EE 204
Lecture 11
Nodal Analysis – Introduction

Definition of Essential Nodes

The essential nodes of the circuit are labeled "0" , "1" , "2" , "3" , "4"

A*ll points* that are connected by a *short* circuit *belong* to the *same essential* node.

All points in the lower part of the circuit are connected by a *short circuit*, they all belong to node "0"

The same applies to nodes "1" , "2", "3" , "4"

For this circuit $\Rightarrow$ 5 essential nodes

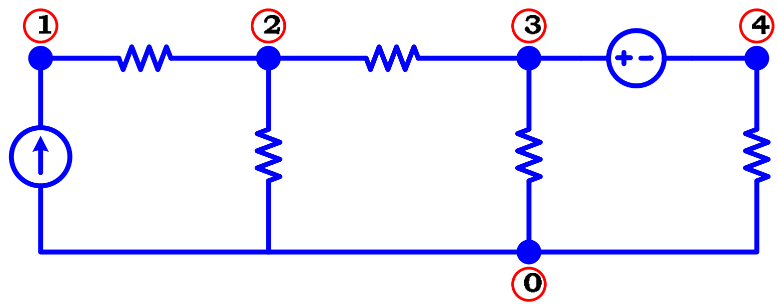

Figure 1

Example 1:

Label the essential nodes starting from "0" onwards.

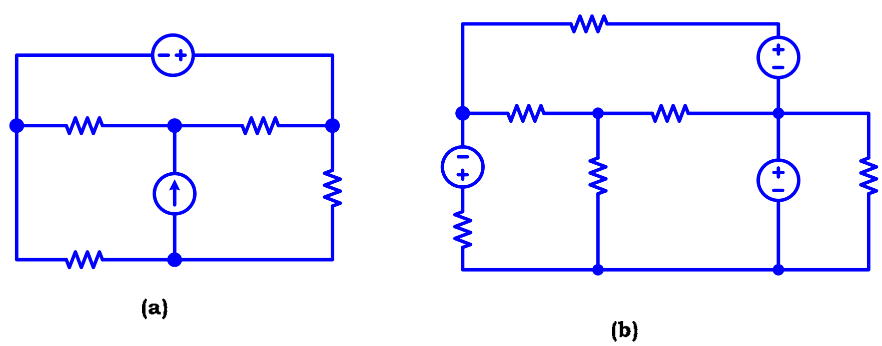

Figure 2

Solution:

a) 4 essential nodes.

b) 6 essential nodes.

We can label the essential nodes as we like.

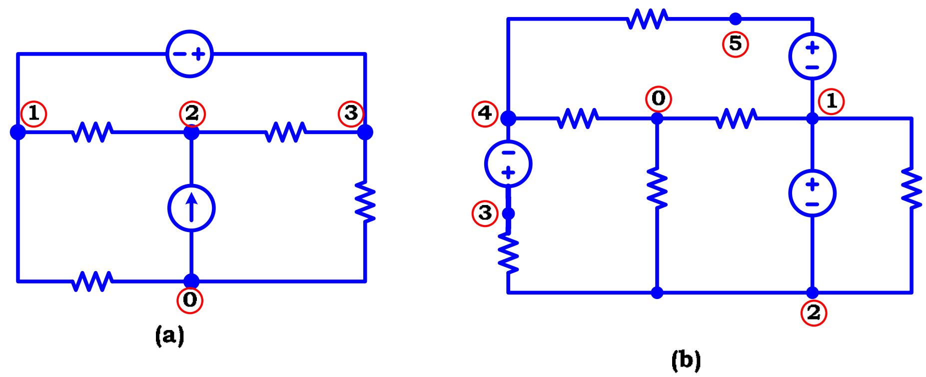

Figure 3

The Reference Node

The node labeled “0” is called a *reference node.*

We will see later that the *reference* node always has *zero Nodal Voltage*.

Other possible *labels* for the reference node are shown in the figure

Figure 4

Nodal Voltages

We associate a voltage with *every essential* node. These voltages are called Nodal Voltages.

$V_0 \ , \ V_1 \ , \ V_2 \ , \ V_3 \qquad \Leftrightarrow$ Nodal Voltages of essential nodes 0, 1 ,2 ,3

The nodal voltage V_i is the voltage drop *from* node “ i ” *to* the reference node “0”

$$\Downarrow$$

$$V_i \equiv V_{i,0}$$

$$\Downarrow$$

$$V_0 \equiv V_{0,0} \qquad V_1 \equiv V_{1,0} \qquad V_2 \equiv V_{2,0} \qquad V_3 \equiv V_{3,0}$$

Reference node $\Rightarrow$ $V_0 \equiv V_{0,0} = 0$ $\Rightarrow$ Reference node *always* has zero nodal voltage.

