EE 204
Lecture 07
Source Transformation

Source Transformation:

Given an ideal voltage source V_s *in series* with a resistor R_s.

$\Downarrow$ (can we replace them with)

An ideal current source I_s in parallel with a resistor R_p?

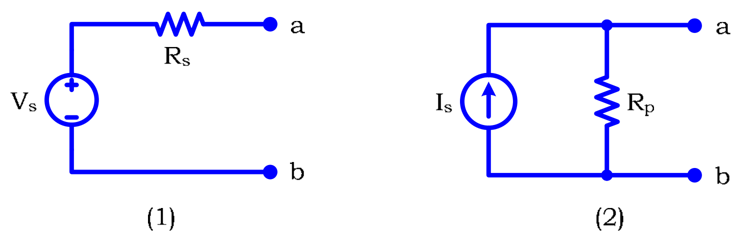

Figure 1

Connect the *same* load resistor R_L across terminal "a-b" in both circuits.

If circuits "1" and "2" are *equivalent* $\Rightarrow$ $I_1 = I_2$ & $V_1 = V_2$

Circuit "1" $\Rightarrow$ $I_1 = \dfrac{V_s}{R_{eq}} = \dfrac{V_s}{R_s + R_L}$ (1)

Circuit "2" $\Rightarrow$ $I_2 = \dfrac{R_p}{R_p + R_L} I_s$ (2) (CDR is used)

$$I_1 = I_2 \quad \Rightarrow \quad \frac{V_s}{R_s + R_L} = \frac{R_p I_s}{R_p + R_L} \qquad (3)$$

If we choose $R_p = R_s$ $\Rightarrow$ $V_s = I_s R_p = I_s R_s$

$$\therefore R_s = R_p \quad \& \quad V_s = I_s R_s$$

$$\Downarrow$$

$$V_s \text{ in series with } R_s \quad \Leftrightarrow \quad I_s \text{ in parallel with } R_s$$

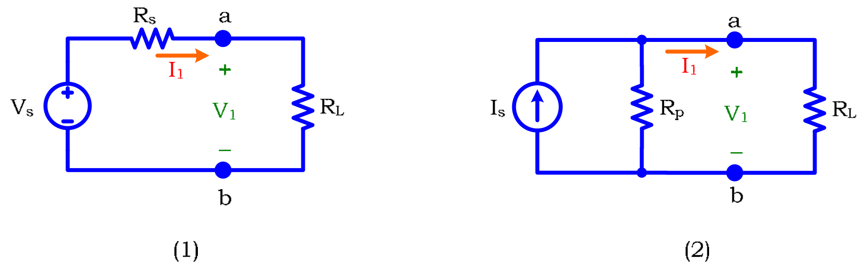

Figure 2

The above conversion is called *Source Transformation* (ST).

Circuits (1) & (2) are *equivalent*. However, they are *not the same*.

When *any load* is connected to terminals “*a*” & “*b*” of circuits (1) & (2)

$\Downarrow$

Load *cannot distinguish* between the two circuits.

Circuits (1) & (2) are *equivalent* from the *outside when accessed from terminals “a” & “b”*.

Circuits (1) & (2) are *different* from the *inside*.

Example 1:

Covert the following circuits using ST.

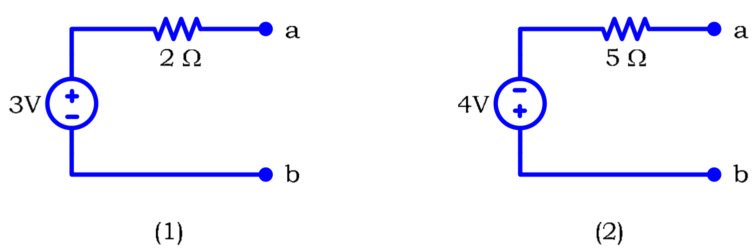

Figure 3

Solution:

Circuit (1):

