

# EE 204

## Lecture 09

### Thevenin Equivalent Circuits

#### Thevenin Equivalent Circuit:

Given an electrical circuit  $\Rightarrow$  split it into circuits A & B

Call circuit B “*the load*”

Notice that circuits A & B are connected by the *two* terminals *a* & *b*

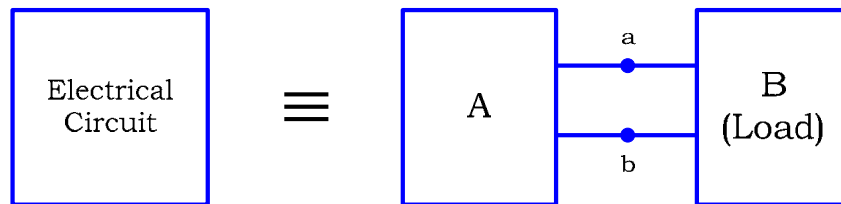


Figure 1

#### Thevenin theorem:

*In general*, it is possible to replace circuit A with a voltage source in *series* with a resistor.

The voltage source is labeled  $V_{th}$  (Thevenin voltage)

The resistance is labeled  $R_{th}$  (Thevenin resistance)

The new circuit is called the *Thevenin Equivalent Circuit* (TEC) of circuit A

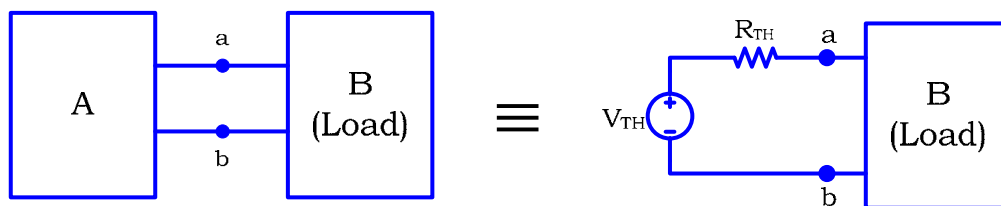


Figure 2

We will consider *four* methods for finding the TEC.

*Only two* methods will be presented in this class.

#### Finding the TEC (Method 1):

Calculate 1)  $V_{th} = V_{oc}$  & 2)  $R_{th} = \frac{V_{oc}}{i_{sc}}$

**1) Calculate  $V_{th}$ :**

First we remove the load (circuit B)

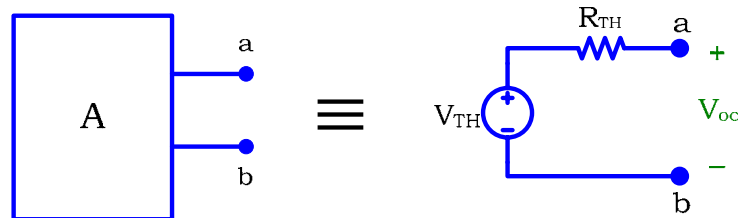
The terminals  $a$  &  $b$  become *open-circuited* (no load)

The resulting voltage across the terminals  $a$  &  $b$  is labeled  $V_{oc}$  (Open-circuit voltage)

Current through the *open circuit* is zero  $\Rightarrow$  no current flows through  $R_{th}$

$$\text{KVL} \Rightarrow -V_{th} + 0R_{th} + V_{oc} = 0 \Rightarrow V_{th} = V_{oc}$$

We calculate the *open circuit voltage* of circuit A and equate it to  $V_{th}$ .



**Figure 3**

**2) Calculate  $R_{th}$  using  $R_{th} = \frac{V_{oc}}{i_{sc}}$**

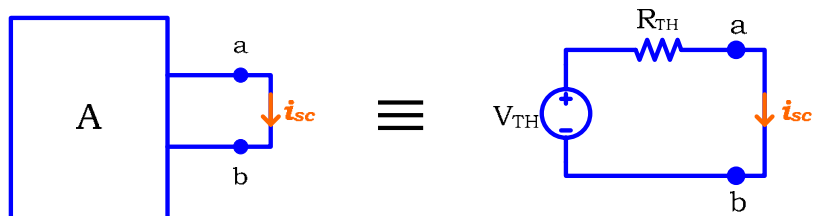
Remove the load and place a *short circuit* across “ $a-b$ ”

