King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE 207 – Signals and Systems

Major I

November 18, 2008

Time Allowed 1 ½ Hours

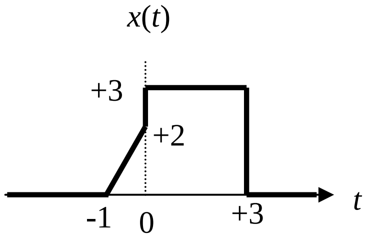
Student Name : _____Key_____________________________________

Student ID Number : ______0_________

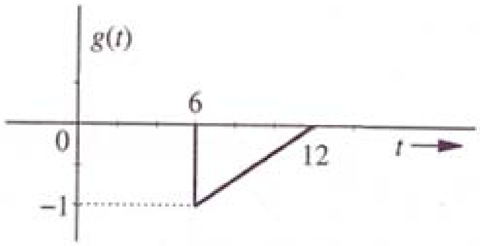
Problem	Max Score	Score
Problem 1	10	
Problem 2	10	
Problem 3	10	
Total	30	

Problem 1:

a. **(3 marks)** Express the given signal in terms of singularity functions



b. **(3 marks)** For the signal $g(t)$ shown in the Figure below, sketch the signal $10g(4\text{-}2t)$. Show all the steps.



c. **(4 marks)** A given system is described by the following relation,

$y(t)=x^4(t+2)+5t$ where $x(t)$ is the input signal and $y(t)$ is the output signal.

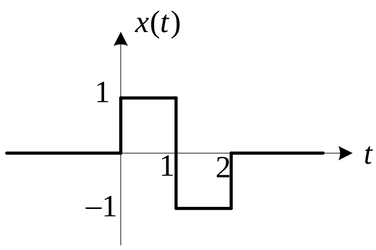
Classify the above system as to linearity, causality, and time invariance (justify your answer).

Problem 2:

a. **(2 marks)** The impulse response of a LTI system is given by: $h(t)=(\frac{1}{\tau_o})e^{-t/\tau_o}u(t)$,

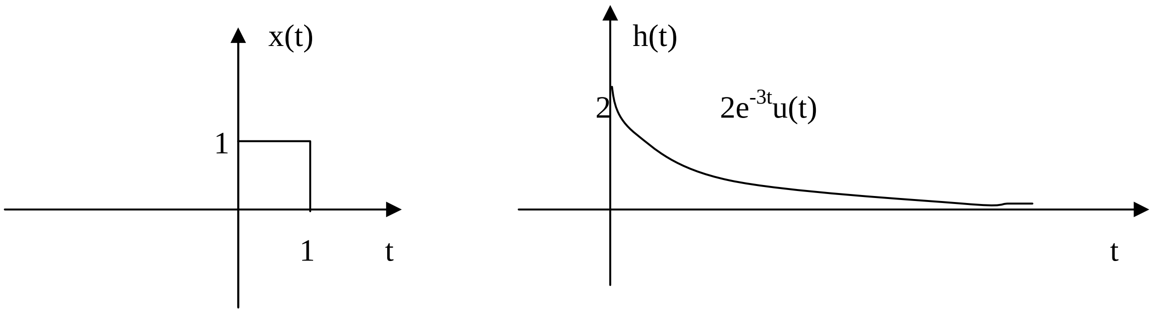
find the step response $a(t)$ [i.e. the response due to a unit step input $x(t)=u(t)$].

b. **(3 marks)** Now consider the following input signal $x(t)$:



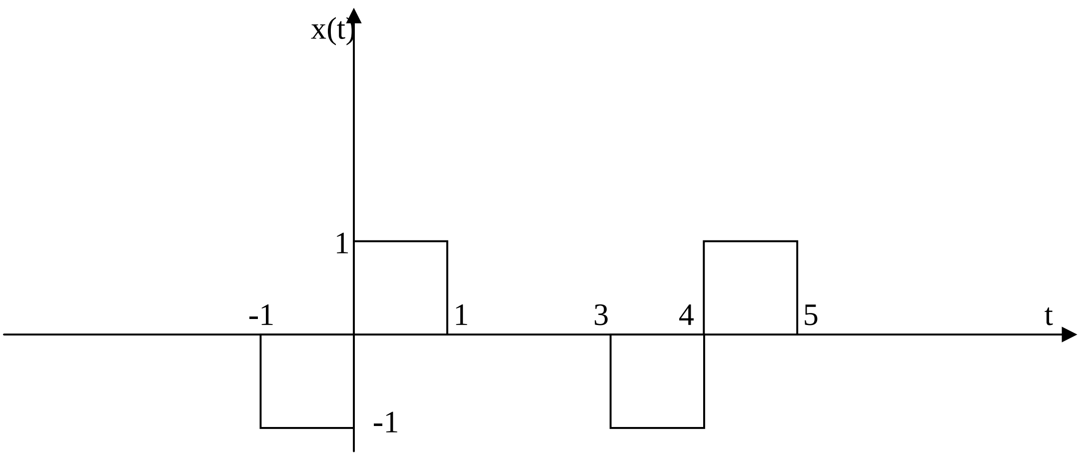
Find the output signal when the input is $x(t)$ above.
[Hint: It is useful to consider the answer to part (a)].

c. **(5 marks)** Determine the convolution y(t) of the following 2 signals, and plot it. Show all your work in details.



