

Name: Kay

Student Number:

Question-1 (2 marks): Write the transfer functions of the systems below. If it is not possible to do that, explain why, a, b, and c are constants:

1-  $\ddot{x} + a\dot{x} + c \cdot x = u^2$

2-  $\ddot{x} = u$

①  $\ddot{x} + a\dot{x} + c \cdot x = u^2$  is a nonlinear system ( $u^2$ ). Nonlinear systems do not have transfer functions.

②  $\ddot{x} = u$  is a linear system, it has a transfer function.

$\mathcal{L}[\ddot{x}] = \mathcal{L}[u] \Rightarrow S^2 X(s) = U(s) \therefore H(s) = \frac{X(s)}{U(s)} = \frac{1}{S^2}$

Question-2 (2 marks): write down the answers of the following expressions:

Expression

Answer

1-  $f(t) \cdot \delta(t) = \dots \dots \dots f(t) \rightarrow$  Dirac Property

2-  $f(t) * \delta(t - t_0) = \dots \dots \dots f(t - t_0)$

Where \* denotes convolution and · denotes multiplication.

Question-3 (2 marks): write down the partial fraction expansion of the following rational function:

function

expansion

$\frac{S}{(S+1)^2} = \frac{A}{(S+1)^2} + \frac{B}{S+1} \Rightarrow A + B(S+1)$

$S = A + B(S+1) \quad S = -1 \quad \therefore A = -1$

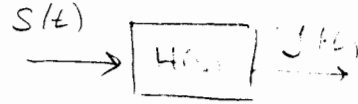
differentiate the above & substitute  $S = -1$

$1 = 0 + B(0+1) \Rightarrow B = 1$

$\therefore \frac{S}{(S+1)^2} = \frac{-1}{(S+1)^2} + \frac{1}{S+1}$

Question-4 (2 marks): A system has the transfer function  $H(S)=1/S$ . If the signal:  $S(t)=\sin(4\pi t)+\cos(6\pi t)$  is used as an input, write down the output signal.

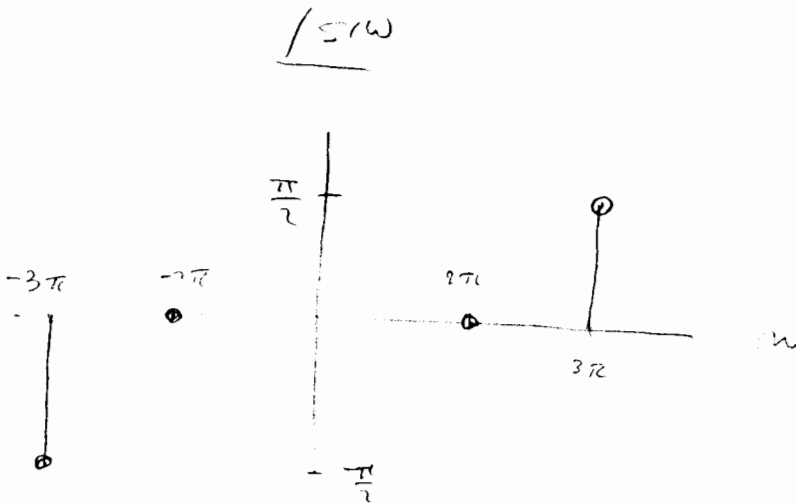
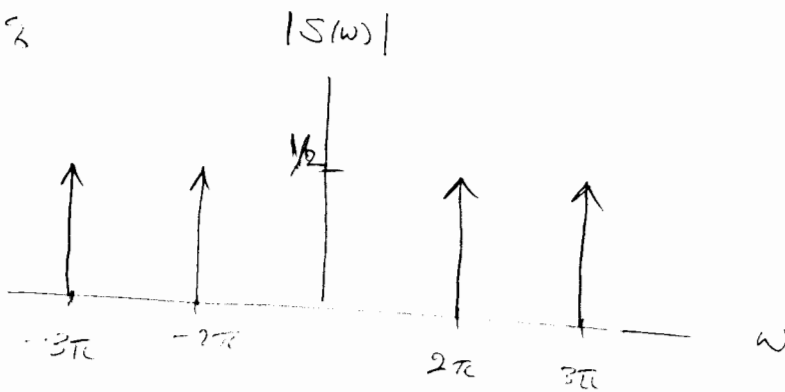
$$H(S) = \frac{1}{S} \quad \text{or} \quad H(\omega) = H(S)|_{S=j\omega} = \frac{1}{j\omega} = \frac{1}{\omega} \angle -90^\circ$$



