A Decentralized, Harmonic, Potential Field-based Controller for Steering Dynamic Agents in a Cluttered Environment

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Abstract: In this paper a separation maintenance controller is developed for continuously steering a purposive, dynamic group of mobile agents away from each other in a confined, cluttered environment. The controller allows conflict-free, simultaneous use of space by the agents. It has a decentralized form that is constructed in conformity with the artificial life approach to behavior synthesis. The G-type control action used individually by the agents to govern their motion (also known as the control protocol) is extracted from a harmonic potential field. The overall controller governing the group (P-type control) has a decentralized, self-organizing, asynchronous nature with a computational effort that linearly grows with the number of agents. The capabilities of the controller are demonstrated by simulation. Proofs that the agents can avoid self and environment collision as well as converge to their targets are provided.

I. Introduction:

The interest in the deployment of teams of robots in the air (unmanned aerial vehicles: UAVs) and/or on the ground (autonomous ground vehicles: AGVs) has significantly increased in the past few years [1,2]. With the advances in technology to miniaturize actuators and sensors and make them more efficient, the use of large teams of mobile sensors and actuator networks (MAS-nets) to do jobs like surveillance, area coverage etc. is becoming popular [3,4]. The immediate challenge facing the efficient deployment of such systems seems to be the development of practical, robust and user-friendly multi-agent, goal-oriented, context-sensitive, intelligent controllers (i.e. planners). A well-designed multi-agent planner improves efficiency, reduce fuel consumption, increase mission life and marginalize the probability of damage or unexpected behavior [5]. Multi-agent planners are also receiving considerable attention in the air traffic control (ATC) area both as aids to human operators in order to manage the complexity of the process or in a fully autonomous mode as regulators of traffic (free-flight [6,7]). There is a long list of desired properties a planner must possess in order to be useful in practice. For example, a multi-agent planner must be able to de-conflict space while allowing all agents to simultaneously proceed to their respective targets along trajectories having acceptable states, differential and integral properties. While under ideal conditions the design of such planners is an involved task, the restrictions imposed by a realistic environment further complicates the job [8]. The planner has to take into considerations the limitations on the sensory and communication modes an agent can assume as well as its dynamics. Tolerance to actuators and sensors failure is an operational necessity. The planner must also be able to efficiently handle the large size of the group and control the growth of the computational effort needed to generate the navigation policy. The environment in both crisp and vague forms is also an important issue that should be factored in the motion generation process [9,37]. Moreover, the generated planning action must not be committal, i.e. it must not assume the happening of certain future events for the existing planing action to be valid at a later time.

Planners based on heuristics [10,11], even if proven efficient, are not acceptable. Any planning action considered in the context of the above described situation has to be provably-correct. Geometric methods for planning are the opposite of heuristic methods. They can be placed in an elegant, provably-correct, algorithmic form [12]. They show good ability to control the growth of complexity when planning is carried-out in high dimensional spaces. However, they remain inherently centralized and, in the opinion of this author, far from being able to deal with swarm-like groups [13]. Up-untill-now there is no record that algorithmic methods can accommodate all the requirements stated above. There is a growing feeling in the area of large scale systems that evolutionary and self-organizing methods [14,15] can be successfully used to carry-out the challenging task of designing a realistic, multi-agent planner. Experimental tests under realistic conditions using airliners seems to support such an opinion, in particular, showing the advantages potential field methods have over classical approaches [36]. The artificial life (AL) approach is a powerful evolutionary paradigm that has proven effective in regulating the behavior of a large group of agents [16]. It works by synthesizing a proper, social, self-controler to be used individually in an asynchronous, sensory-limited manner by the agents making up the group. This controller acts like a control protocol and has the name G-Type controller. The overall control action regulating the behavior of the group (P-Type control) emerges as a result of the interpretation of the G-type control in the context of the environment. The harmonic potential field approach to planning [17-20] is an expression of the AL paradigm. Harmonic potential fields (HPFs) have proven themselves to be effective tools for inducing in an agent an intelligent, emergent, embodied, context-sensitive and goal-oriented behavior. A planning action generated by an HPF-based planner can operate in an informationally-open and organizationally-closed mode [21] enabling an agent to make decisions on-the-fly using on-line sensory data without relying on the help of external agents. HPF-based planners can also operate in an informationally-closed, organizationally-open mode. This makes it possible to utilize existing data about the environment in generating the planning action as well as elicit the help of external agents. A hybrid of the two modes may also be constructed. Such features make it possible to adapt HPFs for planning in a variety of situations. For example in [22] vector-harmonic potential fields were used for planning with robots having second order dynamics. In [23] the approach was configured to work in a pursuit-evasion planning mode, and in [24] the HPF approach was modified to incorporate joint constraints on regional avoidance and direction. The decentralized, multi-agent, planning case was tackled using the HPF approach in [25]. The HPF approach was also found to facilitate the integration of planners as subsystems in networked controllers containing sensory, communication and control modules with a good chance of yielding a successful behavior in a realistic, physical setting [26].

