KING FAHD UNIVERSITY OF PETROLEUM & MINERALSEE 570ELECTRICAL ENGINEERING DEPARTMENT

Dec. 29, 2011

HOMEWORK #7

Due Date: Jan. 14, 2011 (Final exam date)

- Q1 Three white and three black balls are distributed in two urns in such a way that each urn contains three balls. We say that the system is in state i with i = 0, 1, 2, 3 if the first urn contains i white balls. At each step n, we draw one ball from each urn and place it in the other urn (so each urn will always contain 3 balls). Let X_n denote the number of white balls in urn 1. Explain why X_n is a Markov chain and calculate its transition probability matrix.
- Q2 Let P denote the transition probability matrix of a finite state Markov chain. Assume that for some m, all the entries of \mathbf{P}^m are positive. What can you say about the Markov chain? Would all the entries of \mathbf{P}^r all remain positive for r > m? If so, prove your answer or give a counter example.
- Q3 Determine the classes of the following Markov chains (described by their transition matrices) and determine whether they are recurrent or transient:

$$\mathbf{P}_{1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}, \quad \mathbf{P}_{2} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{P}_{3} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4}0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2}0 & 0 & 0 \\ 0 & 01 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q4 Prove that if state *i* is recurrent and state *i* does not communicate with state *j* then $P_{ij} = 0$.

Q5 Consider a markov chain with probability transition matrix

$$\mathbf{P}_1 = \left[\begin{array}{rrrr} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{array} \right]$$

Find the stationary probabilities for this chain.

- **Q6** Three out of four trucks in a given road are followed by a car while one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?
- **Q7** Calculate s_{i3} for i = 1, 2, 3. for Markov chain with probability transition matrix

$$\mathbf{P}_1 = \begin{bmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$