Electric Circuits II

Frequency Selective Circuits (Filters)
Bode Plots

Lecture #40

The material to be covered in this lecture is as follows:

- Introduction to Bode Diagrams
- First order Poles and Zeros
- Straight-Line Amplitude Plots
- Corrected Amplitude Plots
- Straight-Line Phase Angle Plots

After finishing this lecture you should be able to:

- Appreciate the usefulness of Bode Diagrams
- Draw Straight-Line Approximation of the Amplitude Plot of a First Order Pole or Zero
- Draw Straight-Line Approximation of the Phase Angle Plot of a First Order Pole or Zero
- Correct the plots for a First Order Pole and Zero

Introduction to Bode Diagrams

- Bode Diagram or plot is a graphical technique that gives a feel for the frequency response of a circuit
- A Bode Diagram consists of two separate plots:
  1) Shows how amplitude of $H(j\omega)$ varies with frequency,
  2) Shows how the phase angle of $H(j\omega)$ varies with frequency,
- The plots are made on semilog graph paper for greater accuracy in representing the wide range of frequency values.

Real First Order Poles and Zeroes:

Given the following expression: $$H(s)=\frac{K(s+z_1)}{s(s+p_1)} \qquad (40\text{-}1)$$

Replacing $s=j\omega$ becomes $$H(j\omega)=\frac{K(j\omega+z_1)}{j\omega(j\omega+p_1)} \qquad (40\text{-}2)$$

The first step in making Bode Diagram is to put (40-2) in a Standard Form:

$$H(j\omega)=\frac{Kz_1\left(1+j\omega/z_1\right)}{p_1(j\omega)\left(1+j\omega/p_1\right)} \qquad (40\text{-}3)$$

Let $\frac{Kz_1}{p_1} = K_0$ therefore (40-3) becomes $H(j\omega) = \frac{K_0\left(1 + j\,\omega/z_1\right)}{(j\omega)\left(1 + j\,\omega/p_1\right)}$ (40-4)

In polar form:

$$H(j\omega) = \frac{K_0\left|1 + j\,\omega/z_1\right|\angle\psi_1}{|\omega|\angle 90^\circ\left|1 + j\,\omega/p_1\right|\angle\beta_1}$$

$$= \frac{K_0\left|1 + j\,\omega/z_1\right|}{|\omega|\left|1 + j\,\omega/p_1\right|}\angle\left(\psi_1 - 90^\circ - \beta_1\right) \qquad (40\text{-}5)$$

From (40-5) we get: $\left|H(j\omega)\right| = \frac{K_0\left|1 + j\,\omega/z_1\right|}{|\omega|\left|1 + j\,\omega/p_1\right|}$ (40-6) and $\theta(\omega) = \psi_1 - 90^\circ - \beta_1$ (40-7) where:

$\psi_1 = \tan^{-1}\frac{\omega}{z_1}$ (40-8) and $\beta_1 = \tan^{-1}\frac{\omega}{p_1}$ (40-9)

Straight-Line Amplitude Plots

- The amplitude plot involves the multiplication and division of factors associated with the poles and zeros of $H(s)$. We reduce this multiplication and division to addition and subtraction by expressing the amplitude of $H(j\omega)$ in terms of a logarithmic value: the decibel (dB).

$$A_{dB} = 20\log_{10}\left|H(j\omega)\right| \qquad (40\text{-}10)$$

Expressing (40-6) in dB gives: $A_{dB} = 20\log_{10}\dfrac{K_0\left|1+j\omega/z_1\right|}{\omega\left|1+j\omega/p_1\right|}$

$$= 20\log_{10}K_0 + 20\log_{10}\left|1+j\omega/z_1\right| - 20\log_{10}\omega - 20\log_{10}\left|1+j\omega/p_1\right| \qquad (40\text{-}11)$$

- The key to plotting (40-11) is to plot each term separately and then combine the separate plots graphically.
- The plot of $20\log_{10}K_0$ is a horizontal straight line
  - positive if $K_0 > 1$
  - zero $K_0 = 1$
  - negative if $K_0 < 1$
- The plot of $20\log_{10}\left|1+j\omega/z_1\right|$ is approximated by two straight lines

- $20\log_{10}\left|1+j\omega/z_1\right| \rightarrow 0 \quad as \quad \omega \rightarrow 0$ (40-12)
- $20\log_{10}\left|1+j\omega/z_1\right| \rightarrow 20\log_{10}\left(\omega/z_1\right) \quad as \quad \omega \rightarrow \infty$ (40-13)

