mathcal{F}}$	$\pi e^{-2\pi|f|}$

Table of Continuous-time Pulsation Fourier Transform Pairs

$x(t) = \mathcal{F}_\omega^{-1}\left\{X(\omega)\right\} = \int_{-\infty}^{+\infty} x(t) e^{j\omega t} d\omega$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega) = \mathcal{F}_\omega\left\{x(t)\right\} = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$
$x(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$
$x(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(-\omega)$
$x^*(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X^*(-\omega)$
$x(t)$ is purely real	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(f) = X^*(-\omega)$ even/symmetry
$x(t)$ is purely imaginary	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(f) = -X^*(-\omega)$ odd/antisymmetry
even/symmetry $x(t) = x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$ is purely real
odd/antisymmetry $x(t) = -x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$ is purely imaginary
time shifting $x(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega) e^{-j\omega t_0}$
$x(t) e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega - \omega_0)$ frequency shifting
time scaling $x(af)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{\|a\|} X\left(\frac{\omega}{a}\right)$
$\frac{1}{\|a\|} x\left(\frac{f}{a}\right)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(a\omega)$ frequency scaling
$a x_1(t) + b x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$a X_1(\omega) + b X_2(\omega)$
$x_1(t) x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
$x_1(t) * x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X_1(\omega) X_2(\omega)$
$\delta(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	1
$\delta(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$e^{-j\omega t_0}$
1	$\xleftrightarrow{\mathcal{F}_\omega}$	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$2\pi\delta(\omega - \omega_0)$
$e^{-a\|t\|}$ $a > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{2a}{a^2 + \omega^2}$
$e^{-at} u(t)$ $\Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{a + j\omega}$
$e^{-at} u(-t)$ $\Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{a - j\omega}$
$e^{\frac{t^2}{2\sigma^2}}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\sigma\sqrt{2\pi} e^{-\frac{\sigma^2\omega^2}{2}}$
$\sin(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$j\pi\left[e^{-j\phi}\delta(\omega + \omega_0) - e^{j\phi}\delta(\omega - \omega_0)\right]$
$\cos(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi\left[e^{-j\phi}\delta(\omega + \omega_0) + e^{j\phi}\delta(\omega - \omega_0)\right]$
$x(t)\sin(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{j}{2}\left[X(\omega + \omega_0) - X(\omega - \omega_0)\right]$
$x(t)\cos(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{2}\left[X(\omega + \omega_0) + X(\omega - \omega_0)\right]$
$\sin^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi^2\left[2\delta(f) - \delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$
$\cos^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi^2\left[2\delta(\omega) + \delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$
$\text{rect}\left(\frac{t}{T}\right) = 1_{[-\frac{T}{2}, +\frac{T}{2}]}(t) = \begin{cases} 1 & \|t\| \leqslant \frac{T}{2} \\ 0 & \|t\| > \frac{T}{2} \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$T\,\text{sinc}\left(\frac{\omega T}{2}\right)$
$\text{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{\|t\|}{T} & \|t\| \leqslant T \\ 0 & \|t\| > T \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$T\,\text{sinc}^2\left(\frac{\omega T}{2}\right)$
$u(t) = 1_{[0, +\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi\delta(f) + \frac{1}{j\omega}$
$\text{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{2}{j\omega}$
$\text{sinc}(Tt)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{T}\text{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T} 1_{[-2\pi T, +2\pi T]}(f)$
$\text{sinc}^2(Tt)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{T}\text{triang}\left(\frac{\omega}{2\pi T}\right)$
$\frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$(j\omega)^n X(\omega)$
$t^n f(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$j^n \frac{d^n}{df^n} X(\omega)$
$\frac{1}{t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$-j\pi\,\text{sgn}(\omega)$