$$\begin{aligned} \ddot{y}(t) &= \frac{1}{2\pi} \cdot \sin(4\pi t - 90^\circ) + \frac{1}{3\pi} \cos(6\pi t - 90^\circ) \\ &= -0.16 \cdot \cos(4\pi t) + 0.106 \cdot \sin(6\pi t) \end{aligned}$$

Question-5 (2 marks): Draw the spectrum of the signal:  $S(t)=\cos(4\pi t)+\sin(6\pi t)$

Spectrum 2



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Question-1 (2 marks): Write the transfer functions of the systems below. If it is not possible to do that, explain why, a, b, and c are constants:

1-  $x = au + c$

2-  $\ddot{x} + a\dot{x} + c \cdot x = u$

1)  $x = au + c$  is not a linear system, therefore it does not have a transfer function.

2)  $\ddot{x} + a\dot{x} + cx = u$  is a linear system.

$\mathcal{L}[\ddot{x} + a\dot{x} + cx] = \mathcal{L}[u] \Rightarrow s^2 X(s) + a s X(s) + c X(s) = U(s)$

$\therefore \frac{X(s)}{U(s)} = \frac{1}{s^2 + as + c}$

Question-2 (2 marks): write down the answers of the following expressions:

<u>Expression</u>	<u>Answer</u>
1- $f(t) * \delta(t) =$ .....	$f(t)$
2- $f(t) \cdot \delta(t - t_0) =$ .....	$f(t_0)$ $\rightarrow$ sifting property

Where \* denotes convolution and  $\cdot$  denotes multiplication.

Question-3 (2 marks): write down the partial fraction expansion of the following rational function:

<u>function</u>	<u>expansion</u>
$\frac{s}{(s-1)^2}$	$= \frac{A}{(s-1)^2} + \frac{B}{s-1} = \frac{A + B(s-1)}{(s-1)^2}$

$\therefore \frac{s}{(s-1)^2} = \frac{A}{(s-1)^2} + \frac{B}{s-1}$   $\therefore$   $s = +1$   $A = 1$

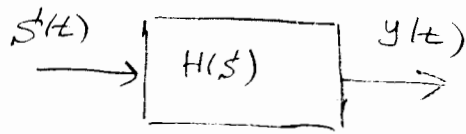
differentiate the above:

$1 \neq 0 + B(1-0) \therefore B = 1$

$\therefore \frac{s}{(s-1)^2} = \frac{1}{(s-1)^2} + \frac{1}{s-1}$

Question-4 (2 marks): A system has the transfer function  $H(S)=1/S$ . If the signal:  $S(t)=\cos(4\pi t)+\sin(6\pi t)$  is used as an input, write down the output signal.

$$H(s) = \frac{1}{s} \quad \therefore \quad H(\omega) = H(s) \Big|_{s=j\omega} = \frac{1}{j\omega}$$



$$\begin{aligned} \therefore y(t) &= \frac{1}{2\pi} \cos(4\pi t - 90) + \frac{1}{3\pi} \sin(6\pi t - 90) \\ &= -16 \sin(4\pi t) - .106 \cos(6\pi t). \end{aligned}$$

Question-5 (2 marks): Draw the spectrum of the signal:  $S(t)=\sin(4\pi t)+\cos(6\pi t)$

spectrum  $\omega$

