2018 Annual American Control Conference (ACC)
June 27–29, 2018. Wisconsin Center, Milwaukee, USA

A Novel and Effective Procedure for an Undergraduate First Control Laboratory Education in State Space-based Motor Identification.

Ahmad A. Masoud (a), Mohammad Abu-Ali (b), Ali Al-Shaikhi (c)

***Abstract*— State space systems and experimental system identification are essential components of control education. This paper suggests an undergraduate physical experiment for directly identifying the state space model of a DC motor in the laboratory. The experiment is designed using standard undergraduate control laboratory equipment. It does not require advanced knowledge in control and it does not place simplifying assumptions on the motor's model. It is easy to understand and has a good informative component that gracefully introduces an undergraduate student to advanced mathematical tools with direct tangible laboratory outcome.**

Keywords: Motor identification, state space, control laboratory, undergraduate education

I. INTRODUCTION

DC motors are important in both academia and industry [1]. They are commonly adopted as the servo-process of choice around which a first laboratory in control is constructed. Investigating the control of these processes is usually preceded by an experiment to determine the transfer function of the motor. It is the norm to identify a first order velocity transfer function from its step response. The argument used is that the electrical time constant of the motor is negligible compared to its mechanical time constant. This assumption is restrictive and may not support all modes a DC machine can operate in. It only applies when a DC motor is in a field control mode. The assumption does not support armature control mode commonly used in laboratory servo-trainers. Moreover, it may not have a positive impact on the subsequent experiments in the laboratory curriculum. For example, a core experiment in a first control laboratory is the effect of the controller's parameters (position and (or) velocity feedback) on phase transition of the motor's response (stable, unstable, over-damped, critically-damped and under-damped). Assuming a first order velocity transfer function means that the position transfer function is second order. Stable second order systems are unconditionally stable for any negative feedback. However, in a physical position control experiment, the motor may exhibit high frequency sustained oscillations even when the feedback is negative. In some cases, the cause of such oscillations is instability with saturation preventing the magnitude from becoming unbounded. Moreover, a first order velocity transfer function of a motor is detrimental to the understanding of the concept of relative stability. At the undergraduate level, this concept is tied to the damping coefficient ζ of a second order system [2, p.421]. Using a second order system as a prototype behavior, the students are usually placed under the impression that the servo must exhibit high oscillations (i.e. ζ becomes very low) prior to becoming unstable. Third order systems [3] can transit from a stable over-damped phase to an unstable phase. This may be demonstrated by the transfer function in (1) with a parameter α. Reducing α from 0.1 to 0.02 changes the response from over-damped to that of an unstable system (Figure-1).

$$H(S,\alpha)=\frac{.03}{S^3+\alpha\cdot S^2+S+.03} \tag{1}$$

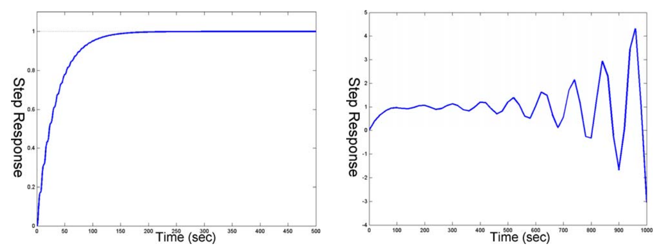

Figure-1: A third order system changing phases

Associating laboratory experience with theoretical lectures is a challenging task especially at the early stages of control education. Developing such an understanding around an isolated view of a first order motor velocity transfer function may not be the best way to approach such a fundamental step in control education. The value of the response of a second order prototype transfer function as a tool of understanding system behavior in the laboratory is expected to significantly improve if the second order response is viewed within the confines of a third order one [4]. This requires the identification of a second-order velocity transfer function of the motor. The procedure taught in a first control laboratory for identifying a velocity transfer function from the motor's step response is inherently limited to a first order system.