Problem 3:

A periodic signal x(t) is shown below.



a. **(2 marks)** Determine the fundamental period T_0 and fundamental frequency f_0 of the signal x(t)
b. **(5 marks)** Determine the Trigonometric Fourier Series representation of x(t) (show your work)
c. **(3 marks)** Determine the power and the energy of the signal x(t).

KEY

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Problem 1:

1 point for shape

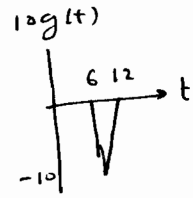
a) $$2r(t+1) - 2r(t) + u(t) - 3u(t-3)$$

← one possible solution
note $r(2t) = 2r(t)$
not true in general.

b) $$10\,g(4-2t) \equiv 10\,g[-2(t-2)]$$

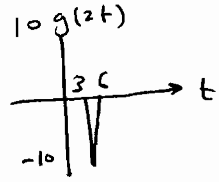
There are four operations.

① amplitude scaling by 10 $\quad 10g(t)$

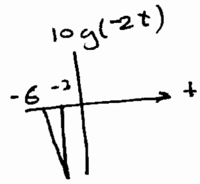


② Time compression by a factor of 2.

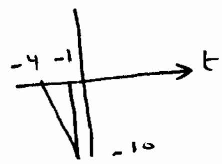
$6 \rightarrow 3 \qquad 12 \rightarrow 6$



③ Time inversion.



④ Time shifting by 2 units to the right.

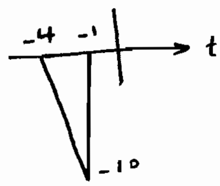


we can check the time operation by

$t' = 4-2t \Rightarrow t'-4 = -2t \Rightarrow t = 2 - \frac{t'}{2}$

t'	6	12
t	-1	-4

and amplitude scale by 10 units



time point $6 \rightarrow -1$

time point $12 \rightarrow -4$

c) • The system is not linear because it is proportional to the fourth power of $x(t)$

$$x_1^4 + x_2^4 \neq (x_1+x_2)^4$$

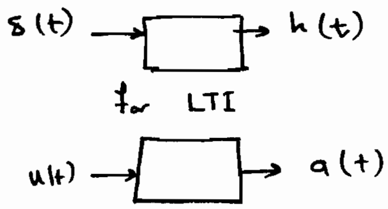
• The system is not causal because for Example the output y at $t = 1$ s depends on the input at $t = 3$ sec.

• The sys. is time variant because the relation changes with time due to the exeplect time term $\underline{\underline{5t}}$

at $t = 1$ sec. $\quad y(t) = x^4(t+2) + 5$ ← The relation is not the same

at $t = 2$ se $\quad y(t) = x^4(t+2) + 10$ ← hence time varying.

Problem 2 :



a) for LTI $\quad a(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$

$$a(t) = \int_{-\infty}^{t} \left(\frac{1}{\tau_0}\right) e^{-\tau/\tau_0} u(\tau)\, d\tau \qquad a(t) = 0 \quad \text{for} \quad t \le 0$$

$$= \frac{1}{\tau_0}\int_0^t e^{-\tau/\tau_0}\, d\tau = -\frac{\tau_0}{\tau_0} e^{-\tau/\tau_0}\Big|_0^t = -\left[e^{-t/\tau_0} - 1\right] \qquad t > 0$$

$$= 1 - e^{-t/\tau_0}$$

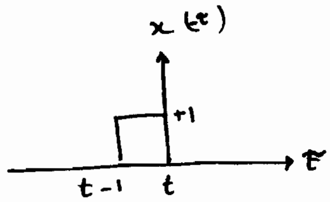
$$\Rightarrow \boxed{a(t) = \left(1 - e^{-t/\tau_0}\right) u(t)}$$

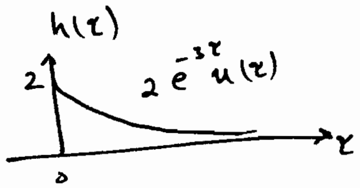
b) $x(t) = u(t) - 2u(t-1) + u(t-2)$

Since the system is linear.

