1. A signal \( x(t) \), has the magnitude spectrum shown in the figure. Sketch the spectrum of the following signals \( x(2t) + x(t) e^{-j2\pi t} \)

From Table 4-1

\[
\begin{align*}
    x(a^2) & \leftrightarrow |a|^{-1} X(\frac{f}{a}) \\
    x(2t) & \leftrightarrow \frac{1}{2} X(\frac{f}{2})
\end{align*}
\]

Using the scale change property (3a)

\[
\begin{align*}
    x(j\pi t) & \leftrightarrow X(f - f_s) \\
    x(j\pi t + b) & \leftrightarrow X(f + f_s)
\end{align*}
\]

Using Frequency Translation Property (5a)

By linearity

\[
\frac{1}{2} X(\frac{f}{2}) + X(f + 12)
\]

2. Find the Fourier Transform of the signal shown.

Hint: The signal can be expressed in the form of \( g(t)\cos(2\pi f_0 t) \), What is \( f_0 \)?

\[
\begin{align*}
    f_0 &= \frac{\text{cycles}}{\text{Time interval}} = \frac{10}{2\pi - \pi} = \frac{10}{\pi} \\
    \omega_0 &= 2\pi f_0 = 2\pi \left( \frac{5}{\pi} \right) = 10 \text{ rad/sec}
\end{align*}
\]

The signal can be found by

\[
\begin{align*}
    \Pi \left(\frac{t - 2\pi}{2\pi}\right) \cos 2\pi \left(\frac{5}{\pi}\right) t \\
    \Pi \left(\frac{t}{\pi}\right) & \leftrightarrow \pi \text{sinc}(\pi f) \quad \text{Eq. 4.1} \\
    \Pi \left(\frac{t}{2\pi}\right) & \leftrightarrow 2\pi \text{sinc}(2\pi f) \\
    \text{also Table 4.1} \quad \text{Time delay} \\
    T\left(\frac{t - 2\pi}{2\pi}\right) & \leftrightarrow 2\pi \text{sinc}(2\pi f) \quad \text{Eq. 4.1}
\end{align*}
\]

Finally using 9.1 (5 b) Modulation

\[
\begin{align*}
    x(t) \cos \omega_0 t & \leftrightarrow \frac{1}{2} X(f - \omega_0) + \frac{1}{2} X(f + \omega_0) \\
    \text{sinc}(2\pi f - \frac{5}{\pi}) e^{-j2\pi f} & \leftrightarrow -3 \quad \text{Eq. 4.1} \\
    \text{sinc}(2\pi f + \frac{5}{\pi}) e^{-j2\pi f} & \leftrightarrow -5 \quad \text{Eq. 4.1}
\end{align*}
\]

Good Luck, Dr. Ali Muqaibel