Existing HPF techniques were used to control one mobile agent only. This work demonstrate that the HPF approach is naturally decentralized. An HPF-based controller may be used in a provably-correct manner, with negligible modifications, as the G-type control in an AL multi-agent controller. The resulting multi-agent controller has decentralized, self-organizing, asynchronous nature with a computational effort that linearly grows with the number of agents.

II. The decentralized controller:

Formally a multi-agent planner that maintains separation must maximize the minimum inter-agent distance as well as the minimum distance between the agents and the clutter populating the environment while guaranteeing that each member reaches its destination. Unfortunately formulating the problem in this manner for a large group of agents leads to an intractable situation. Self-organizing optimization methods [27-30] may be used for such a purpose. They are known for their ability to handle nonlinear functions having large degrees of freedom. Neglecting the fact that these methods are not provably correct and cannot guarantee that a solution can be found if one exist, they do not provide acceptable transient behavior that allows them to serve online as traffic controllers. Instead of seeking a formal and optimal solution to the problem, this paper attempts a practical solution with acceptable properties. The solution sought is built around a decentralized paradigm that employs local interaction and sensing among agents in regulating the group's motion. The artificial life paradigm to behavior synthesis does support this mode of operation. In an AL approach each agent is equipped with a self-controler (G-Type control) that senses its local neighborhood. The self-controlers are similar and relatively simple. The overall control action that is regulating the behavior of the group (P-Type control) is collectively generated as the agents interact in the context they are operating in. Therefore designing a controller for the collective (figure-2) reduces to designing the proper G-Type controller (G_i) which each agent must use. The controller should be designed such that for the hyper-system in (1) overall system conflict is eliminated and goal for each member is attained (Figure-1).

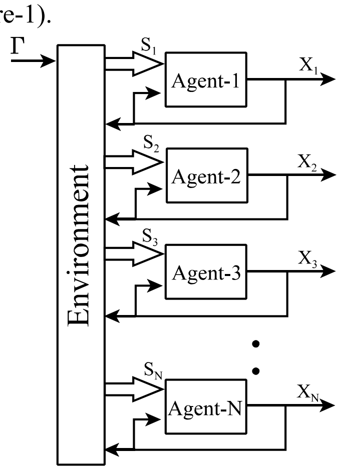

Figure-2: Overall, decentralized system.