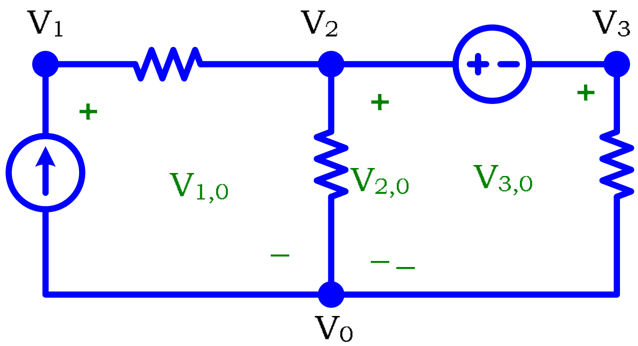

Figure 5

Relationship between Nodal Voltages and Voltages across Elements

Nodal voltages (NV) $\Rightarrow$ V_1 , V_2 , V_3

The reference *nodal voltage* $V_0 = 0$ is not shown (it is always not shown)

Another label is used to mark the reference node in this case.

Voltages across elements (VAE) $\Rightarrow$ V_a , V_b , V_c , V_d , V_e

KVL

$\Downarrow$

$-V_a + V_{10} = 0 \quad \Rightarrow \quad -V_a + V_1 = 0 \quad \Rightarrow \quad V_a = V_1 \quad \Rightarrow \quad V_a = V_1 - V_0$

$-V_1 + V_b + V_2 = 0 \quad \Rightarrow \quad V_b = V_1 - V_2$

$V_c = V_2 \quad \Rightarrow \quad V_c = V_2 - V_0$

$-V_2 + V_d + V_3 = 0 \quad \Rightarrow \quad V_d = V_2 - V_3$

$V_e = V_3 \quad \Rightarrow \quad V_e = V_3 - V_0$

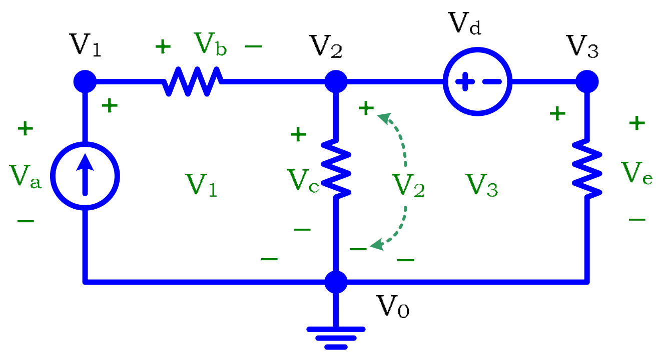

Figure 6

V_x = voltage across element

V_i = Nodal voltage on the (+) side of V_x

V_j = Nodal voltage on the (-) side of V_x

$$\Downarrow$$

$$V_x = V_i - V_j$$

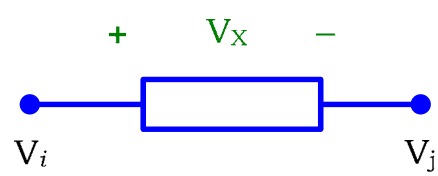

Figure 7

Example 2:

Express the VAE V_a , V_b , V_c , V_d , V_e in terms of the NV V_1 , V_2 , V_3 .

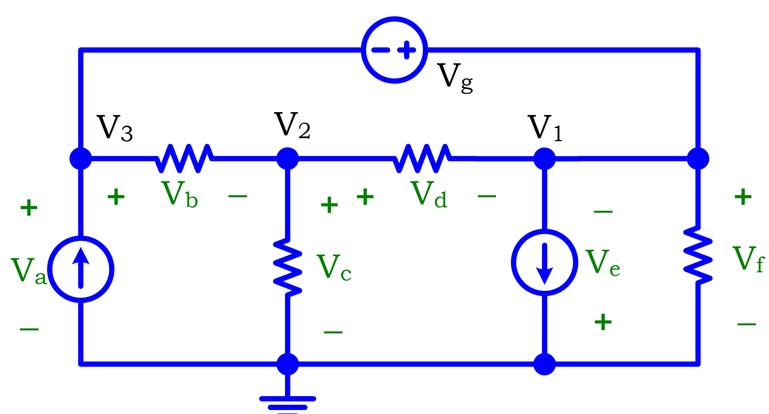

Figure 8

Solution:

$V_a = V_3 - V_0 = V_3 - 0 = V_3$

$V_b = V_3 - V_2$

$V_c = V_2 - 0 = V_2$

$V_d = V_2 - V_1$

$V_e = 0 - V_1 = -V_1$ (why?)

