Time-sensitive, Sensor-based, Joint Planning and Control of Mobile Robots in Cluttered Spaces: A Harmonic Potential Approach

Ahmad A. Masoud, Ali Al-Shaikhi

Abstract— **This paper suggests an integrated navigation control system for time critical missions. The navigation control is derived from a harmonic potential field. It is designed to enable a mobile agent to proceed to a target point in an unknown environment without the need for a dedicated exploration and map-building stage. The agent, en route to the target, collects and processes only the necessary and sufficient sensory data needed to successfully execute the mission. Sensing, processing and all related activities needed to generate mobility are carried-out in real-time at the servo-level. The structure of the navigation control is described in details. Experimental results are provided as a proof of principle.**

I. INTRODUCTION

The interest in first responder robotics is continuing to grow. The aim is to provide a human operator with situational awareness in a timely manner. This mode of operation imposes strict & challenging requirements on a robot. In a first responder situation, a mobile robot should be able to move to a designated area in an unstructured and unknown environment. This should be accomplished under zero *a priori* knowledge without engaging in time-consuming activities reserved to exploration and mapping only. The robot is expected to dedicate all its effort to reaching the assigned target zone using necessary and sufficient information its sensors pick-up while attempting to reach the target. Other requirements complicate the constructions of such robots. For example, a robot of this type must have an agile and robust behavior that is communication-aware. The robot is also expected to operate in a hazardous situation where the probability of damage, even loss, is high. Therefore, a first responder robot is expected to be affordable. The above requirements represent a challenge, to say the least, to existing paradigm for autonomous mobility generation.

Mobility is a composite activity that emerges from the interaction of basis activity modules (Figure-1). One of these modules is concerned with the acquisition of environment data [1]. The data is processed and structured by a representation module [2,3] to create a map. A localization module [4] is then used to make the position of the agent on the map corresponds to its true location in the environment. The guidance module provides the agent with the direction along which it has to proceed in order to reach the target [5,6]. The control module [7,8] converts the reference direction into a control signal that is fed to the agents actuators. Despite the intensive work on each module, many issues relating to how they function are still considered to be an open area of research. There is a growing concern that a modularized view of mobility leads to an overly complicated system with shaky performance. The trend is growing to develop theoretical frameworks that jointly examine the construction of more than one of these modules. Examples of this are: simultaneous localization and mapping [9], direct guidance from sensory (observation) space [10,11], joint guidance and control [12]. To the best of these authors' knowledge, a theoretical framework that jointly tackles all the modules needed for providing an autonomous agent with mobility does not exist. Putting together a complete mobility system seems to be mainly dependant on the experience of the designer [13,14].

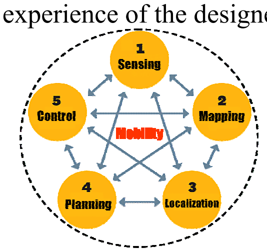

Fig. 1. Components of a mobility structure

The authors strongly believe that modularization is not conducive to achieving an agent mobility that suits a first response situation. They provide in this paper a proof of principle that an integrated view of mobility can meet these requirements. The integration of the mobility elements is carried-out using harmonic potential fields (HPFs) [15,16]. A harmonic potential can efficiently host the representation, guidance and control modules to cerate an integrated navigation action that is implementable at the servo-level. The navigation control can accept virtually unprocessed data even from difficult to use sensors such as ultrasonic sensors [17]. This data may be used to incrementally construct a representation of a safe space [18] for the agent to move through en-route to the target. The representation appears as a pattern imprinted on the gradient field of the HPF. This field is used to guide the robot. Guidance is computed by solving the Laplace boundary value problem, while what the control action attempt to do is to align the velocity of the robot with the gradient field. HPFs have an interesting property that makes the amount of processing commensurate with the sensory update hence achieving an output-sensitive

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Ahmad A. Masoud is with the electrical engineering department, King Fahad University of Petroleum & Minerals, Dhahran, 31261, Saudi Arabia (e-mail: masoud@kfupm.edu.sa)

Ali Al-Shaikhi is with the electrical engineering department, King Fahad University of Petroleum & Minerals, Dhahran, 31261, Saudi Arabia (e-mail: shaikhi@kfupm.edu.sa)

navigation action. That is: if HPFs are disturbed by introducing local constraints, the effect of the newly introduced constrains is localized to the vicinity of the changes.

