EE 570 SOLUTION
HW # 5

TOTAL - 90

Q1 - 6	Q4 - 9	Q7 - 33
Q2 - 6	Q5 - 9	
Q3 - 6	Q6 - 21	

Q1

6

Mean, $\eta(t) = \int_{-\infty}^{\infty} e^{at} f_a(a)\, da$

Autocorrelation, $R(t_1, t_2) = \int_{-\infty}^{\infty} e^{at_1} e^{at_2} f_a(a)\, da$

First order density, $f(x,t) = \dfrac{f_a(a)}{|(e^{at})'|} = \dfrac{f_a(a)}{|t e^{at}|}$

$$= \frac{f_a\left(\frac{1}{t}\ln x\right)}{tx} U(x)$$

Q2

3 a) $P\{x(t) \le 3\} = Q\left(\frac{3-0}{2}\right) = Q(1.5)$ as $X \sim N(0,4)$

3 b) $E\{[x(t+1) - x(t-1)]^2\} = E\{x(t+1)^2\} + E\{x(t-1)^2\} - 2E\{x(t+1)x(t-1)\}$

$$= \sigma_x^2 + \sigma_x^2 - 2R(2)$$

$$= 2(4) - 2(4e^{-4})$$

$$= 8(1 - e^{-4})$$

Q3

i.i.d process implies

6

$$f(x_1;t_1) = f(x_2;t_2)$$

$$f(x_1,x_2;t_1,t_2) = f(x_1;t_1)\cdot f(x_2;t_2)$$

Combining the above two expressions, we get

$$f(x_1,x_2;t_1,t_2) = f(x_1,x_2;t_1+c,t_2+c)$$

Similarly,

$$f(x_1,x_2\dots x_n;t_1,t_2\dots t_n) = f(x_1,x_2\dots x_n;t_1+c,\dots t_n+c)$$

Thus $X(t)$ is Stationary. as its joint pdf remains constant.

Q4

$$Y(n) = (-1)^n X(n)$$

9

$Y(n)$ is not an i.i.d process, as it is distributed differently for different values of n.

$$\eta_x(n) = \frac{1}{2}$$

$$\eta_y(n) = \frac{(-1)^n}{2}$$

Mean of $Y(n)$ is not constant and is a function of n. Hence $Y(n)$ is not Wide-Sense stationary. Correspondingly, it is not Stationary as well.

Q5

$$Y(n) = X(n) - X(n-1)$$

9

$$\eta_y(n) = \eta_x - \eta_x = 0$$

$$\sigma_y^2(n) = \sigma_x^2 + \sigma_x^2 - 2\,E\{x(n)\}\,E\{x(n-1)\}$$

$$= 2\sigma_x^2 - 2\eta_x^2$$

$$\sigma_y^2(n) = 2 - 2\mu^2$$

Thus $Y \sim \mathcal{N}(0, 2-2\mu^2)$

Hence identically distributed.

$$E\{Y(n)\,Y(n+m)\} = E\{[X(n)-X(n-1)][X(n+m)-X(n+m-1)]\}$$

$$E[Y(n)\,Y(n+m)] \begin{cases} \neq 0 & ; \text{for } m = -1, 0, 1 \\ = 0 & ; \text{otherwise} \end{cases}$$

$$E\{Y(n)\}\,E\{Y(n+m)\} = 0$$

$$\therefore E\{Y(n)\,Y(n+m)\} \neq E\{Y(n)\}\,E\{Y(n+m)\}$$

Hence $Y(n)$ is not independently distributed

$Y(n)$ is not an i.i.d process

Q6

3 1. The joint distribution of $X(n)$ and $Y(n)$ is Gaussian and also each of $X(n)$ and $Y(n)$ is Gaussian.

3 2. Joint WSS of $X(n)$ and $Y(n)$ implies that each of $X(n)$ and $Y(n)$ is also WSS and their cross-correlation is a function of time difference.