Advanced techniques are being examined to obtain a better model of a motor [5-9]. These techniques use tools that are not suitable for a first laboratory in control. Direct identification of the state space model of the motor [10,11] seems to have considerable advantages over the traditional approach of first identifying the transfer function [13] then constructing the state space model using realizations. In [12] Basilio and Moreira proposed an experiment for identifying

*Resrach is supported by King Fahad University of Petroleum & Minerals. Authors are with the electrical engineering department, King Fahad University of Petroleum & Minerals, Dhahran, 31261, Saudi Arabia (e-mail: masoud@kfupm.edu.sa (a), ee.m.abuali@gmail.com (b), shaikhi@kfupm.edu.sa (c))

the state space model of a motor-generator set. The experiment, however, is meant for a second level control laboratory. It assumes that the motor's velocity transfer function is first order and uses relatively involved mathematics for an elementary level in control. Moreover, it does not allow the use of a motor's step response in determining the motor's transfer function.

In this paper, we propose an undergraduate state space-based DC motor identification experiment that suits a first laboratory in control. The experiment uses only the velocity step response to compute a second-order or a third-order position transfer function of a motor. Some of the advantages of the suggested procedure are:

1- It does not place simplifying assumptions on the model of the motor.
2- It has good resistance to noise and can process physical signals.
3- It can produce models with good fit of the experimental data.
4- It introduces undergraduate students to relatively advanced techniques in mathematics and signal processing while maintaining a level of complexity that they can handle.
5- The experiment can be easily adapted for use in other laboratories (e.g. electrical circuits).
6- The experiment requires only standard laboratory equipment commonly used in any undergraduate control lab (Servo trainer 33-110 from feedback inc. (Figure-2), Tektronix TDS 2012C , two-channel digital storage oscilloscope + PC interface software, standard laboratory PC with Matlab and MS Excel installed).

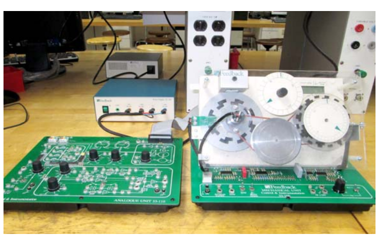
Figure-2: Feedback inc. servo-trainer 33-110

The experiment supports the following objectives:

1- Make experimental state space modeling accessible at the undergraduate level.
2- Introduce undergraduate students to advanced, control-related mathematical tools.
3- Strengthening the relation between basic theoretical concepts in control and experimental observations.
4- Introduce a useful, relatively accurate and easy to use experimental modeling tool whose usefulness extends beyond a control laboratory.

The experiment was successfully conducted by students of the EE380 (control systems-I) in the EE department at KFUPM during one laboratory session.

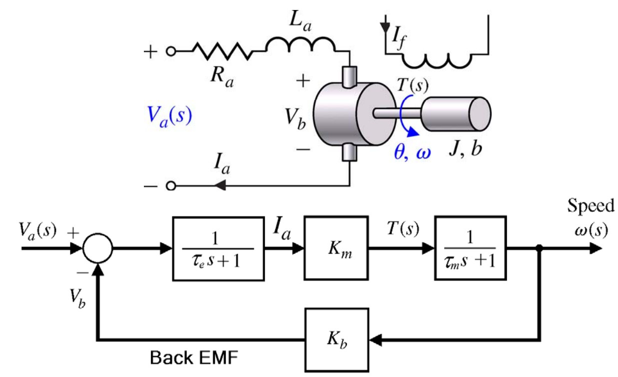

Figure-3: DC motor in armature control mode

II. THEORITICAL BACKGROUND

This section provides a background of the theoretical components the students need to perform the experiment.

A. DC Motor – Armature control

The motor used in the 33-110 servo-trainer is a permanent magnet DC motor. Since the field is constant, the motor is in an armature control mode. The equivalent circuit and block diagram of the motor are shown in Figure-3. The velocity transfer function (2) of the motor is:

$$\mathrm{Hv(S)} = \frac{\mathrm{K_m}}{\tau_e \cdot \tau_m S^2 + (\tau_e + \tau_m)S + (1 + K_b \cdot K_m)} \quad (2)$$

where τ_e, τ_m are the electrical and mechanical time constants of the motor respectively, K_m and K_b are constants that relate to the manner in which the motor functions. As can be seen, the coefficient of the second order dynamic term may not be negligible even if the electrical time constant is small relative to the mechanical time constant. Moreover, the motor and the servo-amplifier are usually constructed as one block. This makes the dynamics of the servo-amplifier an integral part of the dynamics of the motor.