This paper suggests components and workflow for constructing an HPF-based navigation control at the servo-level. The navigation control is able to move the robot along an obstacle-free path in a fully unknown environment without engaging in an exploration and mapping stage. The controller is implemented and tested on the X80 robotics platform. It is restricted to using only the front ultrasonic sensor of the robot. Naïve dead-reckoning is used for localization where the pause of the robot is obtained by directly integrating its linear and angular speeds. All processing is one on-board a host computer. Sensory and control signals are exchanged in real-time with the X80 using a wireless communication link.

II. CONDITIONALLY-INFORMED NAVIGATION

Projecting mobility through the sequence of activities: environment mapping, guidance generation and motion actuation is self-defeating as far as making mobility compatible with a first responder situation. A simple and powerful alternative is to generate a realizable plan to the target given the amount of information that is initially available. The agent neither seeks, nor engages in environment mapping activities. Instead, it proceeds to the target assuming that it knows everything. If an event that would prevent the agent from continuing to use the plan is encountered, the information about that event is used to locally adjust the plan so that the event is mitigated and the continuity of action is restored.

The harmonic potential approach to planning does support the above modality of operation. The approach amasses many critical properties needed for successful integration in a first responder mobility system [21]. The approach is provably-correct, it can operate in a model-based or sensor-based modes [22], it can process vague information [16], it can enforce, in a provably-correct manner, a variety of constraints on motion [15] and it yields analytic trajectories guaranteeing the construction of a provably-correct control. A basic setting of the harmonic approach is:

$$\text{Solve} \qquad \nabla^2 V(x,y) \equiv 0 \qquad x, y \in \Pi$$

$$\text{Subject to:} \qquad V(x_T, y_T) = 0, V(x,y) = 1 \qquad x, y \in \partial\Pi \qquad (1)$$

Motion is safely guided to the target using the gradient dynamical system:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\begin{bmatrix} \partial V / \partial x \\ \partial V / \partial y \end{bmatrix} = \begin{bmatrix} Gx \\ Gy \end{bmatrix} \qquad (2)$$

where Π is the workspace of the robot and $\partial\Pi$ is the boundary of Π.

The HPF approach can localize the disturbance caused by introducing a new environment component. Let V be a harmonic function constructed from the set $\partial\Pi$. Also, let Vp be a harmonic potential constructed from the set $\partial\Pi \cup P$, where P is a newly introduced point obstacle. If B is a spherical region with center P and radius ε, then one can show that an ε maybe found such that

$$|V - Vp| < \delta \qquad \forall x, y \notin B. \qquad (3)$$

where δ is an arbitrarily small positive number. The following example illustrates this property. Figure-2 shows the guidance field from a harmonic potential and the same field perturbed by a localized, newly introduced obstacle. A deviation measure (Figure-3) between the two fields is computed and normalized to unity. As can be seen, the deviation between the two guidance fields quickly diminishes with motion away from the disturbance. This means that far from the disturbance one does not need to recomputed the guidance field.

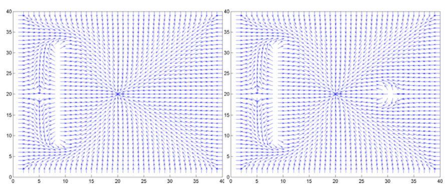

Fig. 2. a guidance field & its slightly perturbed counterpart.

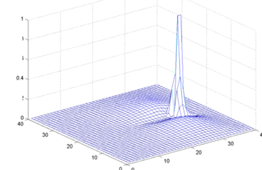

Fig. 3. deviation between the fields in Figure-5.