$$\begin{bmatrix}\dot{\mathbf{x}}_1\\ \dot{\mathbf{x}}_2\\ \cdot\\ \dot{\mathbf{x}}_N\end{bmatrix}=\begin{bmatrix}\mathbf{G}_1(\mathbf{x}_1,\mathbf{S}_1)\\ \mathbf{G}_2(\mathbf{x}_2,\mathbf{S}_2)\\ \cdot\\ \mathbf{G}_N(\mathbf{x}_N,\mathbf{S}_N)\end{bmatrix},\quad x_i\in\Omega,\ x_i\neq x_j\ i=1,..,N,\ i\neq j$$

we have

$$\begin{aligned}&\left|\mathbf{x}_i-\mathbf{x}_j\right|>\mathbf{0} \qquad \mathbf{i}\neq\mathbf{j}\\ &\mathbf{x}_j\cap\Gamma=\phi \qquad \forall\mathbf{t}\end{aligned},$$

$$\mathbf{S}_i=\Gamma\bigcup_{\substack{j=1\\ j\neq i}}^{N_i}\mathbf{x}_j\ , \tag{1}$$

where S_i is the data sensed by the i'th agent (figure-2), N_i is the number of agents observed by the i'th agent, Ω be the free space (workspace) which the agents are allowed to occupy at a certain time, O is the region occupied by the clutter and Γ the boundary of this region (Γ=∂Ω), x_i is the position of the i'th agent at a certain instant in time.

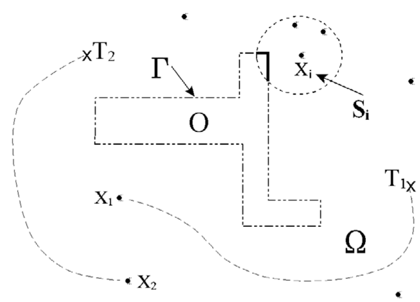

Figure-2: Planner physical setting.

III. The G-Type Controller

While the separation control effort regulating the behavior of the group (P-Type control) is generated by the dynamical system in (1), the focus in an AL implementation is on designing admissible, self-controllers (G-Type controllers) whose social interaction within the confines of the environment yields the desired behavior figure-3,

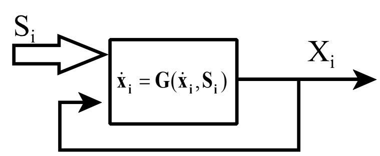

Figure- 5: The G-type controller.

Two cases for constructing the G-Type controller are considered: the goal oriented case where each agent proceeds along a collision-free path towards a target that it chooses independently of the other agents. The second mode considered is a flexible formation mode where the group distributes itself within a confine whose shape and motion are determined by a the leader agent.

A. The goal-oriented mode:

The control effort is derived from the gradient of a harmonic potential used by a single agent to steer itself. As demonstrated by simulation (a proof is also provided in section-IV), the single agent HPF has a social nature that allows it to co-exist with other agents using a similar navigation procedure in the same cluttered space. The reason for that is: the HPF approach treats other agents as obstacles to be avoided. Hence the same procedure used to avoid clutter can also be used to accommodate the presence of the other agents. A basic BVP for generating the self-control action is:

$$\nabla^2 V(x_i) = 0 \qquad x_i \in \Omega .$$
$$V(x_i)\Big|_{x_i \in S_i} = 1 \;\; \& \;\; V(x_i)\Big|_{x_i \in T_i} = 0$$

Where $\qquad \dot{x}_i = -\nabla V(x_i)$. (2)

Other settings of the HPF approach are found in [24,31].

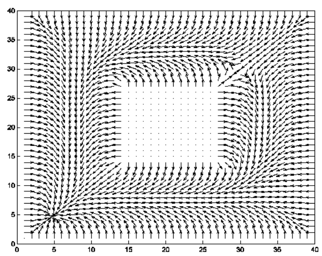

Figure-4: Potential and gradient guidance field for a goal-oriented, G-type controller.