The current that flows in the short circuit is labeled  $i_{sc}$

The voltage across the *short circuit* is zero

$$\text{KVL} \Rightarrow -V_{th} + i_{sc}R_{th} + 0 = 0 \Rightarrow V_{th} = i_{sc}R_{th}$$

$$\therefore R_{th} = \frac{V_{th}}{i_{sc}} \quad [\text{this can be used to calculate } R_{th}]$$



**Figure 4**

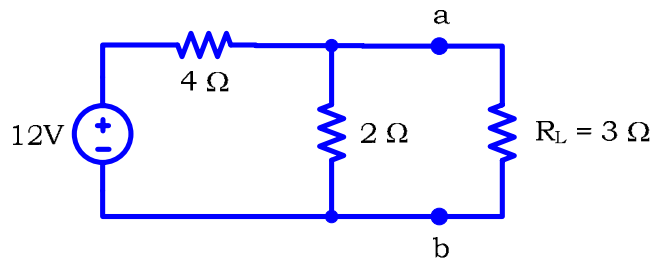
**Summary:**

1) Calculate  $V_{oc} = V_{th}$

2) Calculate  $i_{sc} \Rightarrow$  use  $R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{V_{th}}{i_{sc}}$

**Example 1:**

Calculate the TEC *seen* by the  $3\Omega$  resistor.



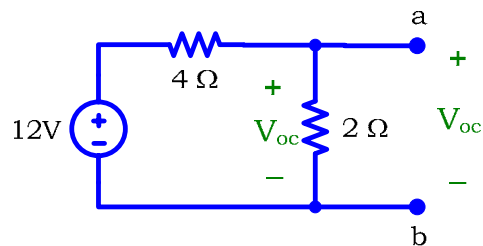
**Figure 5**

Solution:

Remove the  $3\Omega$  load

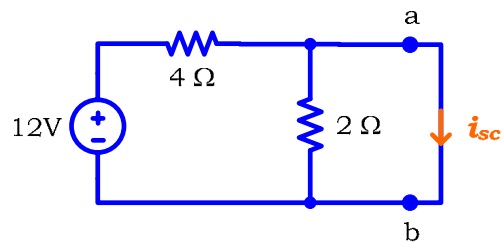
The voltage  $V_{oc}$  is also across the  $2\Omega$  resistor

$$\text{VDR} \Rightarrow V_{oc} = \frac{2}{2+4}(12) = 4V$$



**Figure 6**

Place a short circuit



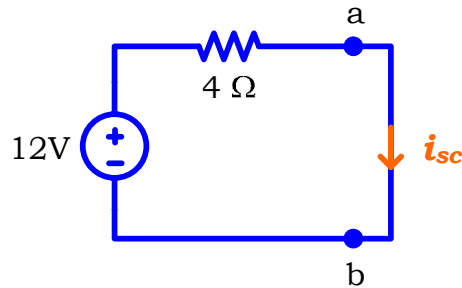
**Figure 7**

$$2\Omega \parallel \text{short circuit} \Rightarrow R_{eq} = \frac{0 \times 2}{0 + 2} = 0\Omega \text{ (short circuit)}$$

[A short circuit in parallel with *any resistance* is equivalent to a short circuit]

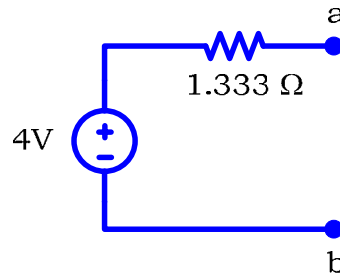
$$\therefore i_{sc} = \frac{12}{4} = 3A$$

$$\therefore R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{4}{3} = 1.333\Omega$$



**Figure 8**

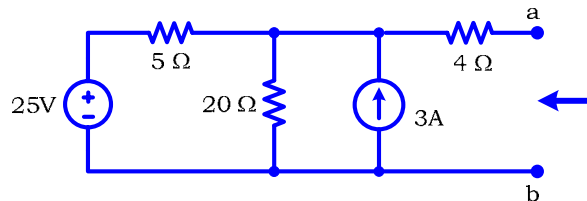
$\therefore$  the TEC seen by the 3Ω resistance is:



**Figure 9**

**Example 2:**

Calculate the TEC to the left of “a–b” [load has already been removed].