Almost all navigation techniques represent environments using objective geometric maps (OE). This is a big source of problem facing the success of any navigation action. The sensor noise and localization errors alone are enough, on their own, to make the registered geometric map significantly deviate from the actual map of the environment. Moreover, geometry is not the event that inhibits mobility. Rather, it is safety. To know when a mobility plan needs adjustment, the agent must subjectively interpret the environment in terms of safety (Figure-4). Subjective safety representations (SE) are easily constructible. All what is required is: if the agent is at a safe place in the environment, then its place on the safety map is marked safe. It ought to be noticed that sensing errors caused by spurious reflections do not endanger robot's safety. These errors will cause a safe location to be marked unsafe. The worst case sensing artifacts and localization error could lead to is partial loss of safe and usable space.

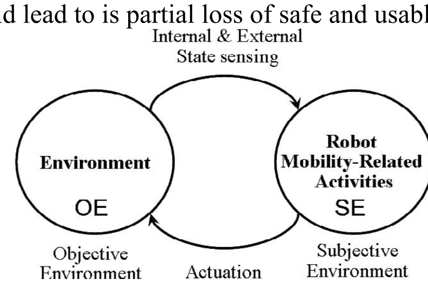

Fig. 4. Objective (OE) and subjective (SE) environments of the robot

An important property of harmonic field planners is that the guidance signal they provide can be converted in a provably-correct manner into a navigation control signal [23] for a generic class of mobile robots that is describable by the system equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = H\left(\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}\right)\begin{bmatrix} \nu \\ \omega \end{bmatrix}, \quad \begin{bmatrix} \nu \\ \omega \end{bmatrix} = G(U) \tag{4}$$

where x and y are the center of the robot in the SE coordinates, θ is the orientation of the robot ν and ω are the tangential and angular speeds of the robot respectively, U is the control signal vector.

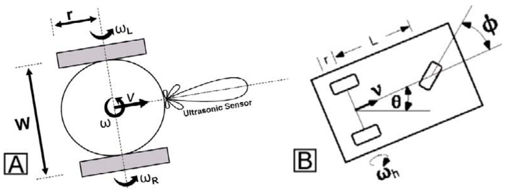

Fig. 5: Differential drive (A) & Car-like (B) robots

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \nu \\ \omega \end{bmatrix}, \quad \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{W} & \frac{-r}{W} \end{bmatrix}\begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}, \tag{5.a}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \nu \\ \omega \end{bmatrix}, \quad \begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} r \cdot \omega_h \\ \tan(\phi)\frac{r \cdot \omega_h}{L} \end{bmatrix}, \tag{5.b}$$

The above model accommodates a wide class of mobile robots including the two important types (Figure-5): the differential drive robot (5.a) and the car-like robot (5.b). ω_L and ω_R are the right and left wheels' speeds of the differential drive robot, W is its width and r is the wheels' radius , ω_h is the speed of the driving wheel of the car-like robot, φ is the steering angle and L is the distance between the center of the driving wheels axis and the steering wheel. It is shown in [23] that if the robot orientation is made to coincide with the guidance orientation at the starting point of motion, the kinematic trajectory will be identical to the dynamic trajectory.

III. FROM GUIDANCE to NAVIGATION CONTROL

Generating the navigation control signal is based on the work in [23]. The aim of the control signal is to synchronize the velocity of the robot with the guidance velocity from the negative gradient of the harmonic potential (Figure-6). The result is a provably-correct navigation control signal that can be made to inherit all the properties of the guidance signal. There are however two problems facing the direct use of the method in [23] in navigating a real mobile robot. The first is related to hardware-friendliness. The second and most important has to do with dynamic sensor update. The structure in [23] is provably correct given a static guidance field. If there is a need to adjust the field due a safety related event, the robot has to stop, re-compute the field, align its orientation with the new guidance vector at its current location then start actuating motion. Interrupting motion by frequent stops is highly undesired; continuity of motion should be preserved. Moreover, as shown below, stopping can be easily bypassed without sacrificing the provably correct nature of the guidance to control conversion process.