$$y(t) = \left(1 - e^{-t/\tau_0}\right) u(t) - 2\left(1 - e^{-(t-1)/\tau_0}\right) u(t-1) + \left(1 - e^{-(t-2)/\tau_0}\right) u(t-2)$$

c)





$$x(t) * h(t) = \begin{cases} 0 & t < 0 \\ \frac{2}{3}\left[1 - e^{-3t}\right] & 0 \le t \le 1 \\ \frac{2}{3}\left[e^{-3(t-1)} - e^{-3t}\right] & 1 \le t \le \infty \end{cases}$$

$\frac{2}{3} e^{-3t}(e^{+3} - 1) \equiv$

Case I

$t < 0$ no overlap (1)

$x(t) * h(t) = 0$

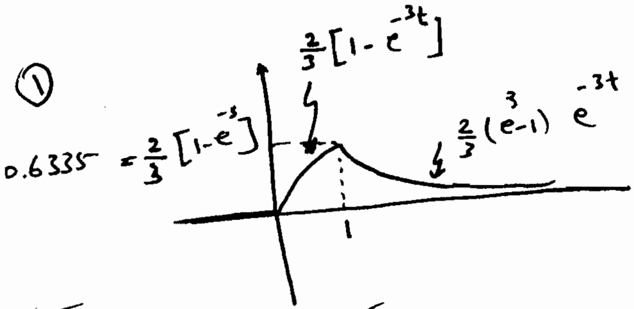
t	0	1
$x(t)*h(t)$	0	$\frac{2}{3}(1-e^{-3})$

Case II partial overlap.

$0 \le t < 1$

$$\int_0^t (1)(2e^{-3\tau})\, d\tau = -\frac{2}{3} e^{-3\tau}\Big|_0^t$$

$$= -\frac{2}{3}\left[e^{-3t} - 1\right] = \frac{2}{3}\left[1 - e^{-3t}\right] \quad (1)$$



Case III full overlap

$$\int_{t-1}^{t} (1)(2e^{-3\tau})\, d\tau = -\frac{2}{3}\left[e^{-3t} - e^{-3(t-1)}\right]$$

$1 \le t \le \infty$

$$= -\frac{2}{3} e^{-3t}\left[1 - e^{3}\right]$$

$$= \frac{2}{3} e^{-3t}\left[e^{3} - 1\right]$$

(2.5) (15)

Problem 3

a) $T_0 = 4$ (sec) , $f_0 = \frac{1}{4} = 0.25$ (Hz) $\Rightarrow w_0 = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/sec (1) (1)

b) Determine the Trignometric Fourier Series representations of x(t)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + \sum_{n=1}^{\infty} b_n \sin n w_0 t$$ (0.5)

- The signal is odd symmetric $\Rightarrow a_n = 0$ for $n: 1 \xrightarrow{to} \infty$ (1)
- By inspection the average value of $x(t) = 0 \Rightarrow a_0 = 0$ (0.5)

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(n w_0 t)\, dt$$ (0.5)

odd * odd = even.

$$= \frac{2}{4} \int_{-2}^{2} x(t) \sin\left(\frac{2\pi n}{4} t\right) dt = \frac{1}{\cancel{2}}\ \cancel{2} \int_0^1 (1) \sin\left(\frac{\pi n}{2} t\right) dt$$

$$= \frac{-2}{n\pi} \cos\left(\frac{n\pi t}{2}\right)\Bigg|_0^1 = \frac{-2}{n\pi}\left[\cos\frac{n\pi}{2} - 1\right]$$

(1.5) $$= \frac{2}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

for odd value of n $\quad \cos\frac{n\pi}{2} = 0$ (1)

$\Rightarrow b_n = \frac{2}{n\pi}$

for even values of n

$n = 2, 6, 10, 14, 18 \ldots$

$b_n = \frac{2}{n\pi}[1-(-1)] = \frac{4}{n\pi}$

$n = 4, 8, 12, 16, 20 \ldots$

$b_n = \frac{2}{n\pi}[1-(+1)] = 0$

$$b_n = \begin{cases} \frac{2}{n\pi} & \text{for odd } n \\ \frac{4}{n\pi} & n = 2, 6, 10, 14, 18, \ldots \\ 0 & n = 4, 8, 12, 16, 20 \end{cases}$$

$$\Rightarrow \boxed{x(t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi t}{2} = \frac{2}{\pi}\sin\frac{\pi}{2}t + \frac{2}{\pi}\sin \pi t + \frac{2}{3\pi}\sin\frac{3\pi t}{2} + \cdots}$$

c) $$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2\, dt = \frac{1}{4}\left[\int_{-1}^{0} (-1)^2 dt + \int_0^1 (1)^2 dt\right] = \frac{1}{2}$$ (2)

E = area under square of the signal. $= \int_{-\infty}^{\infty} |x(t)|^2 dt$ (1)

$= \infty$

we may write

$$x(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right] \sin\frac{n\pi}{2} t$$