EE 204
Lecture 13
Mesh Analysis - Introduction

Definition of Mesh

The circuit contains four windows (meshes).

A mesh is simply a *window* in an electric circuit

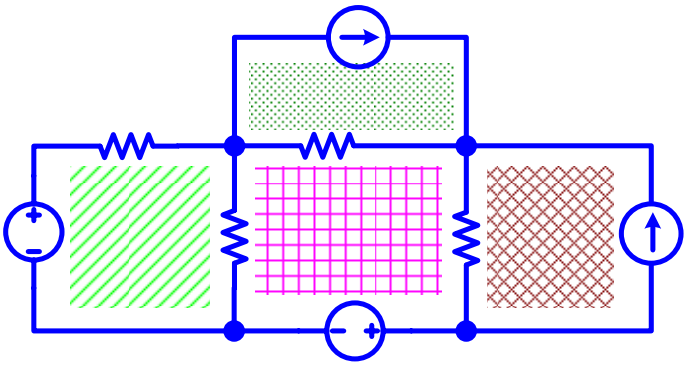

Figure 1

Currents through Elements & Mesh Currents:

The currents i_a , i_b , i_c are currents through elements

KCL at node 1 $\Rightarrow$ $i_a = i_b + i_c$ $\Rightarrow$ $i_b = i_a - i_c$

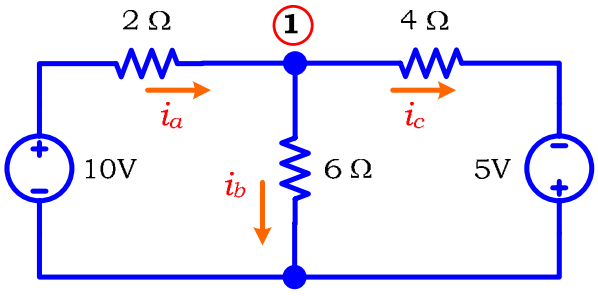

Figure 2

The *imaginary* currents i_1 & i_2 are *mesh* currents

We imagine i_1 to circulate around mesh 1 (CW)

We imagine i_2 to circulate around mesh 2 (also CW)

$i_a = i_1$ (because only mesh current i_1 goes through 2Ω and $10V$)

$i_c = i_2$ (because only mesh current i_2 goes through 4Ω and $5V$)

$i_b = i_a - i_b = i_1 - i_2$ (*two mesh* currents i_1 & i_2 go through 6Ω in *opposite* directions)

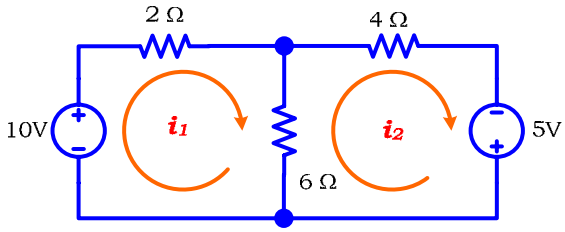

Figure 3

Example 1:

Express the currents through elements (CTE) i_x , i_y , i_z , i_w in terms of mesh currents (MC) i_1 & i_2 .

Solution:

$i_x = i_1$

$i_y = -i_3$

$i_z = i_1 - i_2$

$i_w = i_3 - i_1$

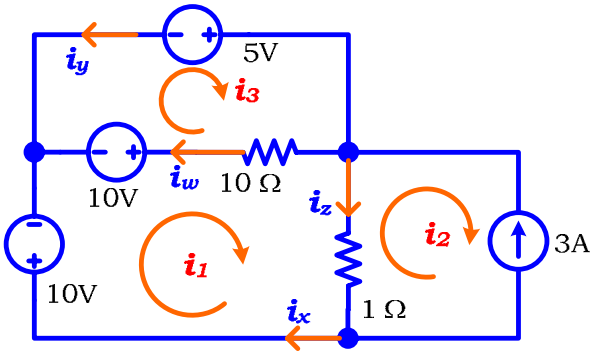

Figure 4

We know all MC $\Rightarrow$ We know all CTE

Number of MC $\leq$ Number of CTE

Mesh Analysis (without Current Sources):

The Mesh Analysis procedure for circuits *without* current sources will be considered first.

We will learn the *basic* Mesh Analysis procedure through a simple example.

Example 2:

Calculate the mesh currents i_1 & i_2.

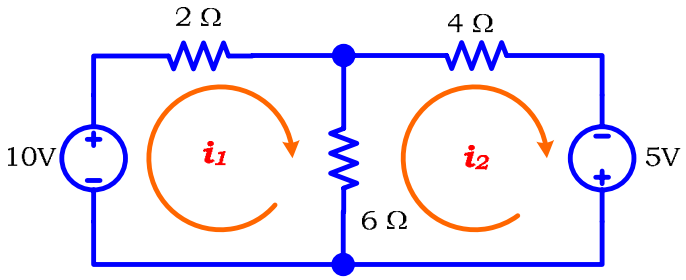

Figure 5

Solution:

Procedure:

1- KVL around mesh 1 $\Rightarrow$ $-10+V_a+V_b=0$

2- Ohm's Law $\Rightarrow$ $-10+2i_a+6i_b=0$

3- KCL $\Rightarrow$ $-10+2i_1+6(i_1-i_2)=0$ [CTE are expressed in terms of MC]

4- Simplify $\Rightarrow$ $8i_1-6i_2=10$ (1)

Repeat the same procedure for the remaining meshes:

1- KVL around mesh 2 $\Rightarrow$ $-V_b+V_c-5=0$

2- Ohm's Law $\Rightarrow$ $-6i_b+4i_c-5=0$

3- KCL $\Rightarrow$ $-6(i_1-i_2)+4i_2-5=0$ [CTE are expressed in terms of MC]

4- Simplify $\Rightarrow$ $-6i_1+10i_2=5$ (2)

Equations (1) & (2) contain only the *mesh unknowns* i_1 & i_2.

Solving (1) & (2) $\Rightarrow$ $i_1 = 2.955\text{A}$ & $i_2 = 2.273\text{A}$

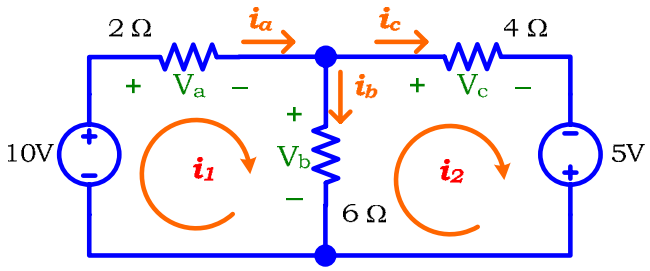

Figure 6

Mesh Analysis procedure: KVL $\Rightarrow$ Ohm's Law $\Rightarrow$ KCL $\equiv$ VOC

Repeat the previous example by combining steps 1 & 2 & 3 into a single step:

Currents through resistors = CTR

Always imagine CTR to be in the *same direction as KVL*

Express the *imagined* CTR in terms of MC

Mesh 1: KVL & Ohm's Law & KCL $\Rightarrow$ $-10 + 2i_1 + 6(i_1 - i_2) = 0$ $\Rightarrow$ $8i_1 - 6i_2 = 10$ (1)

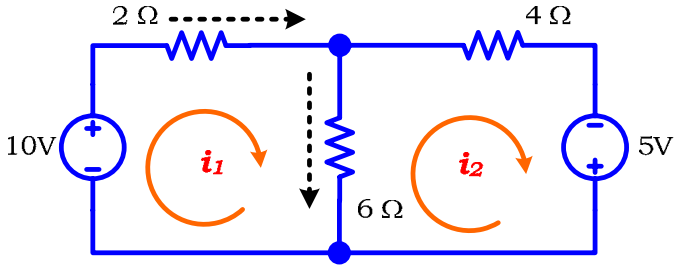

Figure 7

Mesh 2: KVL & Ohm's Law & KCL $\Rightarrow$ $6(i_2 - i_1) + 4i_2 - 5 = 0$ $\Rightarrow$ $-6i_1 + 10i_2 = 5$ (2)

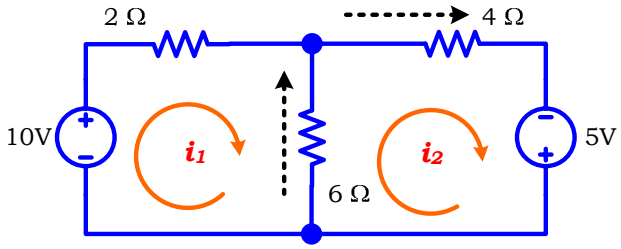

Figure 8

Example 3: Calculate the mesh currents i_1 & i_2 & i_3

Solution:

Mesh 1: $\Rightarrow \quad 4i_1 - 8 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad 10i_1 - 6i_2 = 8 \qquad (1)$

Mesh 2: $\Rightarrow \quad 6(i_2 - i_1) + 8(i_2 - i_3) + 12 = 0 \quad \Rightarrow \quad -6i_1 + 14i_2 - 8i_3 = -12 \qquad (2)$

Mesh 3: $\Rightarrow \quad 2i_3 + 8(i_3 - i_2) + 8 = 0 \quad \Rightarrow \quad -8i_2 + 10i_3 = -8 \qquad (3)$

Solving (1) & (2) & (3) $\Rightarrow \quad i_1 = -1.24\text{A} \;\; \& \;\; i_2 = -3.40\text{A} \;\; \& \;\; i_3 = -3.52\text{A}$

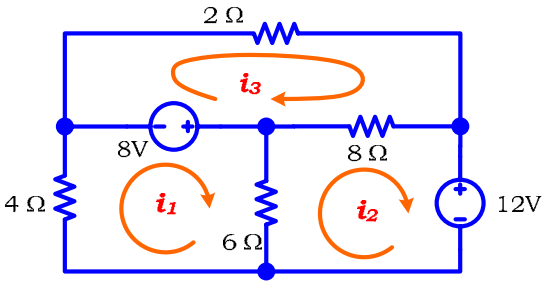

Figure 9