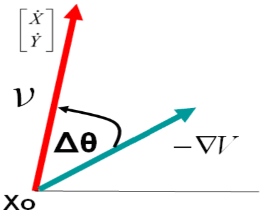

Fig. 6. Controller aligns guidance signal with robot's velocity

First the sine and cosine of the angle between (Figure-6) the velocity of the robot and the guidance vector ($\Delta\theta$) along with the straight line distance to the target (dst)

$$\eta_d = \cos(\Delta\theta) = \frac{Gx \cdot \dot{x} + Gy \cdot \dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}, \tag{6}$$

$$\eta_c = \sin(\Delta\theta) = \frac{Gx \cdot \dot{y} - Gy \cdot \dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}$$

$$dst = \sqrt{(x - x_T)^2 + (y - y_T)^2}$$

A desired angular actuation signal (ωc) must function to synchronize the actual robot speed with the desired guidance vector. The angular synchronizing signal (Figure-7) may be easily computed as:

$$\omega c = \omega d \cdot \begin{bmatrix} \eta_c & \eta_d > 0 \\ +1 & \eta_c > 0 \,\&\, \eta_d < 0 \\ -1 & \eta_c < 0 \,\&\, \eta_d < 0 \end{bmatrix}, \tag{7}$$

where ωd is the maximum desired angular speed the robot should assume.

A desired tangential robot speed (νc) should be maximum when the robot is in phase with the guidance vector. This speed should gradually drop as the speed gets out of phase with guidance. νc should drop to zero when the robot's speed and guidance are in an antipodal configuration which is created by the detection of an event that endangers the safety of the robot (Figure-7). The robot should slow-down when it approaches the target, totally dropping to zero when it is an Rc distance from the target. The speed may be constructed as follows:

$$\nu c = vd \cdot \begin{bmatrix} (\eta_d + 1)/2 & \eta_d < 0 \\ 1 - (|\omega c / \omega d|)/2 & \eta_d \geq 0 \\ dst/Rc & dst \leq Rc \end{bmatrix} \tag{8}$$

where vd is the maximum tangential speed the robot should assume. It ought to be mentioned that even a constant νc with a decelerating target approach profile will produce satisfactory results. After the desired angular and tangential speeds of the robot are obtained, the control signal may be easily generated as:

$$U = Q\left(\begin{bmatrix} \nu c \\ \omega c \end{bmatrix}\right) = G^{-1}\left(\begin{bmatrix} \nu c \\ \omega c \end{bmatrix}\right) \tag{9}$$

The overall structure for converting guidance to control is shown in Figure-8.

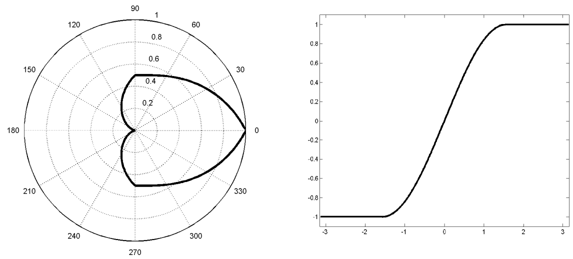

Fig. 7: Desired tangential and angular velocities of the robot respectively

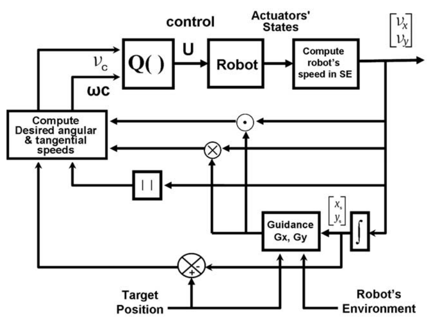

Fig. 8. The suggested, hardware-friendly controller

IV. THE INTEGRATED NAVIGATION STRUCTURE

In this section the modules discussed above are integrated to yield the mobility structure. The structure is described using the flowchart in Figure-9. As mentioned before, the structure is self-contained where the only piece of information it needs from the external user is where the target is located. It provides the robot will full autonomous capabilities with no restrictive assumptions on the robot's space.

