

EE 570
QUIZ #1
SOLUTION

Q1.

$$\begin{aligned}\Pr(\text{Two balls have same colour}) &= \Pr(\text{Both Green}) + \Pr(\text{Both Yellow}) \\ &= \frac{5}{15} \cdot \frac{4}{14} + \frac{10}{15} \cdot \frac{9}{14} \\ &= 0.524\end{aligned}$$

Q2.

a)

$$F_x = Q\left(\frac{x-\mu}{\sigma}\right) = Q\left(\frac{x-0}{\sqrt{2}}\right) = Q\left(\frac{x}{\sqrt{2}}\right)$$

b)

$$\Pr\{X \leq x \mid X \leq 1\} = \frac{\Pr\{X \leq x, X \leq 1\}}{\Pr\{X \leq 1\}}$$

$$\text{Case 1: } x \leq 1 \Rightarrow \frac{\Pr\{X \leq x\}}{\Pr\{X \leq 1\}} = \frac{Q(x/\sqrt{2})}{Q(1/\sqrt{2})}$$

$$\text{Case 2: } x > 1 \Rightarrow \frac{\Pr\{X \leq 1\}}{\Pr\{X \leq 1\}} = 1$$

c)

$$\begin{aligned}\Pr\{0 \leq X \leq 2 \mid X \leq 1\} &= \frac{\Pr\{0 \leq X \leq 2, X \leq 1\}}{\Pr\{X \leq 1\}} \\ &= \frac{\Pr\{0 \leq X \leq 1\}}{\Pr\{X \leq 1\}} = \frac{Q(1/\sqrt{2}) - Q(0)}{Q(1/\sqrt{2})}\end{aligned}$$

Q3.

a)

$$\Pr\{\text{Generating } 1100\} = \Pr\{1\} \cdot \Pr\{1\} \cdot \Pr\{0\} \cdot \Pr\{0\}, \text{ (independent)}$$

$$= q^2 p^2$$

b) $\Pr\{\text{at least 2 zeros}\} = 1 - \Pr\{1 \text{ zero}\} - \Pr\{0 \text{ zeros}\}$

$$= 1 - 4C_1 p q^3 - 4C_0 q^4$$

c) $\Pr\{\text{Receiving } 1011 / \text{Transmitted } 1010\} = \Pr\{R_1/T_1\} \cdot \Pr\{R_0/T_0\} \cdot \Pr\{R_1/T_1\} \cdot \Pr\{R_1/T_0\}$

$$= p \cdot p \cdot p \cdot (1-p) = p^3(1-p)$$

d) $\Pr\{T_1/R_1\} = \frac{\Pr\{T_1 \cdot R_1\}}{\Pr\{R_1\}} = \frac{\Pr\{R_1/T_1\} \cdot \Pr\{T_1\}}{\Pr\{R_1/T_0\} \cdot \Pr\{T_0\} + \Pr\{R_1/T_1\} \cdot \Pr\{T_1\}}$

$$= \frac{p q}{(1-p)p + p q}$$

e) $\Pr\{R_0\} = \Pr\{R_0/T_0\} \cdot \Pr\{T_0\} + \Pr\{R_0/T_1\} \cdot \Pr\{T_1\}$

$$= p p + (1-p) q$$