Table of Z-Transform Pairs

$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$	$\xleftrightarrow{\mathcal{Z}}$	$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	*ROC*
$x[n]$	$\xleftrightarrow{\mathcal{Z}}$	$X(z)$	R_x
$x[-n]$	$\xleftrightarrow{\mathcal{Z}}$	$X(\frac{1}{z})$	$\frac{1}{R_x}$
$x^*[n]$	$\xleftrightarrow{\mathcal{Z}}$	$X^*(z^*)$	R_x
$x^*[-n]$	$\xleftrightarrow{\mathcal{Z}}$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
$\Re\{x[n]\}$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
$\Im\{x[n]\}$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting $\quad x[n-n_0]$	$\xleftrightarrow{\mathcal{Z}}$	$z^{-n_0}X(z)$	R_x
$a^n x[n]$	$\xleftrightarrow{\mathcal{Z}}$	$X\left(\frac{z}{a}\right)$	$|a|R_x$
downsampling by N $\quad x[Nn] \quad N\in\mathbb{N}_0$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{N}\sum_{k=0}^{N-1} X\left(W_N^k z^{\frac{1}{N}}\right) \quad W_N = e^{-\frac{j2\omega}{N}}$	R_x
$ax_1[n] + bx_2[n]$	$\xleftrightarrow{\mathcal{Z}}$	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
$x_1[n]x_2[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2\pi j}\oint X_1(u)X_2\left(\frac{z}{u}\right)u^{-1}du$	$R_x \cap R_y$
$x_1[n] * x_2[n]$	$\xleftrightarrow{\mathcal{Z}}$	$X_1(z)X_2(t)$	$R_x \cap R_y$
$\delta[n]$	$\xleftrightarrow{\mathcal{Z}}$	1	$\forall\, z$
$\delta[n-n_0]$	$\xleftrightarrow{\mathcal{Z}}$	z^{-n_0}	$\forall\, z$
$u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-1}$	$|z| > 1$
$-u[-n-1]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-1}$	$|z| < 1$
$nu[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{(z-1)^2}$	$|z| > 1$
$n^2 u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z+1)}{(z-1)^3}$	$|z| > 1$
$n^3 u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$|z| > 1$
$(-1)^n$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z+1}$	$|z| < 1$
$a^n u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-a}$	$|z| > |a|$
$-a^n u[-n-1]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-a}$	$|z| < |a|$
$a^{n-1}u[n-1]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{z-a}$	$|z| > |a|$
$na^n u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{az}{(z-a)^2}$	$|z| > |a|$
$n^2 a^n u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{az(z+a)}{(z-a)^3}$	$|z| > |a|$
$e^{-an}u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-e^{-a}}$	$|z| > |e^{-a}|$
$\begin{cases} a^n & n = 0,\dots,N-1 \\ 0 & otherwise \end{cases}$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$|z| > 0$
$\sin(\omega_0 n)\,u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z\sin(\omega_0)}{z^2 - 2\cos(\omega_0)z + 1}$	$|z| > 1$
$\cos(\omega_0 n)\,u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z-\cos(\omega_0))}{z^2 - 2\cos(\omega_0)z + 1}$	$|z| > 1$
$a^n \sin(\omega_0 n)\,u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{za\sin(\omega_0)}{z^2 - 2a\cos(\omega_0)z + a^2}$	$|z| > a$
$a^n \cos(\omega_0 n)\,u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z-a\cos(\omega_0))}{z^2 - 2a\cos(\omega_0)z + a^2}$	$|z| > a$
$nx[n]$	$\xleftrightarrow{\mathcal{Z}}$	$-z\frac{d}{dz}X(z)$	R_x
$\frac{x[n]}{n}$	$\xleftrightarrow{\mathcal{Z}}$	$-\int_0^z \frac{X(z)}{z}dz$	R_x
$\frac{\prod_{i=1}^m (n-i+1)}{a^m m!}a^m u[n]$	$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{(z-a)^{m+1}}$	

Please note : $\frac{z}{z-1} = \frac{z^{-1}}{1-z^{-1}}$

Table of Discrete Time Fourier Transform (DTFT) Pairs

$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$

$x[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$
$x[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{-j\omega})$
$x^*[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X^*(e^{-j\omega})$

