EE 570
Quiz #3
Solution

(6 points)

Q1.

$$ACF = IFT\{PSD\}$$

$$R_{xx}(0) = \frac{1}{2\pi}\int_{90}^{110} 4\, d\omega = \frac{40}{\pi}$$

$$\mu_x^2 = \lim_{\tau\to\infty} R_{xx}(\tau) = \lim_{j\omega\to 0} j\omega\, S_x(\omega) = 0$$

$$\Rightarrow \mu_x = 0$$

$$\sigma_x^2 = R_{xx}(0) = \frac{40}{\pi}$$

$$\Rightarrow \sigma_x = \sqrt{\frac{40}{\pi}}$$

$$P_r\{|x(10)| \le 5\}) = F_x(5) - F_x(-5)$$

$$= 1 - Q\left(\frac{5-0}{\sigma_x}\right) - 1 + Q\left(\frac{-5-0}{\sigma_x}\right)$$

$$= -Q\left(\frac{5}{\sigma_x}\right) + 1 - Q\left(\frac{5}{\sigma_x}\right)$$

$$= 1 - 2Q\left(\frac{5}{\sigma_x}\right) = 1 - 2Q\left(\frac{5\sqrt{\pi}}{\sqrt{40}}\right)$$

(14 points)

Q2.

1. (3 points)

$$\mu_I(t) = E\left[\cos(120\pi t + \theta)\right]$$

$$\mu_I(t) = \frac{1}{2\pi}\int_0^{2\pi} (\cos 120\pi t + \theta) = 0 \quad \text{—— (a)}$$

$$R_{II}(t_1, t_2) = E\left[\cos(120\pi t_1 + \theta)\cos(120\pi t_2 + \theta)\right]$$

$$= \frac{1}{2} E\left[\cos(120\pi(t_1 - t_2))\right] + \frac{1}{2} E\left[\cos\left[120\pi(t_1 + t_2) + 2\theta\right]\right]$$

$$R_{II}(t_1, t_2) = \frac{1}{2}\cos\left(120\pi(t_1 - t_2)\right) + 0$$

$$= R_{II}(t_1 - t_2) \quad \text{—— (b)}$$

From (a) & (b), $I(t)$ is a WSS process.

2.

(3 points)

$$Y(t) = \{X(t) - X(t-T)\} + \{I(t) - I(t-T)\}$$

$\therefore$ Effect of $I(t) = I(t) - I(t-T)$

$$= \cos(120\pi t + \theta) - \cos(120\pi t - 120\pi T + \theta)$$

Thus, to get rid of the effect of $I(t)$

$$120\pi T = 2n\pi$$

$$T = \frac{n}{60} \quad ; \quad n = 0, \pm 1, \pm 2, \ldots$$

3.

(3 points)

$$Y(t) = Z(t) - Z(t-T)$$

Substituting $Z(t)$ with $\delta(t)$ and $Y(t)$ with $h(t)$

we get, $h(t) = \delta(t) - \delta(t-T)$

$$\Rightarrow \quad Y(t) = h(t) * Z(t)$$

'$*$' stands for convolution

4.

(3 points)

$$S_{YY}(\omega) = |H(\omega)|^2 S_{ZZ}(\omega)$$

$$H(\omega) = 1 - e^{-j\omega T}$$

$$|H(\omega)|^2 = \left(1 - e^{-j\omega T}\right)\left(1 - e^{j\omega T}\right) = 2 - \left[e^{j\omega T} - e^{-j\omega T}\right]$$

$$= 2 - 2\cos\omega T = 4\sin^2 \omega T/2$$

$$\therefore \quad S_{YY}(\omega) = 4\sin^2 \omega T/2 \left\{S_{XX}(\omega) + S_{II}(\omega)\right\}$$

5. The strategy succeeds in eliminating the
(2 points) interference, but at the same time corrupts
the original signal $x(t)$ as an echo $x(t) - x(t-T)$.