Solution

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
FALL 2011 (111)
EE 570 Stochastic Processes
QUIZ #2

Name: **ID:**

(4 points) **Q1.** Let $X(n)$ be a WSS process such that $X(1) \sim \mathcal{U}(0,1)$ and $E[X(10)X(21)] = 0.3$.

(2) 1. Evaluate $\Pr\{0.1 \le X(3) \le 0.4\}$

(2) 2. Evaluate $E[X(32)X(21)]$

1. $F_X(0.4) - F_X(0.1) = 0.4 - 0.1 = \underline{0.3}$

2. $R_{XX}(32-21) = R_{XX}(11) = E[X(10)X(21)] = \underline{0.3}$

Q2. Let $X(n)$ be an independent discrete time random process defined by

(4 points)

$$\begin{cases} X(n) \sim \mathcal{N}(0,1) & \text{if } n \text{ is even} \\ X(n) \sim \mathcal{U}(-\sqrt{3}, \sqrt{3}) & \text{if } n \text{ is odd} \end{cases}$$

(2) 1. Is $X(n)$ stationary? Justify your answer.

(2) 2. Is $X(n)$ wide sense stationary? Justify your answer.

1. PDF of $X(n)$ is dependent on n. Hence, not Stationary.

2. Mean of $x(n)$, $\mu_X(n) = \begin{cases} 0 & ; n \text{ is even} \\ 0 & ; n \text{ is odd} \end{cases}$

$\therefore$ Constant Mean

Covariance of $X(n)$, $R_{XX}(n+m, n) = \begin{cases} E[x^2(n)] & ; m = 0 \\ E[x(n+m)]\,E[x(n)] & ; m \neq 0 \end{cases}$

$= \begin{cases} \sigma_x^2 & ; m = 0 \\ 0 & ; m \neq 0 \end{cases}$

$\therefore$ Covariance is a function of time difference, m.

(12 points) **Q3.** Let $X(n)$ and $Y(n)$ be zero-mean jointly Gaussian and jointly wide-sense stationary processes with correlations $R_{xx}(\tau)$, and $R_{yy}(\tau)$, respectively, and with cross-correlation $R_{xy}(\tau)$. Define the process $Z(n)$ by

$$Z(n) = X(n) + Y(n)$$

Answer the following questions. Whenever this is needed, express your answer in terms of the 1st and 2nd order statistics of $X(n)$ and $Y(n)$.

(2) 1. Is $X(n)$ stationary? why?

(2) 2. Is the process $Z(n)$ Gaussian?

(2) 3. Find the mean of the process $Z(n)$.

(2) 4. Find the autocorrelation of $Z(n)$

(2) 5. Is $Z(t)$ wide-sense stationary?

(2) 6. Find the joint distribution of the random variables $Z(1)$ and $Z(2)$.

1. X(n) is stationary, as it is both Gaussian and wide-sense stationary.

2. Z(n) is Gaussian, as it is a linear combination of two Gaussian processes.

3. $\mu_z(n) = \mu_x(n) + \mu_y(n) = \underline{0}$

4. $R_{zz}(n+m, n) = E\left(\{X(n+m) + Y(n+m)\}\{X(n) + Y(n)\}\right)$
$= \underline{R_{xx}(m) + R_{xy}(m) + R_{yx}(m) + R_{yy}(m)}$

5. Z(n) is wide-sense stationary as its mean is constant and its covariance is a function of the time difference.

6. $$\underline{f_{z_1,z_2}} = \frac{1}{2\pi\sqrt{(\det C)}} \cdot \exp\left(-\frac{1}{2} \cdot z^T C^{-1} z\right)$$

where $$C = \begin{bmatrix} R_{zz}(0) & R_{zz}(1) \\ R_{zz}(1) & R_{zz}(0) \end{bmatrix}$$

$$\det C = R_{zz}^2(0) - R_{zz}^2(1)$$