B. Suggested state space identification procedure

While the state space approach is a relatively advanced concept to a first laboratory in control, all what a student needs to know is that it is an alternative to using transfer functions for modeling systems and it has the form (3,4):

$$\dot{X} = A \cdot X + B \cdot Va \quad (3)$$
$$Y = C \cdot X + D \cdot Va \quad (4)$$

where $X = [\theta \quad \dot{\theta} \quad Ia]^T$, $C = [0 \quad 1 \quad 0]$ and $D = [0]$, θ is the position of the motor, $\dot{\theta}$ is its velocity, Ia is the motor armature current and Va is the armature voltage used as the input to control the motor. Va is a constant for a step input. The 3x3 matrix A and 3x1 vector B need to be computed in order to identify the transfer function of the motor. The students may be told that the transfer function is obtained from the formula:

$$H(S) = C(S \cdot I - A)^{-1}B + D \quad (5)$$

which can simply be performed by using the matlab command: ss2tf(A,B,C,D).

If the values of the quantities $\theta(t), \dot{\theta}(t), \ddot{\theta}(t), \mathrm{Ia(t)}, \dot{\mathrm{I}}\mathrm{a(t)}$ are known, one may rewrite equation 3 in the form:

$$\dot{\mathrm{X}}(\mathrm{t}) = \begin{bmatrix} \mathrm{X^T(t)} & 0 & 0 & \mathrm{Va} & 0 & 0 \\ 0 & \mathrm{X^T(t)} & 0 & 0 & \mathrm{Va} & 0 \\ 0 & 0 & \mathrm{X^T(t)} & 0 & 0 & \mathrm{Va} \end{bmatrix} \cdot \Lambda \quad (6)$$

or

$$\mathrm{Z(t)} = \mathrm{Q(t)} \cdot \Lambda \quad (7)$$

where $\Lambda = \begin{bmatrix} \mathrm{a_1} & \mathrm{a_2} & \mathrm{a_3} & \mathrm{B^T} \end{bmatrix}^\mathrm{T}$ and the a's are the rows of the matrix A

$$\mathrm{A} = \begin{bmatrix} \mathrm{a_1} \\ \mathrm{a_2} \\ \mathrm{a_3} \end{bmatrix}. \quad (8)$$

Assuming there exist L independent measurements of the above quantities (L>12) at different instants in time $\{t_1,..t_L\}$, one may construct the expanded system

$$\begin{bmatrix} \mathrm{Z(t_1)} \\ : \\ Z(\mathrm{t_L}) \end{bmatrix} = \begin{bmatrix} \mathrm{Q(t_1)} \\ : \\ \mathrm{Q(t_L)} \end{bmatrix} \cdot \Lambda \quad (9)$$

$$\Lambda = \begin{bmatrix} \mathrm{Q(t_1)} \\ : \\ \mathrm{Q(t_L)} \end{bmatrix}^+ \cdot \begin{bmatrix} \mathrm{Z(t_1)} \\ : \\ Z(\mathrm{t_L}) \end{bmatrix} \quad (10)$$

where the superscript + indicate the Moore-Penrose pseudo inverse. This operation is realized using the matlab command pinv(*). Equation (7) is solved assuming that all coefficients of the A and B matrices are unknown. This is usually not needed since many of the coefficients can be *a priori* determined from the physical nature of the system. It is important when computing equation (9) that the initial conditions are set to zero (i.e. $\theta(0)=0, \dot{\theta}(0)=0 \,\&\, \mathrm{Ia}(0)=0.$).