B. The separation mode:

The other mode for which a G-Type controller is constructed is the formation separation mode. Unlike the goal-oriented mode where the target point is given and the group need only to lay a conflict-free path to it, the separation mode requires the group to jointly generate the target point for each agent as well as lay a safe trajectory to that point. The HPF approach may still be used to generate a self-controller for this case. The BVP generating the potential is similar to the one in (2) with no target point having a potential preset to zero. The control action that dynamically distribute the agent in specified space may be derived from the BVP:

$$\nabla^2 \mathbf{V}(\mathbf{x}_i) = \mathbf{0} \qquad x_i \in \Omega . \qquad (3)$$
$$\mathbf{V}(\mathbf{x}_i)\Big|_{\mathbf{x}_i \in \mathbf{S}_i} = \mathbf{1}$$

The above BVP may appear to be of little use since by the maximum principle, the solution of V in Ω is a constant. This means that the gradient field degenerates everywhere in Ω. The pontential field from an environment similar to the one in figure-4 is shown in figure-5.

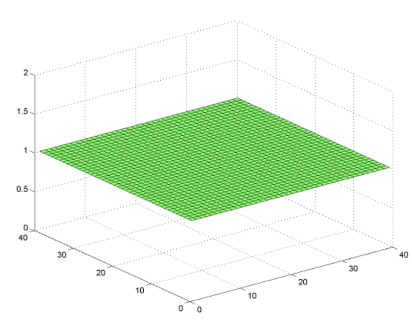

Figure-5: Potential field degenerates in the formation case.

A careful examination of the solution of (3) reveals that only the magnitude of the gradient field ($A(x_i)$) degenerates while the phase field ($Q(x_i)$) remains stable and computable. The component of the BVP in (3) that corresponds to the phase field may be derived as follows:

$$\nabla V(x_i) = A(x_i)Q(x_i), \qquad (4)$$
$$\nabla^2 V(x_i) = \nabla \cdot \nabla V(x_i)$$
$$= \nabla A(x_i)^T Q(x_i) + A(x_i)\nabla \cdot Q(x_i) .$$

The gradient of the magnitude of ∇V in (4) drops to an infinitesimally small positive constant ϵ while A converges to unity. In this case the laplacian becomes:

$$\nabla^2 \mathbf{V}(\mathbf{x}_i) = \nabla \cdot Q(x_i) .$$

Since the potential is restricted to a constant value at S_i, Q will have no component tangent to S_i (i.e. $n \times Q(x_i)=0$, $xi \in S_i$), where n is a unit vector normal to S_i. Therefore, the boundary value problem that may be used to generate Q is:

$$\nabla \cdot Q(x_i) \equiv 0 \qquad x_i \in \Omega$$
$$n \times Q(x_i) = 0, \quad n^T G(x_i) = 1. \qquad x_i \in S_i.$$
$$\dot{x}_i = \alpha \cdot Q(x_i) \qquad (5)$$

where α is a positive constant. The field, Q, generated by solving the above BVP is observed to possess field lines that emanate normal to S_i and move into Ω meeting at critical points inside the region (figure-6). Among other things these points show the tendency to form far from clutter and other agents occupying Ω. This makes it possible to utilize Q as the G-type separation control. As can be seen stable equilibrium points spontaneously form equally far from the obstacles in the environment The reason equilibrium points form inside Ω has to do with the fact that all the flows at the boundary are forced to be inside Ω, the continuity condition (∇·Q=0) will fail at some areas in Ω. This results in stable and unstable equilibrium points being formed. A Quantitative study of these points in terms of how far from the closest object they will form is expected to be mathematically involved and will be kept for future work. However, a qualitative examination show that these points are comparable to maximizing the minimum distance from the obstacles.

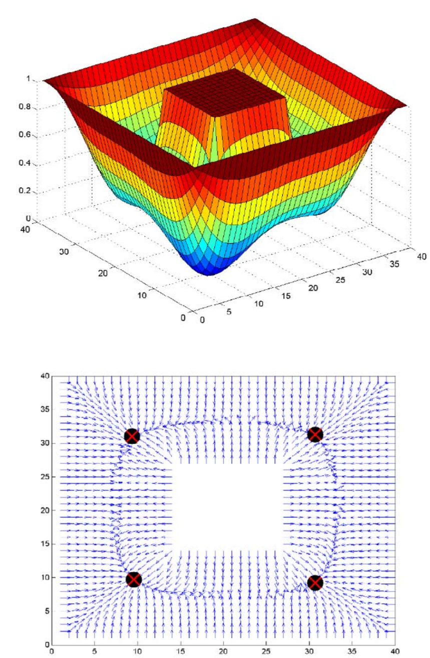

Figure-6: Separation G-type control, harmonic phase field and gradient guidance field.