$R_p = R_s = 2\Omega$

$$I_s = \frac{V_s}{R_s} = \frac{3}{2} = 1.5A$$

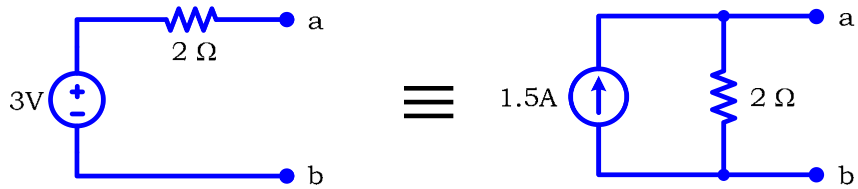

Figure 4

Circuit (2):

$$R_p = R_s = 5\Omega$$

$$I_s = \frac{V_s}{R_s} = \frac{4}{5} = 0.8A \quad \text{(downwards. Why?)}$$

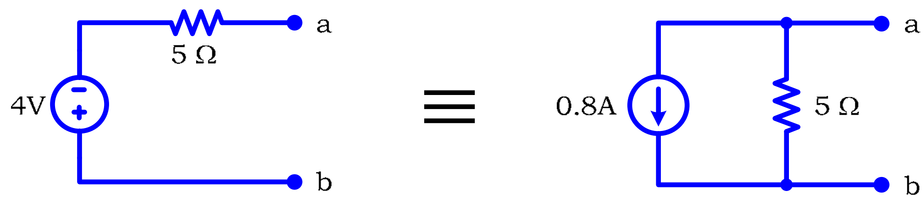

Figure 5

Example 2:

Covert the following circuits using ST.

Figure 6

Solution:

Circuit (1):

$$R_s = R_p = 3\Omega \qquad \& \qquad V_s = R_s I_s = 3(3) = 9V$$

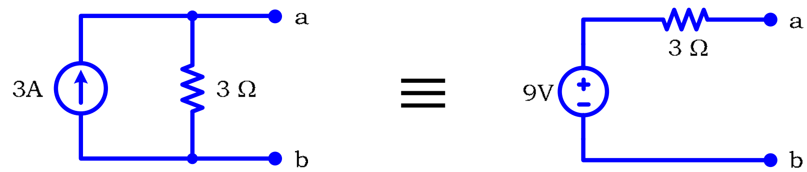

Figure 7

Circuit (2):

$R_s = R_p = 5\Omega$ & $V_s = R_s I_s = 5(2) = 10V$ (upper voltage polarity is negative. Why?)

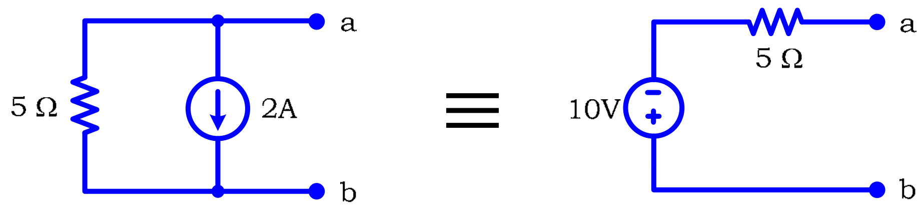

Figure 8

You need to be careful when using ST, as we will see in the next example.

Example 3:

Covert the following circuits using ST.

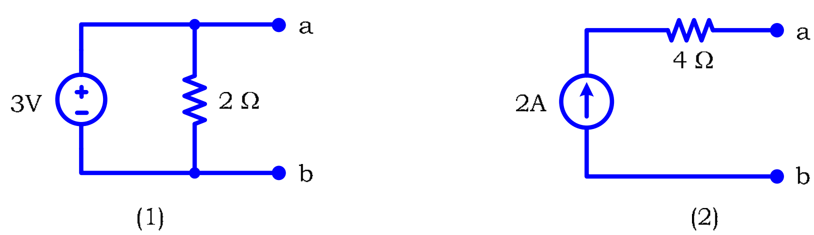

Figure 9

Solution:

Circuit (1):

A resistor *in parallel* with a voltage source (*not in series*) $\Rightarrow$ ST *does not* apply.

A resistor in parallel with a voltage source $\Rightarrow$ equivalent to a voltage source.