C. Differentiation

The 33-110 servo-trainer allows direct measurements of $\theta(t), \dot{\theta}(t), \& \,\mathrm{Ia(t)}$. The angular acceleration and the derivative of the armature current have to be computed using numerical differentiation. Robust differentiation of natural signals [14] is not easy to teach at the undergraduate level. Also directly using Eulers discritization (11) to compute the derivatives will not produce satisfactory results

$$\ddot{\theta}(t) \approx \frac{\dot{\theta}(t) - \dot{\theta}(t-\Delta T)}{\Delta T}. \quad (11)$$

There are reasonably accurate differentiation formulae that are usable by an undergraduate student. For example, the formulae in (12,13) do produce good results [17] with (13) being more accurate than (12).

$$\ddot{\theta}(t) \approx \frac{\dot{\theta}(t+\Delta T) - \dot{\theta}(t-\Delta T)}{2 \cdot \Delta T} \quad (12)$$

$$\ddot{\theta}(t) \approx \frac{-\dot{\theta}(t+2\cdot\Delta T) + 8\cdot\dot{\theta}(t+\Delta T) - 8\cdot\dot{\theta}(t-\Delta T) + \dot{\theta}(t-2\cdot\Delta T)}{12\cdot\Delta T} \quad (13)$$

D. Noise reduction

The data acquired is noisy; the source of noise is mainly the power electronics and the signal encoders. Advanced noise removal techniques [15] may not be suitable for an undergraduate experiment. Performing noise removal is subject to stringent requirements at such a basic level. First, the filter has to be simple and easy for the students to understand and work with. Also, it must be supported by matlab. Most importantly, it must not significantly distort the measured signals. Simple convolutional FIR and IIR filters cannot be used since they distort the registration of motor's dynamics in the signal and lead to significant errors.

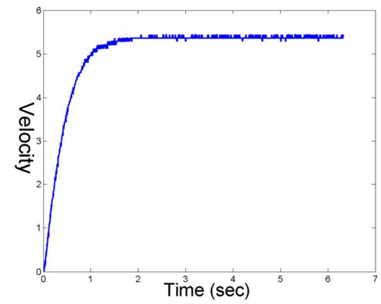

Figure-4: velocity step response of the motor

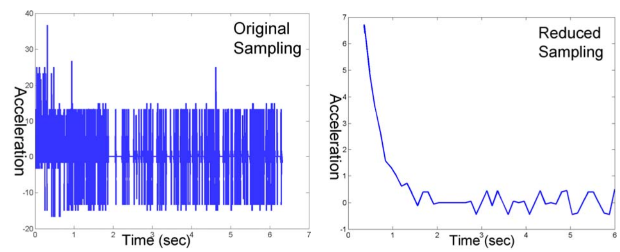

Figure-5: Mitigating noise effect on differentiation by sample rate reduction

Here, a sample reduction approach is used to alleviate the effect of noise without disturbing the informational content of the measured signals. Sampling a signal every ΔT seconds is equivalent to applying a Nyquist lowpass filter in the frequency domain with a bandwidth W=1/ΔT. Sample reduction increases ΔT and reduces W. This causes the removal of high frequency components without affecting the informational content of the low frequency ones. A typical signal captured at a rate of 4 ms contains about 2000 samples. For a second order model of the position transfer function, only 6 samples are needed and for the third order model only 12 samples are enough to solve equation (9). This means that significant reduction in the signal bandwidth of at least 100 times is possible without affecting its usability in identifying the state space model. Figure-4 shows the speed signal from the step response. It is sampled at 4 ms and contains approximately 1500 samples.

Figure-5 shows the result of direct differentiation to obtain the acceleration signal using equation (13) and differentiation after a 1:30 sample reduction (new ΔT=120 ms). The significant reduction in noise is obvious. The power circuit of the 33-110 servo-trainer seems to use an H-bridge for velocity control along with pulse width modulation {PWM}. As a result, the armature current signal contains a high level of noise (Figure-9) and the amplifier-motor model significantly deviate from a linear one. Despite the above, the sample reduction method is still able to produce acceptable results.

III. DATA CONDITIONING

This section describes the conditioning of the Excel sheets data prior to importing it to matlab .

A. Synchronization: A measurement excel sheet contains the time trace, step input and the quantity that is measured. Determine the entry in the excel sheet at which the step input changes value and delete all the samples prior to where this change occurs. Also, make sure that the data records are truncated so that their length is equal to the length of the smallest record.

B. Position measurements unwrapping: Due to the position encoder, position measurements experience a sudden jump from +10 volts to -10 volts and vice versa depending on the direction of motion. If a sudden change from positive to negative is encountered and the value of the measurement is close to 10 volts, the difference in value is added to the subsequent samples. After the unwrapping is finished, shift the whole data record by a constant value so that the value of the first sample is zero.