- On a **log frequency scale**, $20\log_{10}\left(\omega/z_1\right)$ is a straight line with a slope of $20dB/decade$. (A decade is a 10-to-1 change in frequency)
- The two straight lines intersect the 0 dB axis at $\omega = z_1$. This value of ω is called the corner frequency.
- The plot of $-20\log_{10}\omega$ is a straight line having a slope of $-20dB/decade$ that intersects the 0 dB axis at $\omega = 1$. Fig. 40-1 shows a straight-line approximation of the amplitude plot of a first order zero.

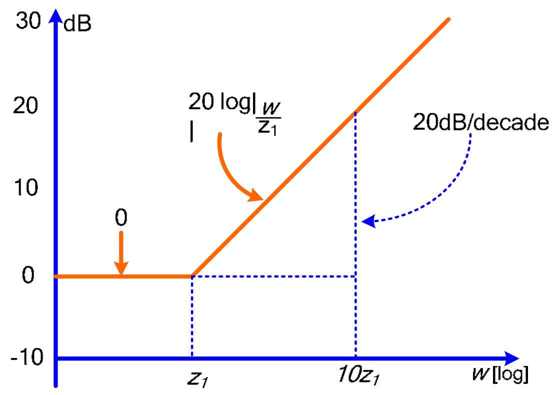

Fig. 40-1 A Straight-line Approximation of the amplitude plot of a first-order zero

- The plot of $-20\log_{10}\left|1+j\omega/p_1\right|$ is approximated by two straight lines
  - $-20\log_{10}\left|1+j\omega/p_1\right| \rightarrow 0 \quad as \quad \omega \rightarrow 0$ (40-14)
  - $-20\log_{10}\left|1+j\omega/p_1\right| \rightarrow -20\log_{10}\left(\omega/p_1\right) \quad as \quad \omega \rightarrow \infty$ (40-15)
- On a log frequency scale, $-20\log_{10}\left(\omega/p_1\right)$ is a straight line with a slope of $-20\,dB/decade$
- The two straight lines intersect on the 0 dB axis at $\omega = p_1$, (the corner frequency). Fig. 40-2 shows a straight-line approximation of the amplitude plot of a first order pole.

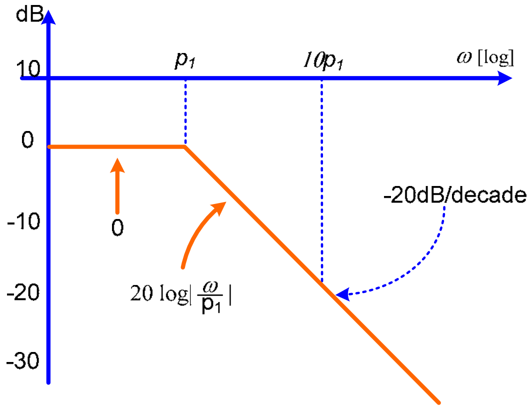

Fig. 40-2 A Straight-line Approximation of the amplitude plot of a first-order pole

Example 40-1

Plot the amplitude straight-line approximation of the TF for Equation (40-6). Take $z_1 = 0.1 rad/s$, $K_0 = \sqrt{10}$, $p_1 = 5 rad/s$