IV. Analysis: the goal-oriented case:

Proposition-1: If $x_i(0) \in \Omega$, the motion steered by the gradient dynamical system in (2) will always remain inside Ω (i.e. $x_i \cap S_i \equiv \phi \ \ \forall t, \forall i$) .

Proof: Consider the part of Ω near an a forbidden region (S_i). Let $n(x_i)$ be a vector that is normal to the surface of the obstacle. Let S_i' be a region created by infinitesimally expanding the forbidden region S_i such that $S_i \subset S_i'$. The radial derivative of $V(x_i)$ along S_i may be computed as:

$$\frac{\partial V}{\partial n} = \frac{V(x_i) - V(x_i')}{\Delta r} \tag{6}$$

where x_i' is taken as the minimum distance between x and S_i' and Δr is a positive differential element. Since by the maximum principle the value of the potential in Ω is less than 1, and x_i' lies inside Ω, the radial derivative of the potential along $n(x_i)$ is negative, i.e.

$$n(x_i)^T \nabla V(x_i) < 0 \tag{7}$$

Since motion is steered using the negative gradient of V, the agent will be pushed away form S_i and x_i will remain inside Ω.

Proposition-2: If the G-type controller of the multi-agent system in (1) is selected as the HPF planner in (2) then every agent is guaranteed to converge to the target (T_i),

$$\lim_{t\to\infty} x_i = T_i \qquad i=1,2, ..N \tag{8}$$

Proof: Since $V_i(x_i)$ is shown to be a valid Liapunov function candidate (LFC) [32], i.e.

$$V_i(x_i) = 0 \quad \text{at and only at } x_i=T_i \quad \&$$
$$V_i(x_i) > 0 \quad \text{elsewhere,} \tag{9}$$

Their summation is also an LFC:

$$V(X) = \sum_{i=1}^{N} V_i(x_i) \tag{10}$$

$$V(X) = 0 \quad \text{at and only at } X=T \qquad \&$$
$$V(X) > 0 \quad \text{elsewehre,}$$

where $X=[x_1\ x_2\ ...\ x_N]^t$, and $T=[T_1\ T_2\ ...T_N]^t$. The time derivative of V(X) is:

$$\dot{V}(X) = -\sum_{i=1}^{N} \nabla V_i(x_i)^t \dot{x}_i \tag{11}$$

substituting $\qquad \dot{x}_i = -\nabla V_i(x_i)$

yields: $\qquad \dot{V}(X) = -\sum_{i=1}^{N} \left|\nabla V_i(x_i)\right|^2$

Harmonic functions are also proven to be Morse, i.e. all zero measure, isolated points at which the gradient field vanishes are unstable equilibrium points (see appendix). This leaves the stable equilibrium, target points (T_i's) as the only points in the minimum invariant set of the system. By applying the LaSalle invariance principle [33] it can be easily shown that each agent will converge to its respective target.

V. Results:

The ability of agents equipped with an HPF-based, G-type controller to cooperatively solve the planning problem while treating space as a scarce resource is tested. Five agents positioned opposite to each other are required to move to a specified target from a starting point selected so that a high probability of conflict scenario is established. In figure-7 the five agents utilizing a full communication graph attempt to solve the planning problem they are faced with. As can be seen, all agents reach their destination safely maintaining at all time a nonzero, minimum inter-agent distance (DM).In figure-8 the agents attempt to deal with the same situation. However, this time instead of using a full communication graph, each agent only communicates its position to its closest neighbor. Again the agents were able to safely resolve the conflict and arrive at their destination

In figure-9, the computational effort needed by the planner is examined in terms of the time needed to complete the steering process in the separation mode. The number of agents (Na) is varied from two to five and time needed to complete the steering process is recorded for each. Figure-10 shows the time needed to perform the steering process versus the number of agents. The time is normalized using that of the case Na=2. As can be seen, the computational time linearly grows with the number of agents. A full communication graph is used.