C. Data smoothing & differentiation: The length of the data record should be selected so that the system has reached steady state. The sampling rate is reduced prior to differentiation. A rate of around 100 ms is sufficient for smoothing the data and retaining enough samples to determine the state space model coefficients. The reduced sample record is then differentiated using equation (13) leaving the first and the last two samples.

D. From state space to Transfer function: Equation (14) shows a typical state space result. The first row of the A matrix should be [0 1] and the first element of the B matrix should be [0]. Due to numerical issues, the computed values experience a little deviation. The velocity transfer function obtained by the matlab command ss2tf(A,B,C,D) is shown in equation (15). The position transfer function obtained by selecting C=[1 0] is shown in equation (16).

$$A = \begin{bmatrix} .0042 & 1.0325 \\ -.0327 & -2.3145 \end{bmatrix}, \quad B = \begin{bmatrix} -.0371 \\ 2.1751 \end{bmatrix} \qquad (14)$$
$$C = [0 \quad 1], \quad D = [0]$$

$$Hv(S) = \frac{2.1751 \cdot S + .0103}{S^2 + 2.3187 \cdot S + .0435} = \frac{2.1751 \cdot (S + .0048)}{(S + .0189) \cdot (S + 2.2998)}. \qquad (15)$$

$$Hp(S) = \frac{-0.0371 \cdot S + 2.1599}{S^2 + 2.3187 \cdot S + .0435}. \qquad (16)$$

The small real pole and zero are caused by numerical issues. Therefore, one may cancel them to obtain the approximate velocity and position transfer functions in equations (17) and (18) respectively

$$\tilde{H}v(S) = \frac{2.1751}{S + 2.2998} \qquad (17)$$

$$\tilde{H}p(S) = \frac{2.1751}{S \cdot (S + 2.2998)} \qquad (18)$$

IV. RESULTS

This section presents basic experimental results obtained from the Feedback Servo trainer 33-110. The results demonstrate the stability of the experiment. Emphasis is placed on the reduced state space model with position and velocity as the state vector of the motor. We also demonstrate the applicability of the experiment to the extended model of the motor with position, velocity and armature current taken as the components of the state vector.

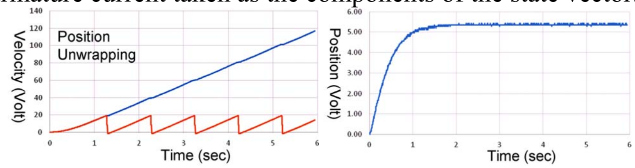

Figure-6: raw position and velocity step response of the motor

A. A typical case:

Here a step input voltage of magnitude 5.92 is applied to the trainer. The value of the step input is selected so that the motor operates in the linear region. The raw position (wrapped & unwrapped) and velocity measurements are shown in figure-6. The final reading from the speed meter is 1824 rpm which corresponds to a 5.36 volt reading from the tacho-generator. A data record of 6 seconds is obtained at a sampling rate of 4 ms. The classical approach for identifying a first order velocity transfer function from the velocity step response yields the transfer function in (19) with time constant τ equal to .4241 seconds and motor constant 0.9054

