King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE 207 – Signals and Systems

Major I

November 18, 2008

Time Allowed 1 ½ Hours

Student Name : _____Key______________________________
Student ID Number : ______0_________

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Problem 1:

a. (3 marks) Express the given signal in terms of singularity functions

\[ x(t) \]

b. (3 marks) For the signal \( g(t) \) shown in the Figure below, sketch the signal \( 10g(4-2t) \).

Show all the steps.

c. (4 marks) A given system is described by the following relation,

\[ y(t) = x^4(t+2)+5t \] where \( x(t) \) is the input signal and \( y(t) \) is the output signal.

Classify the above system as to linearity, causality, and time invariance (justify your answer).

Problem 2:

a. (2 marks) The impulse response of a LTI system is given by:

\[ h(t) = \frac{1}{T_o} e^{-t/T_o} u(t) \]

find the step response \( a(t) \) [i.e. the response due to a unit step input \( x(t) = u(t) \)].

b. (3 marks) Now consider the following input signal \( x(t) \):

\[ x(t) \]

Find the output signal when the input is \( x(t) \) above.

[Hint: It is useful to consider the answer to part (a)].
c. **(5 marks)** Determine the convolution \( y(t) \) of the following 2 signals, and plot it. Show all your work in details.

\[
\begin{align*}
x(t) & = 2e^{-3t}u(t) \\
h(t) & = u(t)
\end{align*}
\]

**Problem 3:**

A periodic signal \( x(t) \) is shown below.

\[
\begin{align*}
x(t) & = 1 - 1 - 1 1 3 4 5 \quad t
\end{align*}
\]

a. **(2 marks)** Determine the fundamental period \( T_0 \) and fundamental frequency \( f_0 \) of the signal \( x(t) \)

b. **(5 marks)** Determine the Trigonometric Fourier Series representation of \( x(t) \) (show your work)

c. **(3 marks)** Determine the power and the energy of the signal \( x(t) \).
Problem 1:

a) \( 2r(t+1) - 2r(t) + u(t) = 3u(t-3) \)

b) \( 10 \log(4.2t) \equiv 10 \log[-2(t-2)] \)

- There are four operations.
  1. Amplitude scaling by 10 \( 10 \log(t) \)
  2. Time compression by a factor of 2.
  3. Time inversion.
  4. Time shifting by 2 units to the right.

For the given operations, we can check the time operation by

\[
\begin{align*}
t' &= 4.2t \\
t &= 2 - \frac{t'}{2}
\end{align*}
\]

<table>
<thead>
<tr>
<th>t</th>
<th>6</th>
<th>12</th>
<th>-1</th>
<th>-4</th>
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<tr>
<td>t'</td>
<td>12</td>
<td>24</td>
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- Amplitude scale by 10 units

- Time point 6 \(\rightarrow\) -1
- Time point 12 \(\rightarrow\) -4

- The system is not linear because it is proportional to the fourth power of \( x(t) \)

\[ x_1 + x_2 \neq (x_1 + x_2)^4 \]

- The system is not causal because for example, the output \( y \) at \( t = 1.5 \) depends on the input at \( t = 3 \) sec.

- The system is time variant because the relation changes with time due to the exponential time term \( e^{t} \)

\[
\begin{align*}
at t = 1 \text{ sec} & \quad y(t) = x^4(t+2) + 5 \quad \text{rel. not the same} \\
at t = 2 \text{ sec} & \quad y(t) = x^4(t+2) + 10 \quad \text{hence time varying}
\end{align*}
\]
Problem 2:

a) For LTI, \( a(t) = \int_{-\infty}^{t} h(z) \, dz \)

\[
\mathcal{A}(\tau) = \int_{-\infty}^{\tau} \left( \frac{1}{\tau_0} \right) e^{-z/\tau_0} u(t) \, dz
\]

\[
= \frac{1}{\tau_0} \int_{0}^{t} e^{-z/\tau_0} \, dz = -\frac{\tau}{\tau_0} e^{-\tau/\tau_0} \quad \text{for} \quad t \leq 0
\]

\[
= 1 - e^{-t/\tau_0}
\]

\[
\Rightarrow \quad a(t) = \left( 1 - e^{-t/\tau_0} \right) u(t)
\]

b) \( x(t) = u(t) - 2u(t-1) + u(t+2) \)