$V_f = V_1 - 0 = V_1$

$V_g = V_1 - V_3$

if we know *all* NV $\Rightarrow$ we know *all* VAE

Nodal Analysis Procedure (without voltage sources)

If the circuit contains *no voltage* sources, the nodal analysis is very easy to apply.

The Nodal analysis *procedure* is best illustrated by an example.

Example 3:

Calculate the nodal voltages V_1 & V_2

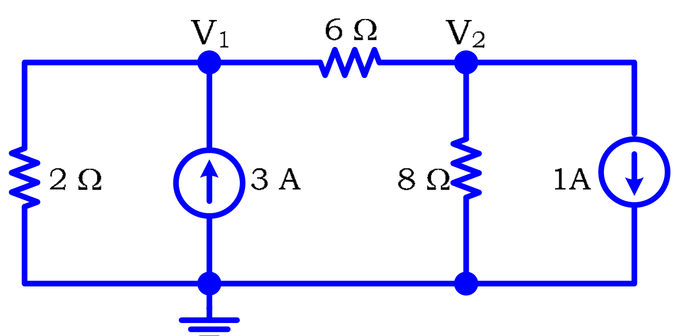

Figure 9

Solution:

The procedure is started with KCL.

Current leaving a node we use (+) & Current entering a node we use (-).

Step1: KCL at node 1 $\Rightarrow$ $i_a - 3 + i_b = 0$

Step 2: Ohm's Law $\Rightarrow$ $\frac{V_a}{2} - 3 + \frac{V_b}{6} = 0$

Step 3: Replace VAE with NV $\Rightarrow$ $\frac{V_1 - 0}{2} - 3 + \frac{V_1 - V_2}{6} = 0$

Step 4: Simplify $\Rightarrow$ $3V_1 - 18 + V_1 - V_2 = 0$ $\Rightarrow$ $4V_1 - V_2 = 18$ (1)

Repeat the same procedure for the remaining nodes:

KCL at node 2 $\Rightarrow$ $-i_b + i_c + 1 = 0$

Ohm' Law $\Rightarrow$ $-\frac{V_b}{6} + \frac{V_c}{8} + 1 = 0$

Replace VAE with NV $\Rightarrow$ $-\frac{V_1 - V_2}{6} + \frac{V_2 - 0}{8} + 1 = 0$

Simplify $-4V_1 + 4V_2 + 3V_2 + 24 = 0$ $\Rightarrow$ $-4V_1 + 7V_2 = -24$ (2)

Solve the resulting equations simultaneously:

(1) + (2) $\Rightarrow$ $6V_2 = -6$ $\Rightarrow$ $V_2 = -1V$

Substitute in (1) $\Rightarrow$ $4V_1 - (-1) = 18$ $\Rightarrow$ $V_1 = \frac{17}{4}V$

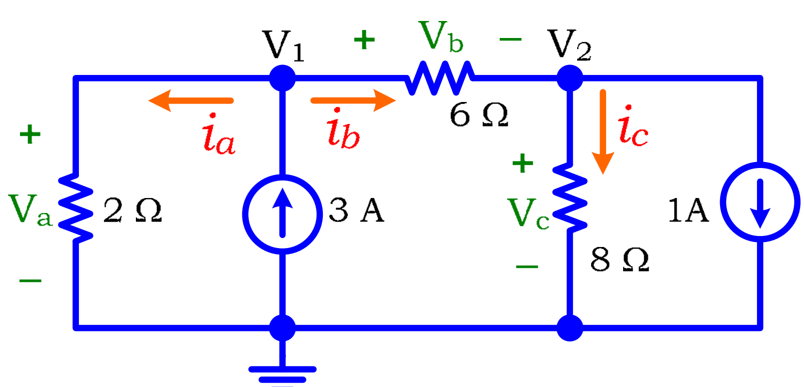

Figure 10

Nodal analysis $\Rightarrow$ step 1) K<u>C</u>L $\Rightarrow$ step 2) <u>O</u>hm's Law $\Rightarrow$ step 3) K<u>V</u>L