$$Hv(S) = \frac{0.9054}{0.4241 \cdot S + 1} \qquad (19)$$

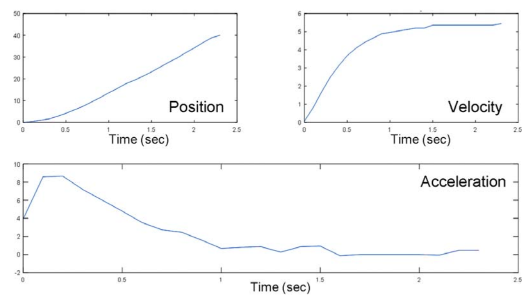

Figure-7: Under-sampled position, velocity and acceleration signals

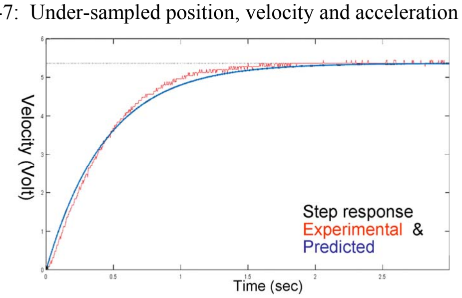

Figure-8: Velocity step response from TF and actual velocity response

Figure-7 shows a sample-reduced data record of 2.4 seconds of the position, speed and acceleration obtained by differentiating the speed using formula (13) after reducing the sampling rate 25 times to 100 ms. The state space system that results from solving equation (9) is shown in equation (20) and the velocity transfer function is shown in (21). The corresponding time constant of the motor is .4428 and the motor coefficient is 2.1685. The results are close to those obtained from the classical approach. The velocity step response from the transfer function reasonably approximates the actual velocity response (Figure-8).

$$A = \begin{bmatrix} 0.0 & 1.00 \\ -.0192 & -2.267 \end{bmatrix}, \quad B = \begin{bmatrix} 0.00 \\ 2.1685 \end{bmatrix} \qquad (20)$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$Hv(S) = \frac{2.1685 \cdot S}{S^2 + 2.267 \cdot S + .0192} \approx \frac{2.1685}{S + 2.2585} \qquad (21)$$

Table-1 shows the effect of the record length and sample reduction ratio on the motor's estimated k and τ.

Table-1: Effect of record length and sample reduction ration on the estimated transfer function, Vin=5.92

ΔT	Record Length		
	1.9 s	2.4 s	2.9 s
80 ms	k = 1.8334 τ = 0.4939	k = 1.9378 τ = 0.4672	k = 1.996 τ = 0.4536
100 ms	k = 1.9816 τ = 0.4569	k = 2.0449 τ = 0.4428	k = 2.0967 τ = 0.4318
120 ms	k = 2.1698 τ = 0.4173	k = 2.2031 τ = 0.4110	k = 2.2338 τ = 0.4053

B. Sensitivity to reference input

The sensitivity of the state space identification procedure to the nonlinearities in the servo-trainer is tested. The results in Table-1 are repeated for a low reference input (Vin=1.24 volt). At this input level the effect of the static and Colomb friction, deadzone nonlinearity and low signal amplifier distortion should be non-negligible. The results of the estimated motor coefficient and time constant are shown in Table-2 for different record lengths and sampling period. The same results are computed for a high reference input (Vin=8.8 volt) where the effect of the saturation nonlinearity and amplifier high signal distortion is significant (Table-3). As can be seen in both cases, the results remain reasonably consistent with those reported in Table-1.

Table-2: Effect of record length and sample reduction ration on the estimated transfer function, Vin=1.24

ΔT	Record Length		
	1.9 s	2.4 s	2.9 s
100 ms	k = 1.9750 τ = 0.5553	k = 2.3073 τ = 0.4753	k = 2.2075 τ = 0.4968

Table-3: Effect of record length and sample reduction ration on the estimated transfer function, Vin=8.8

ΔT	Record Length		
	1.9 s	2.4 s	2.9 s
100 ms	k = 1.8662 τ = 0.4628	k = 1.9350 τ = 0.4463	k = 1.9871 τ = 0.4346

C. Extended state space model

The armature current contains a high amount of noise and introduces a strong element of nonlinearity in the model. This is caused by the power electronic circuit using an H-bridge and employing pulse width modulation in controlling the speed of the motor [16].

The sample reduction procedure can still produce acceptable results. However, it is observed that how well the model fits the experimental results is dependent on choosing the sample reduction ratio that suits the magnitude of the step input. While tuning the sample reduction ratio to get the best fit is difficult to perform, it is strongly felt that it may not be suitable to include at a first control laboratory.