First, the structure needs to be initialized by specifying the subjective coordinates which data is recorded with respect to. The horizon of the safety map is a square domain (Γ) of width D. A Cartesian coordinate system (x,y) is assumed so that the origin is at the center of the domain. Initially, the robot is assumed to lie at the origin (x=0,y=0) of the SE coordinates with orientation along the x-axis (θ=0). The perimeter (D) of SE is supplied along with the target in SE which has to be the image of the target in OE.

The guidance field is then globally computed given the initial information available. The wheels' speeds of the robot and the ultrasonic sensor measurements are recorded. If no obstacles are within the sensor range of the robot, the pre-computed guidance information is used. If an obstacle facing the robot is detected, it is mapped into the subjective environment of the robot. The guidance field is then locally recomputed around the added environment component and the modified guidance information is obtained. Using the robot's wheels' speeds the pose of the robot in SE is updated. This information is combined with the guidance signal to compute the control signals. If the robot stops short of reaching its target, then the cause of the problem has to be the partial field computation stage. Since motion did halt, computing the guidance field in real-time is no longer important. Therefore, the full field computation stage is invoked to correct this problem.

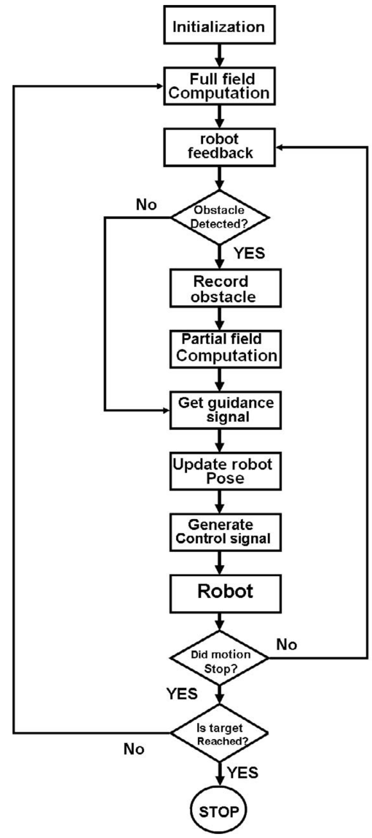

Fig. 9. The ultrasonic navigation control structure

V. EXPERIMENTAL RESULTS

An inexpensive platform (X80 differential drive UGV, Figure-10) is used to experimentally validate the structure. The servo navigation signal is sent to the robot via a wireless link that was not designed for real-time operation. Only sensor-based experiments are reported. The obstacles of the environment are constructed from cable drums. The structure of cable drums creates considerable scattering of the ultrasonic signal.

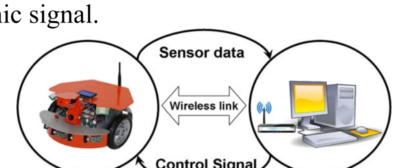

Fig. 10. The X80 differential drive mobile robot platform

Advanced deadreckoning techniques [19], even precise optical deadreckoning [20], do exist. However, here, basic deadreckoning is used by directly computing the robot's pose from its wheels' speeds:

$$\begin{bmatrix} \nu \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{r}}{2} & \frac{\mathrm{r}}{2} \\ \frac{\mathrm{r}}{\mathrm{W}} & \frac{-\mathrm{r}}{\mathrm{W}} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix},$$

$$\nu x = \nu \cdot \cos(\theta)\,,\ \nu y = \nu \cdot \sin(\theta) \tag{10}$$

$$\mathrm{x}(\mathrm{t}+\Delta\mathrm{T}) = \mathrm{x}(\mathrm{t}) + \Delta\mathrm{T}\cdot \nu\mathrm{x}(\mathrm{t})\,,$$

$$\mathrm{y}(\mathrm{t}+\Delta\mathrm{T}) = \mathrm{y}(\mathrm{t}) + \Delta\mathrm{T}\cdot \nu\mathrm{y}(\mathrm{t})$$

$$\theta(t+\Delta\mathrm{T}) = \theta(t) + \Delta\mathrm{T}\cdot\omega(t) \quad \mathrm{x}(0)=0,\ \mathrm{y}(0)=0,\ \theta(0)=0$$