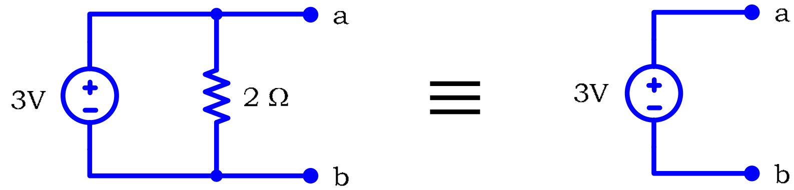

Figure 10

Circuit (2):

A resistor *in series* with a current source (*not in parallel*) $\Rightarrow$ ST *does not* apply.

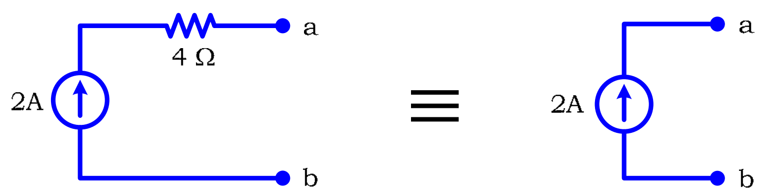

Figure 11

A resistor in series with a current source $\Rightarrow$ equivalent to a current source.

Example 4:

Use ST to calculate:

a) i_1

b) i_2

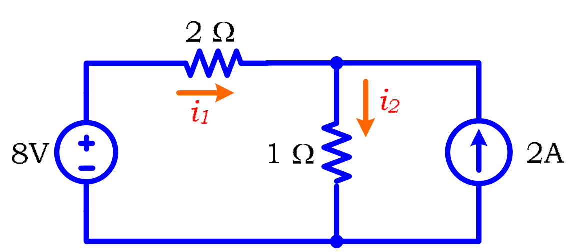

Figure 12

Solution:

It is a good idea to first label some points on the circuit.

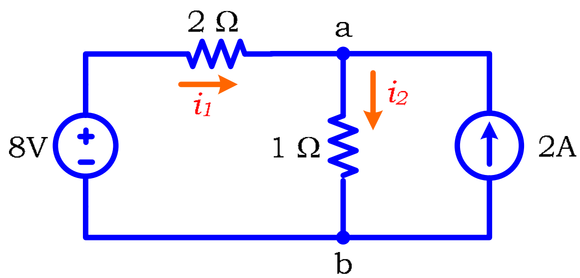

Figure 13

Apply ST to the $2A$ & 1Ω combination $\Rightarrow$ $V = IR = 2(1) = 2V$

Notice that i_2 cannot be drawn. It disappears. Why?

The current through the 1Ω of the *transformed* circuit is *not* i_2. Why?

Reason: $R_p = R_s$ means the two resistors have the *same* value. It *does not* mean we have the same resistor!!

$\because$ 2Ω & 1Ω in series $\Rightarrow$ current through the 1Ω of the *transformed* circuit is i_1.

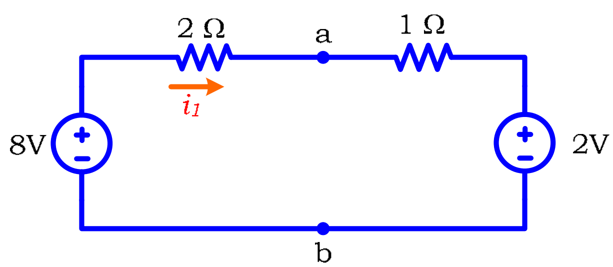

Figure 14

2Ω & 1Ω in series $\Rightarrow$ 3Ω

$8V$ & $2V$ in series $\Rightarrow$ $8 - 2 = 6V$

a) $i_1 = \dfrac{6}{3} = 2A$

b) KCL at node "a" (of the original circuit) $\Rightarrow$ $i_2 = i_1 + 2 = 2 + 2 = 4A$

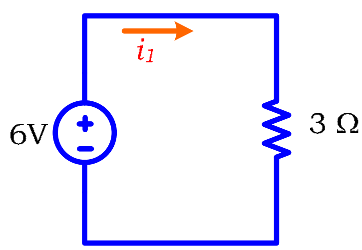

Figure 15

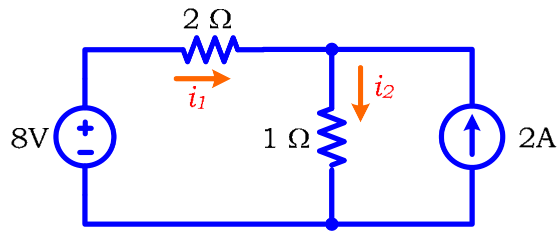

Figure 12

Example 5:

Use ST to calculate V.