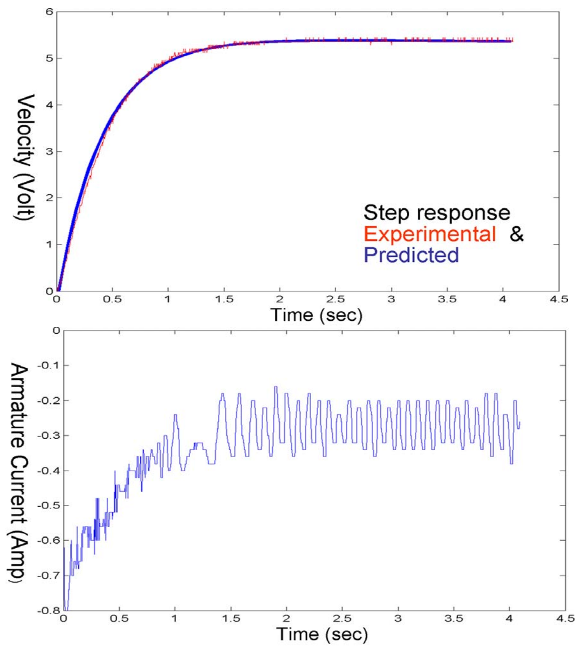

Figure-9: measured velocity step response, the fitted response and the corresponding armature current, Vin=5.92.

The data is reprocessed to estimate the motor's state space model with state $X = [\theta \quad \dot{\theta} \quad Ia]^T$. A record of 4.3 seconds is used at a 4 ms sampling rate. The experimental state space model and the approximated transfer function obtained for Vin=5.94 are shown in equation (22). A sample reduction ratio of 1:30 is used. Figure-9 shows the measured velocity step response, the fitted response from the approximate transfer function along side the armature current. Despite the high level of noise in the current, the approximate velocity transfer function provides an excellent fit of the experimental data.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -.0101 & -2.4731 & .6196 \\ -.0031 & -.0717 & -.2191 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2.3373 \\ .082 \end{bmatrix} \qquad (22)$$

$$C = \{0 \quad 1 \quad 0\}$$

$$Hv(S) \approx \frac{2.337 \cdot S + .5630}{S^2 + 2.692 \cdot S + .597}$$

The extended state experiment was repeated for Vin 1.24 and 8.8. The results obtained are consistent with the reduced state case.

V. EXPERIMENT DELIVERY & STUDENT FEEDBACK

A third year undergraduate student, the second author Mr. M. Abu-Ali, was directly involved in designing and implementing the experiment. A semester prior to the development of the experiment, the student took the EE380 control course and its laboratory.

The experiment is about six pages in length. It contains the traditional components of: objectives, equipment, introduction, experimental procedure, analysis and guidelines to write the report. The introduction provides the theoretical background needed to understand and perform the experiment. To perform the needed procedures, e.g.

phase unwrapping of the position measurements, the students were provided with snippets of the needed MATLAB code. Since the experiment heavily relies on being familiar with the functions of the Tektronix TDS 2012C oscilloscope, handouts describing in a step by step manner how to use the needed oscilloscope functions were generated and distributed to the students.

-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
Strongly disagree		Disagree		Neutral			Agree		Strongly Agree	

Figure-10: scale used in evaluating the questions

The experiment only requires the students to identify experimentally the reduced state space model with position and velocity as the components of the state vector. They were encouraged to try the procedure for the extended state space model or to identify an RLC circuit. After the students performed the experiment and wrote the report, a questionnaire was distributed in order to get their feedback about the experiment. Below are some of the questions the questionnaire contains:

1. The objective of the experiment is clear
2. The introduction is helpful in understanding and performing the experiment
3. The level of experiment is suitable for an undergraduate course in control
4. The equipment in the lab are enough to effectively perform the experiment
5. The experiment is strongly related to the control theory covered in class
6. The experiment is practical and will be helpful in the future
7. The experiment introduced me to useful advanced mathematical tools.

The students were asked to evaluate each question using a numerical scale from -5 to +5. This scale mirrored the five options (Figure-11): strongly agree (SA), agree (A), neutral (N), disagree (D) and strongly disagree (SD). The questionnaire also contained a section for student comments. 18 students responded and the results are shown in table-4. In general, the experiment seems to have been well-received by the students.

VI. CONCLUSIONS

A student-friendly procedure is suggested for experimentally identifying the state space model of a DC motor. The experiment places no restrictions on the motor's model. It is easy to perform using standard laboratory equipment and has a good informative component that gracefully introduces an undergraduate student to advanced mathematical tools with direct tangible laboratory outcome. The suggested experimental procedure may be modified for use in advanced control laboratory and industrial control. It can also be used in laboratories other than control, e.g. advanced circuit laboratories.