The velocity control signals that are to be applied to the robot's wheels are computed from the desired angular and tangential velocities of the robot using the equation:

$$\begin{bmatrix} \omega C_R \\ \omega C_L \end{bmatrix} = \frac{1}{r} \cdot \begin{bmatrix} 1 & \frac{W}{2} \\ 1 & -\frac{W}{2} \end{bmatrix} \begin{bmatrix} vc \\ \omega c \end{bmatrix} \qquad (11)$$

Only one ultrasonic sensor (the front sensor) of the robot is used to collect data about the environment. The main lobe of the sensor is aligned along the principal axis of the robot. The sensor produces the continuous output S(t). Ideally, S(t) provides a measurement of the distance between the sensor and the closest obstacle that lies along the principal axis of the robot. The maximum range of the sensor is 2.55 meter. A maximum reading is an indicator that either no obstacle exist along the principal axis or the obstacle is out of sensor range. The subjective environment of the robot is recorded at a resolution Δ using an NxN matrix DSE(i,j) (Δ=D/N). If DSE(i,j) is marked by 1, the location indexed by i & j is considered unsafe. If it is marked by 0, the location is considered possibly safe. DSE is constructed as follows (S=2.55, i.e. no obstacles detected is treated as a special case): first the matrix is initialized

$$\text{DSE}(1,i)=\text{DSE}(N,i)=\text{DSE}(i,1)=\text{DSE}(i,N)=1 \quad i=1,..N, \qquad (12)$$
$$\text{DSE}(i,j)=0 \quad i=2,..N-1,\ j=2,..N-1$$

At a certain instant in time, given a robot's pose (x,y,θ), DSE is populated as follows:

$$\text{Io} = \left[\frac{x + (S+R)\cdot\cos(\theta+\delta)}{\Delta}\right], \quad \text{Jo} = \left[\frac{y + (S+R)\cdot\sin(\theta+\delta)}{\Delta}\right] \qquad (13)$$

$$\text{DSE}(\text{Io}+m,\text{Jo}+n)=1 \quad n=-\text{Im},..\text{Im},\ m=\text{Im},..\text{Im} \quad N>\text{Io}+m>1,\ N>\text{Jo}+n>1$$

where R is the distance from the center of the robot to where the sensor is located, Im is a nonnegative integer used as a safety margin surrounding the sensed obstacle, [X] is the rounding integer function of the real number X, 1>>δ>0 and m,n are positive integers used to specify a safety zone around the point (Io,Jo).

Using only the wheels speeds ($\Omega = [\omega_R \quad \omega_L]^T$), the target location in SE ($X_T = [x_T \quad y_T]^T$) and readings from the sensor (S(t)), a control velocity signal ($\Omega c = [\omega c_R \quad \omega c_L]^T$) is synthesized

$$\Omega c = F(\Omega, X_T, S(t))$$

such that $$\lim_{t\to\infty} X \to X_T \ \ \&\ \ R(t) \cap O \equiv \phi \quad \forall t. \qquad (14)$$

where X_T is the target position, R(t) and O are the regions occupied by the robot and obstacles respectively in OE.

The ability of the navigation controller to move the X80 to a target point under zero initial information about the environment using only one front ultrasonic sensor was tested for many obstacles' configurations. Cable drums are used to construct an obstacle course, which the robot should pass through in order to reach the target. Figure-11 shows the environment along with snapshots of the robot on its way to the target. As can be seen, using only impoverished sensing and modest localization, the robot manages to reach the target from the first attempt.