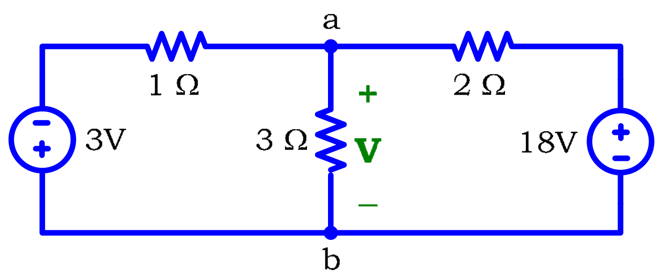

Figure 16

Solution:

Apply ST to ($3V$ in series with 1Ω) & ($18V$ in series with 2Ω)

$$I_{s1} = \frac{3}{1} = 3A \qquad \& \qquad I_{s2} = \frac{18}{2} = 9A$$

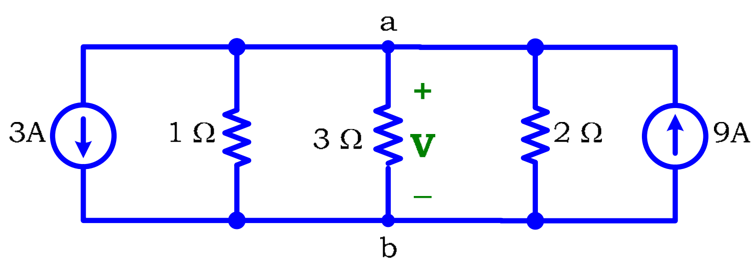

Figure 17

$$1\Omega \| 3\Omega \| 2\Omega \quad \Rightarrow \quad R_{eq} = \frac{1}{1 + \frac{1}{3} + \frac{1}{2}} = 0.545\Omega$$

$$3A \| 9A \quad \Rightarrow \quad I_{eq} = 9 - 3 = 6A$$

$$\therefore V = I_{eq} R_{eq} = 6(.545) = 3.273V$$

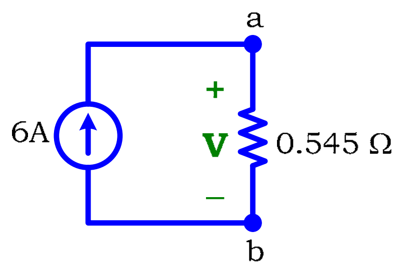

Figure 18

Example 5: Using ST, calculate:

a) i_1

b) i_2

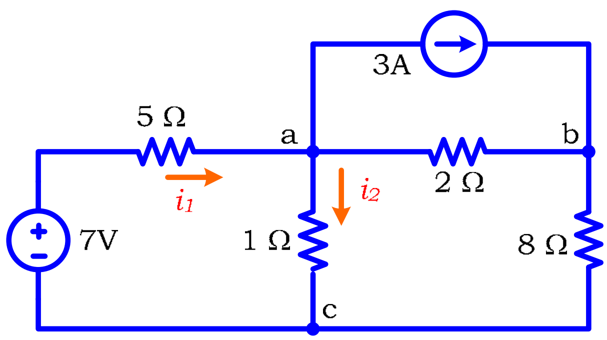

Figure 19

Solution:

ST $\Rightarrow$ $V = 3 \times 2 = 6V$

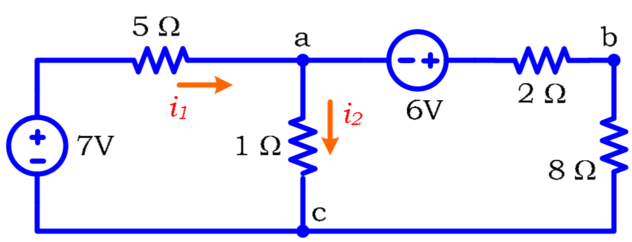

Figure 20

$2\Omega \| 8\Omega \quad \Rightarrow \quad 10\Omega$

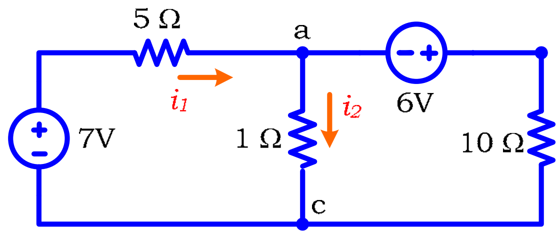

Figure 21

$$\text{ST} \quad \Rightarrow \quad I = \frac{6}{10} = 0.6A$$

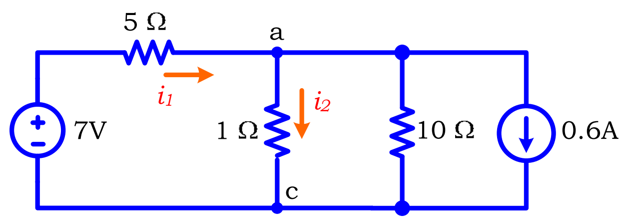

Figure 22

$$1\Omega \| 10\Omega \quad \Rightarrow \quad \frac{10\times 1}{10+1} = \frac{10}{11} = 0.909\Omega$$

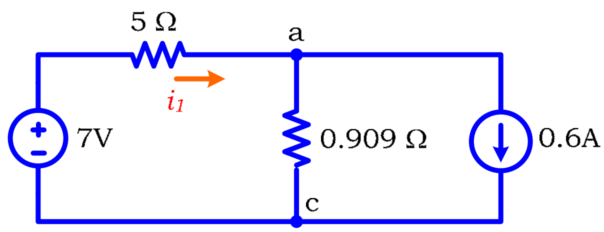

Figure 23

$$\text{ST} \quad \Rightarrow \quad V = 0.909 \times 0.6 = 0.545V$$

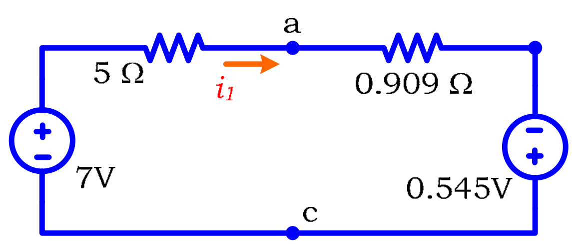

Figure 24

$$R_{eq} = 5 + 0.909 = 5.909\Omega \qquad \& \qquad V_{eq} = 7 + 0.545 = 7.545V$$

a) $\therefore i_1 = \dfrac{7.545}{5.909} = 1.277A$

b) KVL (in the original circuit) $\Rightarrow -7 + 5i_1 + 1i_2 = 0 \Rightarrow -7 + 5(1.277) + 1i_2 = 0 \Rightarrow i_2 = 0.615A$

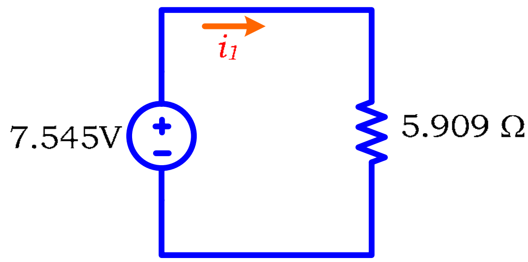

Figure 25

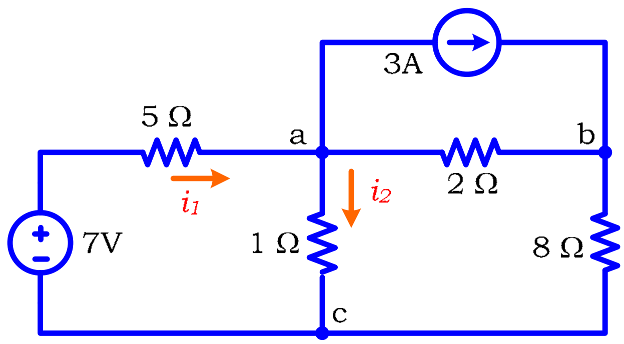

Figure 19