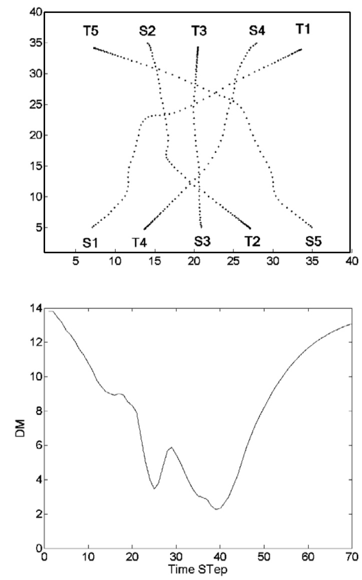

Figure-7: Goal-oriented mode, full graph communication: a. trajectories, b. minimum distance

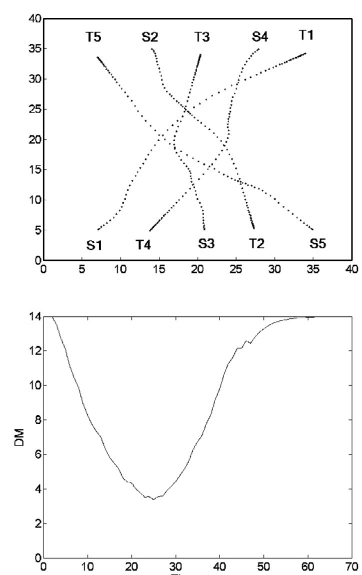

Figure-8: Goal-oriented mode, nearest neighbor communication: a. trajectories, b. minimum distance

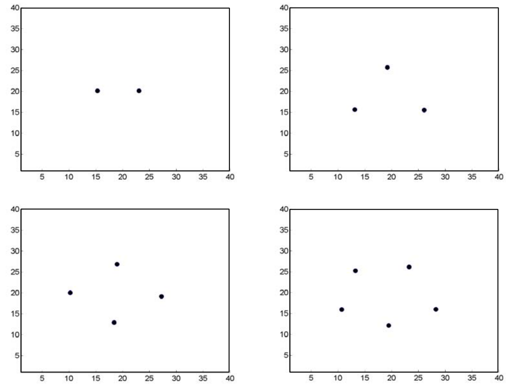

Figure-9: Separation mode, final constellation versus number of agents

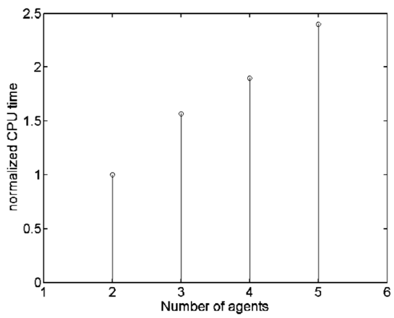

Figure-10: simulation time versus number of agents

In figure-11 the performance of the controller is examined in the presence of clutter for the separation mode. It is observed that all the attributes of the controller in the free space case were preserved when clutter is present. The agents distributed themselves in a final configuration that seems to maximize the minimum inter-agent distance as well as the distance to the nearest obstacle. Also, a strictly increasing with time minimum separation distance profile is observed.