Nodal analysis $\Rightarrow$ COV

Example 3:

Derive the nodal equations (do not simplify and do not solve)

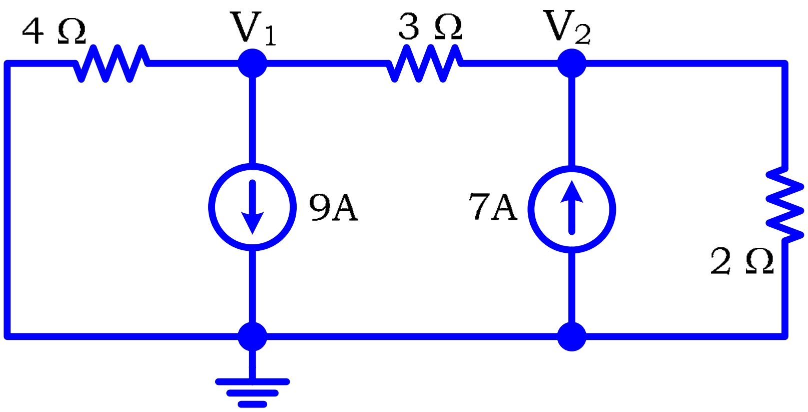

Figure 11

Solution:

This time, we will combine steps 2 & 3 into *a single step*.

The voltages across resistances will not be shown explicitly.

Node 1:

KCL $\Rightarrow$ $i_a + 9 + i_b = 0$

Ohm's Law then KVL $\Rightarrow$ $\frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0$ $\Rightarrow$ eqn. (1)

Node 2:

KCL $\Rightarrow$ $-i_b - 7 + i_c = 0$

Ohm's Law then KVL $\Rightarrow$ $-\frac{V_1 - V_2}{3} - 7 + \frac{V_2 - 0}{2} = 0$ $\Rightarrow$ eqn. (2)

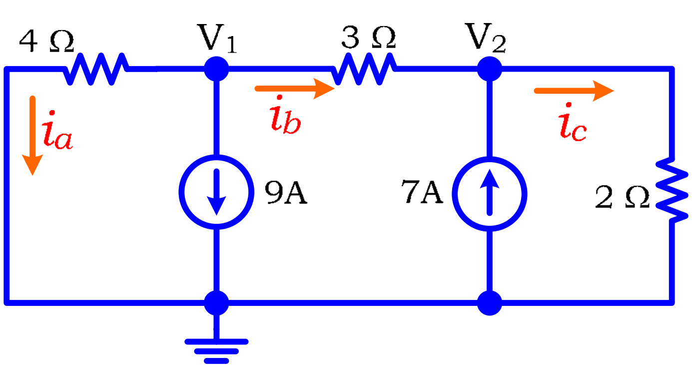

Figure 12

Example 4:

Repeat the previous example by combining steps 1, 2 , 3 into a single step.

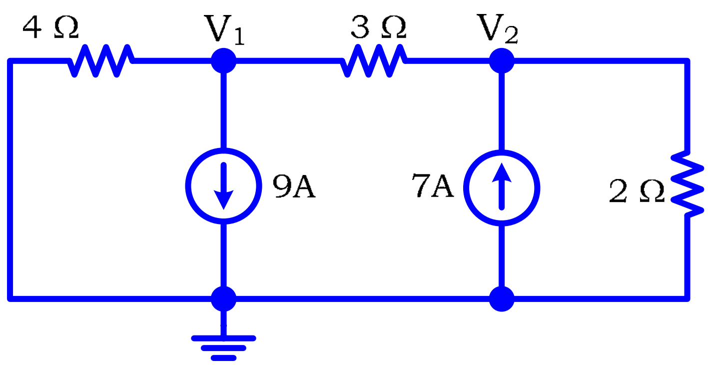

Figure 13

Solution:

This time we will not show currents through resistances or voltages across resistances.

Important: we will *imagine* currents through *resistors* to be ***leaving*** the node under consideration.

Node 1:

$$\frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \qquad \Rightarrow \qquad \text{eqn. (1)}$$

Node 2:

$$\frac{V_2 - V_1}{3} - 7 + \frac{V_2 - 0}{2} = 0 \qquad \Rightarrow \qquad \text{eqn. (2)}$$

We obtained the same equations directly.