$x[n]$ is purely real	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ even/symmetry
$x[n]$ is purely imaginary	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$ odd/antisymmetry
even/symmetry $x[n] = x^*[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$ is purely real
odd/antisymmetry $x[n] = -x^*[-n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})$ is purely imaginary

time shifting $x[n-n_0]$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j\omega})e^{-j\omega n_0}$
$x[n]e^{j\omega_0 n}$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{j(\omega-\omega_0)})$ frequency shifting
downsampling by N $x[Nn]$ $N \in \mathbb{N}_0$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{N}\sum_{k=0}^{N-1} X(e^{j\frac{\omega-2\pi k}{N}})$
upsampling by N $\begin{cases} x\left[\frac{n}{N}\right] & n = kN \\ 0 & otherwise \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$X(e^{jN\omega})$

$ax_1[n] + bx_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
$x_1[n]x_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X_1(e^{j\omega}) * X_2(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X_1(e^{j(\omega-\sigma)})X_2(e^{j\sigma})d\sigma$
$x_1[n] * x_2[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$X_1(e^{j\omega})X_2(e^{j\omega})$

$\delta[n]$	$\overset{DTFT}{\Longleftrightarrow}$	1
$\delta[n-n_0]$	$\overset{DTFT}{\Longleftrightarrow}$	$e^{-j\omega n_0}$
1	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\delta}(\omega-\omega_0) = \sum_{k=-\infty}^{+\infty} \delta(\omega-\omega_0+2\pi k)$

$u[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-e^{-j\omega}} + \frac{1}{2}\tilde{\delta}(\omega)$
$a^n u[n]$ $(|a| < 1)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{\left(1-ae^{-j\omega}\right)^2}$

$\sin(\omega_0 n + \phi)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{j}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) - e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$
$\cos(\omega_0 n + \phi)$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k) + e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$

$\frac{\sin(\omega_c n)}{n} = \omega_c \operatorname{sinc}(\omega_c n)$	$\overset{DTFT}{\Longleftrightarrow}$	$\tilde{\operatorname{rect}}\left(\frac{\omega}{\omega_c}\right) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$
Window : $\operatorname{rect}\left(\frac{n}{M}\right) = \begin{cases} 1 & |n| \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega(M+\frac{1}{2})]}{\sin(\omega/2)}$
MA : $\operatorname{rect}\left(\frac{n}{M} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$
MA : $\operatorname{rect}\left(\frac{n}{M-1} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M-1 \\ 0 & \text{otherwise} \end{cases}$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{\sin[\omega M/2]}{\sin(\omega/2)}e^{-j\omega(M-1)/2}$

$nx[n]$	$\overset{DTFT}{\Longleftrightarrow}$	$j\frac{d}{d\omega}X(e^{j\omega})$
$x[n] - x[n-1]$	$\overset{DTFT}{\Longleftrightarrow}$	$(1 - e^{-j\omega})X(e^{j\omega})$
$\frac{a^n \sin[\omega_0(n+1)]}{\sin \omega_0}u[n]$ $|a| < 1$	$\overset{DTFT}{\Longleftrightarrow}$	$\frac{1}{1-2a\cos(\omega_0 e^{-j\omega})+a^2 e^{-j2\omega}}$

Some remarks

$$\tilde{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega+2\pi k) \qquad\qquad \tilde{\operatorname{rect}}(\omega) = \sum_{k=-\infty}^{+\infty} \operatorname{rect}(\omega+2\pi k)$$

Parseval :

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega$$

Table of Laplace Transform Pairs

$f(t) = {}^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds$	$\xleftrightarrow{\mathcal{L}}$	$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$
$f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s)$
$f(t-a) \quad t \geqslant a > 0$	$\xleftrightarrow{\mathcal{L}}$	$a^{-as}F(s)$
$e^{-at}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s+a)$
$f(at) \quad a > 0$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{a}F(\frac{s}{a})$
$af_1(t) + bf_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$aF_1(s) + bF_2(s)$
$f_1(t)f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s) * F_2(s)$
$f_1(t) * f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s)F_2(s)$
$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
1	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
t	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
$1 - e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
$\frac{1}{a}e^{-\frac{t}{a}}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{1+as}$
$\frac{1}{a}\left(1 - e^{-at}\right)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s+n+1}$
$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
$f'(t) = \frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
$f''(t) = \frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s) - sf(0) - f'(0)$
In general : $\quad f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
$\int_0^t f(u)du$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$