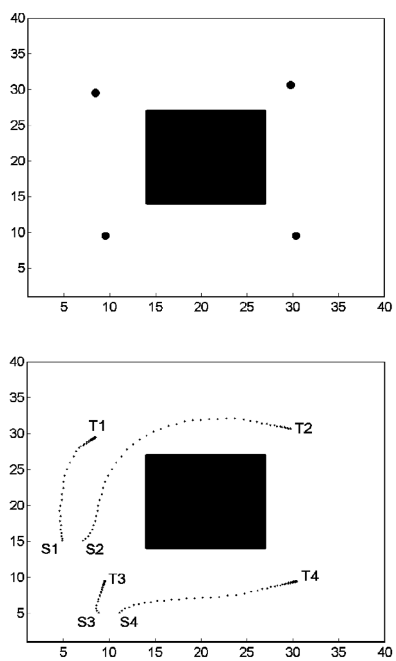

Figure-11: Separation mode in the presence of clutter: a. final constellation, b. trajectories.

VI. Conclusions

This paper demonstrates another useful feature of harmonic potential field-based planners, that is: the social nature of such planners. This feature allows an agent steered by such a method to share, in a conflict-free manner, the same space with other agents using the same planner. Constructing a multi-agent controller in this manner has many advantages. While the system can operate in an asynchronous, decentralized mode, it can also operate in a centralized, synchronous mode that has a computational effort linear in the number of agents being controlled. The controller does exhibit an excellent ability to self-organize as well as a noncommittal planning action. The use of HPF planners enables the overall controller to enforce a variety of constraints on the collective in a provably-correct manner.

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Appendix

A. *Definition*: Let V(X) be a smooth (at least twice differentiable), scalar function ($V(X): R^N \to R$). A point Xo is called a critical point of V if the gradient vanishes at that point ($\nabla V(Xo)=0$); otherwise, Xo is regular. A critical point is Morse, if its Hessian matrix (H(Xo)) is nonsingular. V(X) is Morse if all its critical points are Morse [34].

B. *Proposition*: If V(X) is a harmonic function defined in an N-dimensional space (R^N) on an open set Ω, then the Hessian matrix at every critical point of V is nonsingular, i.e. V is Morse.

Proof: There are two properties of harmonic functions that are used in the proof:

1- a harmonic function (V(X)) defined on an open set Ω contains no maxima or minima, local or global in Ω. An extrema of V(X) can only occur at the boundary of Ω,

2- if V(X) is constant in any open subset of Ω, then it is constant for all Ω.

Other properties of harmonic functions may be found in [35].

Let Xo be a critical point of V(X) inside Ω. Since no maxima or minima of V exist inside Ω, Xo has to be a saddle point. Let V(X) be represented in the neighborhood of Xo using a second order Taylor series expansion:

$$V(X) = V(Xo) + \nabla V(Xo)^T (X - Xo) + \frac{1}{2}(X - Xo)^T H(Xo)(X - Xo) \qquad |X-X0|<<1. \tag{12}$$

Since Xo is a critical point of V, we have:

$$V' = V(X) - V(Xo) = \frac{1}{2}(X - Xo)^T H(Xo)(X - Xo) \qquad |X-X0|<<1. \tag{13}$$

Notice that adding or subtracting a constant from a harmonic function yields another harmonic function , i.e. V` is also harmonic. Using eigenvalue decomposition:

$$V' = \frac{1}{2}(X - Xo)^T U^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} U(X - Xo) \tag{14}$$

$$= \frac{1}{2}\xi^T \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \lambda_N \end{bmatrix} \xi \quad = \quad \frac{1}{2}\sum_{i=1}^{N} \lambda_i \xi_i^2$$

where U is an orthonormal matrix of eigenvectors, λ's are the eigenvalues of H(Xo), and $\xi=[\xi_1\ \xi_2\ ..\xi_N]^T = U(X-Xo)$. Since V` is harmonic, it cannot be zero on any open subset Ω; otherwise, it will be zero for all Ω, which is not the case. This can only be true if and only if all the λ_i's are nonzero. In other words, the Hessian of V at a critical point Xo is nonsingular. This makes the harmonic function V also a Morse function.