Example 5:

Write the nodal equations directly (do not simplify or solve)

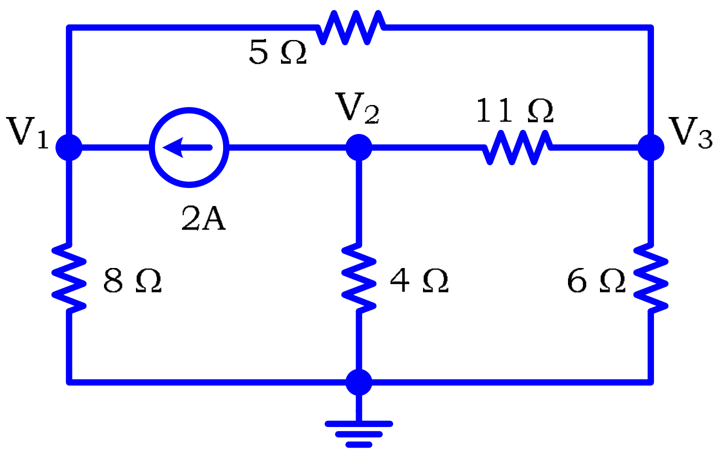

Figure 14

Solution:

Again imagine currents through resistors to be *leaving* the node under consideration.

Node 1 $\Rightarrow$ $\frac{V_1}{8} - 2 + \frac{V_1 - V_3}{5} = 0$

Node 2 $\Rightarrow$ $2 + \frac{V_2}{4} + \frac{V_2 - V_3}{11} = 0$

Node 3 $\Rightarrow$ $\frac{V_3 - V_2}{11} + \frac{V_3 - V_1}{5} + \frac{V_3}{6} = 0$

The last equation has *no constant* term, because *no current sources* are connected to node 3.

Example 6:

a) Calculate the nodal voltages V_1 , V_2 and V_3

b) Use the above results to calculate i_1 & i_2

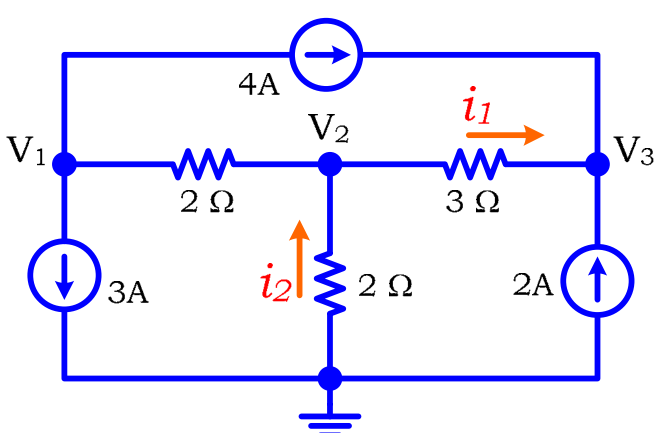

Figure 15

Solution:

a)

We will *ignore* the currents i_2 & i_3 and again *imagine* currents through resistors to be *leaving* the node *under consideration*.

After all if i_1 and i_2 *were not shown* in the circuit, we should still obtain the *same* nodal voltages.

Node 1 $\Rightarrow$ $3+4+\dfrac{V_1-V_2}{2}=0$ $\Rightarrow$ $V_1-V_2=-14$ (1)

Node 2 $\Rightarrow$ $\dfrac{V_2-V_1}{1}+\dfrac{V_2-0}{2}+\dfrac{V_2-V_3}{3}=0$ $\Rightarrow$ $-6V_1+11V_2-2V_3=0$ (2)

Node 3 $\Rightarrow$ $\dfrac{V_3-V_2}{3}-4-2=0$ $\Rightarrow$ $-V_2+V_3=18$ (3)

Solving (1), (2), (3) simultaneously:

$V_1=-30V$ & $V_2=-16V$ & $V_3=2V$

[After you solve the equations, you *should always* check that the results satisfy those equations]

b)

$$i_1=\frac{V_2-V_3}{3}=\frac{-16-2}{3}=-6A$$

$$i_2=\frac{0-V_2}{2}=\frac{0-(-16)}{2}=8A$$

From the examples done in this class it easy to conclude the following:

1) $N_u=N_{ess}-1$

N_u = Number of unknown nodal voltages

N_{ess} = Number of essential nodes

2) $N_u \le N_{ele}$

N_{ele} =Number of unknown voltages across elements

Thus the nodal analysis is efficient because the number of unknown voltages is *reduced*.

3) The procedure remains the *same* if no voltage sources are present in the circuit. We will deal with this case in the next class.