**Figure 10**

The current through the 4Ω resistor is zero, because of the open circuit.

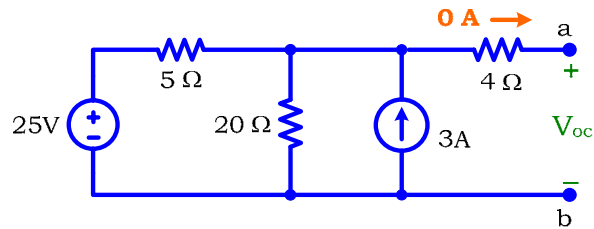


Figure 11

Voltage drop across  $4\Omega$  is zero (why?)

KVL  $\Rightarrow$  Voltage across the  $3A$  &  $20\Omega$  is  $V_{oc}$

We can calculate  $V_{oc}$  by any method we choose, let us use KVL, KCL & Ohm's law.

[Also, the mesh analysis is *efficient* in this case, because we have only *one actual* unknown. Why?].

The  $3A$  current completely goes to the left (why?)

Assume current  $i$  through  $5\Omega$

KCL  $\Rightarrow$  current through  $20\Omega$  is  $(i+3)$

KVL  $\Rightarrow -25 + 5i + 20(i+3) = 0 \Rightarrow i = -\frac{7}{5}A$

Ohm's law  $\Rightarrow V_{oc} = 20(i+3) = 20(-\frac{7}{5}+3) = 32V$

$\therefore V_{th} = V_{oc} = 32V$

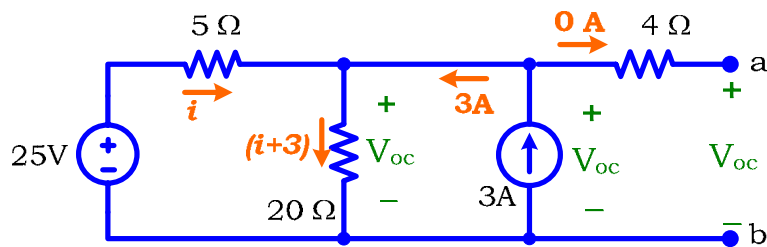


Figure 12

Place a short circuit across " $a-b$ "

We can calculate  $i_{sc}$  by any method we choose

Let us use the NA

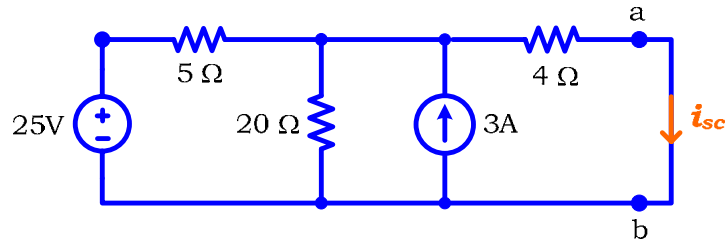


Figure 13

Using the reference node as shown  $\Rightarrow$  only  $V_2$  is unknown.

$$\text{KCL at node 2} \Rightarrow \frac{V_2 - 25}{5} + \frac{V_2}{20} - 3 + \frac{V_2 - 0}{4} = 0 \Rightarrow V_2 = 16V$$

$$\therefore i_{sc} = \frac{V_2 - 0}{4} = \frac{16}{4} = 4A$$

$$\therefore R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{32}{4} = 8\Omega$$

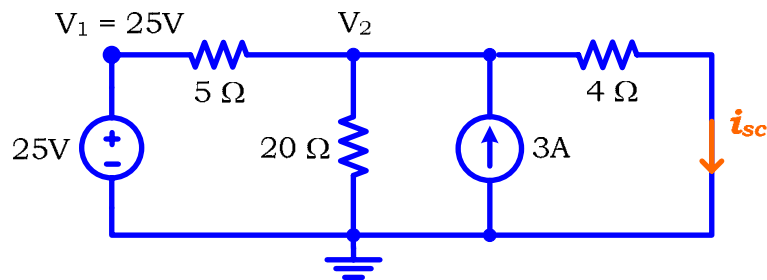


Figure 14

The resulting TEC is:

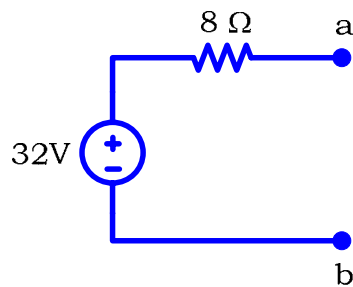


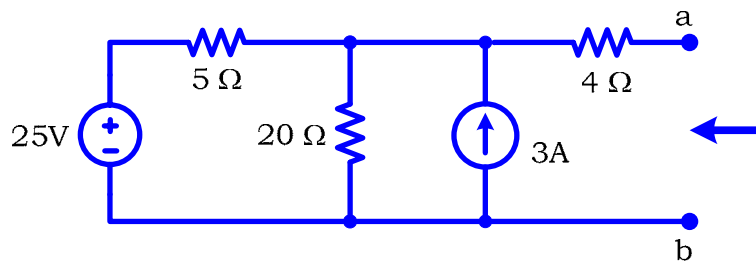
Figure 15

### Finding the TEC (Method 2):

We can also use ST to find the TEC. This is the second method.

### Example 3:

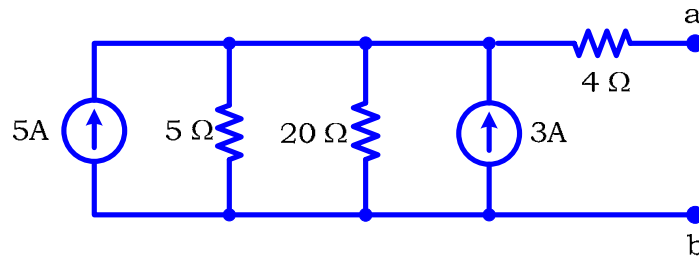
Repeat the previous example using ST to find the TEC.



**Figure 10**

Solution:

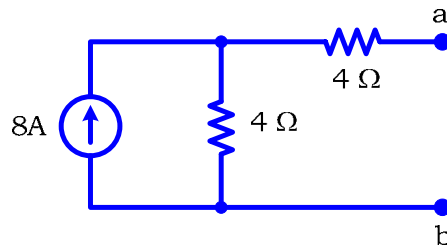
$$ST \Rightarrow I = \frac{25}{5} = 5A$$



**Figure 16**

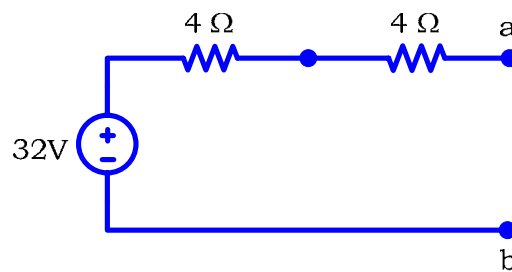
$$3A \parallel 5A \Rightarrow 8A$$

$$5\Omega \parallel 20\Omega \Rightarrow \frac{5 \times 20}{5 + 20} = \frac{100}{25} = 4\Omega$$



**Figure 17**

$$ST \Rightarrow V = 8 \times 4 = 32V$$



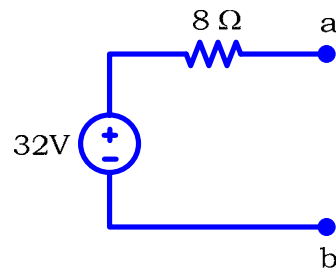
**Figure 18**

$$4 + 4 = 8\Omega$$

The circuit is reduced to a voltage source in series with a resistor.

$$\therefore V_{th} = 32V \quad \& \quad R_{th} = 8\Omega$$

Same answer as before.



**Figure 19**

Using ST, we are able to find  $V_{th}$  &  $R_{th}$  *simultaneously*.

Methods 1 & 2 are *not always* applicable. They have certain limitations.

In the coming classes, we will present the *other two* methods for finding the TEC and also discuss the *limitations* and *advantages* of the *four* methods.