ACKNOWLEDGMENT

The authors gratefully acknowledge the assistance of king Fahad University of Petroleum and Minerals.

Table-4: The results from the student evaluation

Question #	SA	A	N	D	SD	mean
1	10	7	1	0	0	3.56
2	5	6	5	2	0	1.94
3	9	5	3	1	0	2.95
4	13	1	2	2	0	3.33
5	6	7	4	1	0	2.39
6	7	5	6	0	0	2.83
7	7	7	3	1	0	2.67
Overall average	46%	30%	19%	5%	0%	2.81

REFERENCES

[1] George W. Younkin, "Industrial Servo Control Systems: Fundamental and Applications", 2nd edition, Marcel Dekker, Inc. 2003, ISBN 0-8247-0836-9

[2] Richard Dorf, Robert Bishop, "Modern Control Systems", 12th edition, Pearson 2011,

[3] Clement, P., "A note on third-order linear systems", Automatic Control, IRE Transactions on, Issue 2, June 1960, pp.151

[4] Monzingo, R, "On approximating the step response of a third-order linear system by a second-order linear system", Automatic Control, IEEE Transactions on (Volume:13 , Issue: 6), 739, Dec 1968

[5] Dorin G. Sendrescu, "DC Motor Identification Based on Distributions Method," Ann. Univ. Craiova, vol. 9, no. 36, pp. 41-49, 2012.

[6] Arif A. AL-qassar, Mazin Z. Othman, "Experimental Determination of Electrical and Mechanical Parameters of DC Motor Using Genetic Elman Neural Network", Journal of Engineering Science and Technology Vol. 3, No. 2 (2008) 190 - 196

[7] Mohammed S. Z. Salah, "Parameters Identification of A Permanent Magnet DC Motor", M.Sc, Electrical engineering, IUG, 2009

[8] S. Udomsuk, K-L. Areerak, K-N. Areerak and A. Srikaew, "Parameters Identification of Separately Excited DC Motor using Adaptive Tabu Search Technique", Advances in Energy Engineering (ICAEE), 2010 International Conference on, Beijing 19-20 June 2010,pp. 48-51.

[9] Wei Wu, "DC Motor Parameter Identification Using Speed Step Responses", Hindawi Publishing Corporation Modeling and Simulation in Engineering, Volume 2012, Article ID 189757, 5 pages

[10] P. M. MAÈ KILAÈ , "State space identification of stable systems", INT. J. CONTROL, 1999, VOL. 72, NO. 3, 193-205

[11] Mats Viberg, "Subspace-based Methods for the Identification of Linear Time-Invariant Systems", Automatica. Vol. 31. No. 12. pp. 1835-1851. 1995

[12] J. C. Basilio and M. V. Moreira, "State–Space Parameter Identification in a Second Control Laboratory", IEEE Transactions on Education, Vol. 47, No. 2, MAY 2004, pp. 204-210

[13] Hugues Garnier, "Direct Continuous-time Approach to System Identification: Overview and Benefits for Practical Applications", European Journal of Control, Vol. 24, 2015, pp. 50-62

[14] Andreas Griewank and Andrea Walther, "Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation", Second Edition, Humboldt-Universität zu Berlin, Berlin, Germany, 2008, ISBN: 978-0-89871-659-7

[15] Stergios Stergiopoulos, "Advanced Signal Processing Handbook: Theory and Implementation for Radar, Sonar, and Medical Imaging Real Time Systems", December 21, 2000 by CRC Press, ISBN 9781420037395

[16] V.K. Verma, C.R. Baird, V.K. Aatre, "Pulse-width-modulated speed control of d.c. motors", Journal of the Franklin Institute Volume 297, Issue 2, February 1974, Pages 89-101, doi:10.1016/0016-0032(74)90057-

[17] Milton Abramowitz, Irene Stegun, "Handbook of Mathematical Functions",United States Depatment of Commerce,1964, Library of congress catalog card number 64-60036