Fig-11: Environment & robot snapshots, barriers example

Snapshots of the subjective environment superimposed on the guidance field taken at different time instants are shown in Figure-12. The absence of initial information about the environment is obvious. The snapshots clearly illustrate the nature of conditionally-informed navigation.

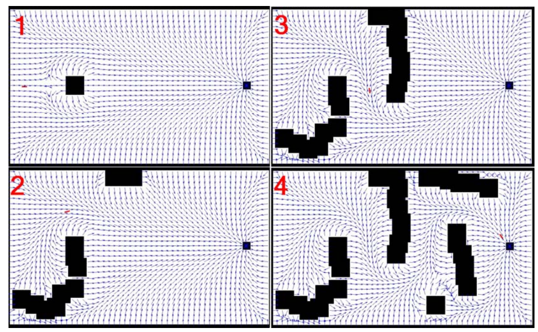

Fig. 12 : snapshots of the guidance fields, barriers example

Figure-13 shows the trajectory the robot took superimposed on both the subjective and objective environments of the robot. As can be seen, there is a significant difference between the actual geometric environment of the robot and the final safety map the robot recorded. Despite this, the differential and integral properties of the trajectory are very good especially when one takes into consideration that it was generated under zero initial information.

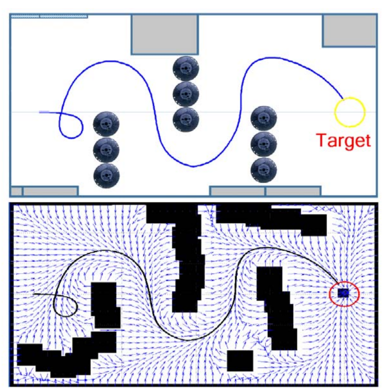

Fig. 13: trajectory overlaid on the objective and subjective environments, barriers example

The signal from the ultrasonic sensor is shown in Figure-14. The signal is highly irregular with a lot of discontinuities. Despite this, the orientation profile of the robot (Figure-15) is excellent and the control signals (right and left wheels' speeds) are well-behaved (Figure-16).

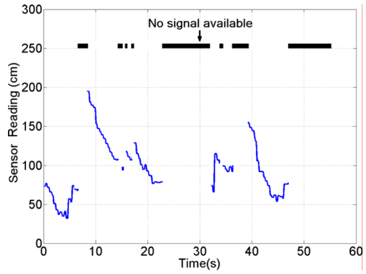

Fig.14: ultrasonic sensor signal, barriers example

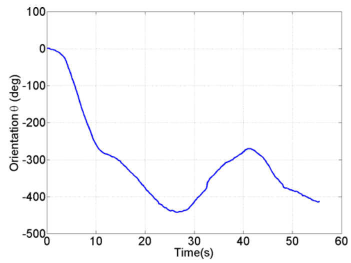

Fig. 15: Robot orientation, barriers example

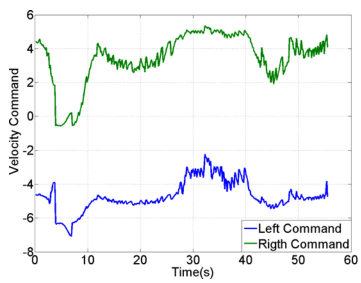

Fig. 16: Control signals, barriers example

VI. CONCLUSIONS

This paper provides a proof of principle that the potential field approach is not only suitable for motion planning at the servo-level of a robot, it also have the ability to provide provide a provably-correct, ultrasonic sensor-based servo-level navigation control signal. The suggested structure, despite the simplicity of the processing used, can successfully tackle challenging environments both in terms of geometry and sensory signal distortion. The paper also demonstrates the centrality of the guidance module (motion planner) to the overall mobility structure. The work in this paper provides a strong reason to re-examine the belief that accurate, geometrical mapping of the environment is a perquisite to satisfactorily navigate an unstructured environment. Experimental results show that proceeding towards the target under zero *a priori* information while collecting only the data needed to guarantee safety can produce trajectories with good differential, state and integral characteristics.

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