3 3. $X(n)$ is WSS and Gaussian and hence it is Stationary.

3 4. $Z(n) = X(n) \cdot Y(n)$

$$f_z(z) = \int \frac{1}{|J(x,y)|} f_{XY}(x, z/x)\, dx$$

As f_{XY} is Gaussian and integral of Gaussian is a Poisson distribution.

Hence $Z(n)$ is not Gaussian.

5. ⑥ ⑤

$$\eta_z(n) = E[X(n)Y(n)] = R_{XY}(0)$$

$$R_{ZZ}(n+m,n) = E[X(n+m)\,Y(n+m)\,X(n)\,Y(n)]$$
$$= E[X(n+m)\,Y(n+m)]\,E[X(n)\,Y(n)]$$
$$+ E[X(n+m)\,X(n)]\,E[Y(n+m)\,Y(n)]$$
$$+ E[X(n+m)\,Y(n)]\,E[Y(n)\,X(n+m)]$$

$$R_{ZZ}(m) = R_{XY}^2(0) + R_{XX}(m)\,R_{YY}(m) + R_{XY}(m)\,R_{YX}(m)$$

③ 6. As $Z(n)$ has a constant mean and an autocorrelation which is a function of the time difference, it is wide-sense stationary.

Q7.

③ 1. $X(t)$ is WSS and Gaussian implies that it is stationary and hence identically distributed.

③ 2. As $X(t)$ is stationary, $Y(t) = X^2(t)$ is also stationary and hence identically distributed.

3.

3

$$E[X(n+m)X(n)] = R_{XX}(\tau) \neq 0$$

$$E[X(n+m)]\,E[X(n)] = 0 \times 0 = 0$$

$$E[X(n+m)X(n)] \neq E[X(n+m)]\,E[X(n)]$$

Hence $X(t)$ is not independent.

4. $Y(t) = X^2(t)$ is also not independent as

3 $X(t)$ is not an independent process.

5.

$$f_Y(y) = \left(\frac{f_X(\sqrt{y})}{2\sqrt{y}} - \frac{f_X(-\sqrt{y})}{2\sqrt{y}}\right) U(y)$$

6

$$= \frac{1}{\sqrt{2\pi y \sigma_x^2}} \exp\left(-\frac{y}{2\sigma_x^2}\right) U(y)$$

6.

$$y_1 = x_1^2 \quad , \quad y_2 = x_2^2$$

6

$$J(x_1, x_2) = \begin{vmatrix} 2x_1 & 0 \\ 0 & 2x_2 \end{vmatrix} = 4x_1x_2 = 4\sqrt{y_1y_2}$$

$$f_{Y_1Y_2}(y_1, y_2; t_1, t_2) = \sum_{ij} \frac{f_{X_1X_2}(x_{1i}, x_{2j})}{|J(x_{1i}, x_{2j})|}$$

$$= \frac{1}{4\sqrt{y_1y_2}} \Big[f(\sqrt{y_1}, \sqrt{y_2}; t_1, t_2) + f(\sqrt{y_1}, -\sqrt{y_2}; t_1, t_2) + f(-\sqrt{y_1}, \sqrt{y_2}; t_1, t_2) + f(-\sqrt{y_1}, -\sqrt{y_2}; t_1, t_2) \Big]$$

7.

6

$$\eta_y(t) = \sigma_x^2$$

$$R_{YY}(t, t+k) = E\left[X^2(t)\, X^2(t+k)\right]$$

$$= E\left[X^2(t)\right] E\left[X^2(t+k)\right] + 2\, E\left[X(t)\, X(t+k)\right]^2$$

$$= (\sigma_x^2)^2 + 2\, R_{XX}^2(k)$$

$$= R_{YY}(k)$$

Thus, $Y(t)$ is WSS

8. $Y(t)$ is WSS and also identically distributed.

3 Hence, it is stationary.