Since the system is linear,

\[
y(t) = (1 - e^{-t/\tau_0}) u(t) - 2(1 - e^{-t/\tau_0}) u(t-1) + (1 - e^{-t/\tau_0}) u(t-2)
\]

c) \( x(t) = u(t) \) and \( h(t) = 2 e^{-t/\tau_0} u(t) \)

\[
x(t) \preceq h(t) = \begin{cases} 
0 & t < 0 \\
\frac{2}{3} \left[ 1 - e^{3t} \right] & 0 \leq t \leq 1 \\
\frac{2}{3} e^{3(t+1)} & 1 \leq t < \infty
\end{cases}
\]

Case I: \( t < 0 \) no overlap

\( x(t) \preceq h(t) = 0 \)

Case II: \( 0 \leq t < 1 \) partial overlap

\[
\int_{0}^{t} \left( 2 e^{-z/\tau_0} \right) u(t) \, dz = -\frac{2}{3} e^{2t} \quad \text{for} \quad t < 0
\]

\[
= \frac{2}{3} \left[ e^{3} - 1 \right] = \frac{2}{3} \left[ 1 - e^{3t} \right]
\]

Case III: \( 1 \leq t \leq \infty \) full overlap

\[
\int_{t-1}^{t} \left( 2 e^{-z/\tau_0} \right) u(t) \, dz = \frac{2}{3} \left[ e^{3} - 1 - e^{-3(t+1)} \right]
\]

\[
= \frac{2}{3} \left[ e^{3} - 1 \right] - \frac{2}{3} e^{3(t+1)}
\]

\[
= \frac{2}{3} \left[ 1 - e^{3} \right] - \frac{2}{3} \left( 1 - e^{3} \right)
\]

\[
= \frac{2}{3} \left[ 1 - e^{3} \right]
\]

\[
0.633 = \frac{2}{3} \left[ 1 - e^{3} \right]
\]

\[
0.633 \approx \frac{2}{3} \left[ 1 - e^{3} \right]
\]
Problem 3

a) \[ T_0 = 4 \text{ sec}, \quad f_0 = \frac{1}{4} = 0.25 \quad \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad/sec} \]

b) Determine the Trigonometric Fourier Series representations of \( x(t) \)

\[ x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \]

- The signal is odd symmetric \( \Rightarrow a_n = 0 \) for \( n \geq 1 \)
- By inspection the average value of \( x(t) = 0 \) \( \Rightarrow a_0 = 0 \)

\[ b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin (n\omega_0 t) \, dt \]

\[ = \frac{2}{4} \int_{T_0} x(t) \sin \left( \frac{2\pi n t}{4} \right) \, dt = \frac{1}{2} \int_{0}^{1} x(t) \sin \left( \frac{\pi n t}{2} \right) \, dt \]

\[ = -\frac{2}{n\pi} \cos \left( \frac{\pi n t}{2} \right) \bigg|_{0}^{1} = -\frac{2}{n\pi} \left[ \cos \frac{n\pi}{2} - 1 \right] \]

\[ = \frac{2}{n\pi} \left[ 1 - \cos \frac{n\pi}{2} \right] \quad \text{for odd values of } n \quad \cos \frac{n\pi}{2} = 0 \]

\[ b_n = \frac{2}{n\pi} \quad \text{for odd values of } n \]

\[ b_n = \frac{4}{n\pi} \quad \text{for even values of } n \]

\[ b_n = \frac{2}{n\pi} \left[ 1 - (-1)^n \right] = \begin{cases} \frac{4}{n\pi} & n = 2, 6, 10, 14, 18, \ldots \\ \frac{2}{n\pi} & n = 4, 8, 12, 16, 20, \ldots \\ 0 & n = 1, 3, 5, 7, \ldots \end{cases} \]

\[ c) \quad P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 \, dt = \frac{1}{4} \left[ \int_{-1}^{0} (\cdot)^2 \, dt + \int_{0}^{1} (\cdot)^2 \, dt \right] = \frac{1}{2} \]

\[ E = \text{area under Square of the signal} = \int_{-\infty}^{\infty} |x(t)| \, dt = \infty \]

we may write

\[ x(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left[ 1 - \cos \frac{n\pi}{2} \right] \sin \frac{n\pi}{2} \, t \]