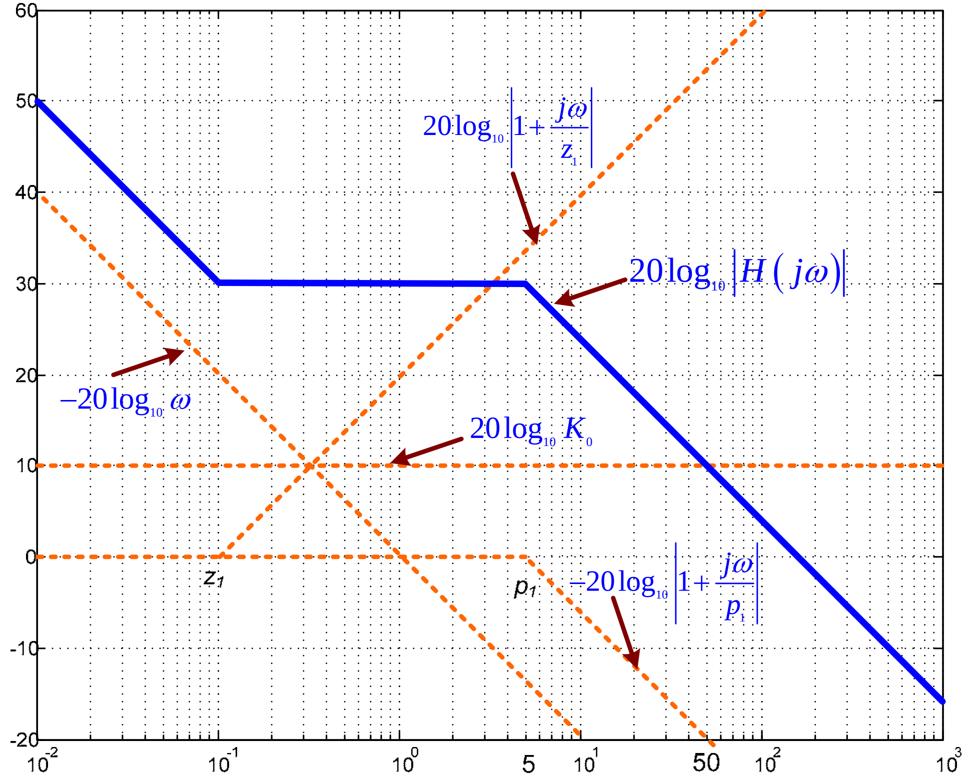

Fig. 40-3 A Straight-line Approximation of the amplitude plot for Equation (40-6)

Example 40-2

For the circuit in Fig. 40-4, $L=100mH$, $C=10mF$, $R=11\Omega$

a) Compute the transfer function, H(s)
b) Construct a straight-line approximation of the Bode amplitude plot.
c) Calculate $20\log_{10}\left|H(j\omega)\right|$ at $\omega=50\,{}^{rad}/_{s}$ rad/s and $\omega=1000\,{}^{rad}/_{s}$
d) Plot the values computed in (c) on the straight-line graph
e) Suppose $V_i(t)=5\cos(500t+15^\circ)V$ then use the Bode Plot you construct to predict amplitude of $v_o(t)$ in the steady-state

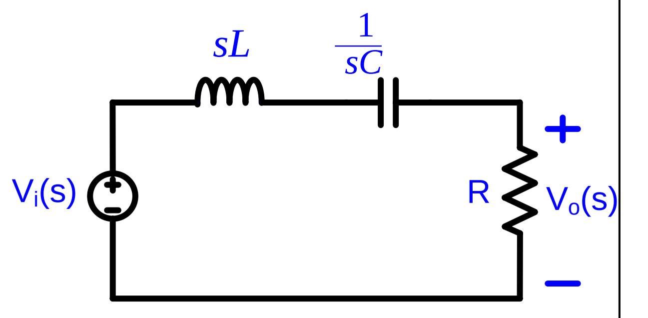

Fig. 40-4 The circuit for Example 40-2

Solution:

a) $V_i(s)=\left(Ls+\frac{1}{sC}+R\right)I(s)=\frac{L}{s}\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)I(s)$

$V_o(s)=RI(s)\Rightarrow I(s)=\frac{V_o(s)}{R}$ thus $V_i(s)=\frac{L}{Rs}\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)V_o(s)$ therefore $H(s)=\dfrac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

b) $H(s)=\dfrac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}=\dfrac{110s}{S^2+110S+1000}=\dfrac{110s}{(s+10)(s+100)}$ replacing $s=j\omega$

$$H(j\omega)=\frac{110}{(10)(100)}\frac{j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}=\frac{0.11j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$$

$$A_{dB}=20\log_{10}\left|H(j\omega)\right|=20\log_{10}0.11+20\log_{10}\left|j\omega\right|-20\log_{10}\left|1+\frac{j\omega}{10}\right|-20\log_{10}\left|1+\frac{j\omega}{100}\right|$$

c) we have $H(j50)=\dfrac{0.11(j50)}{(1+j5)(1+j0.5)}=0.9648\angle-15.25^\circ$,

$20\log_{10}\left|H(j50)\right|=20\log_{10}0.9648=-0.3116dB$,

$H(j1000)=\dfrac{0.11(j1000)}{(1+j100)(1+j10)}=0.1094\angle-83.72^\circ$,

$20\log_{10}0.1094=-19.22dB$.

d) See Fig. 40-5

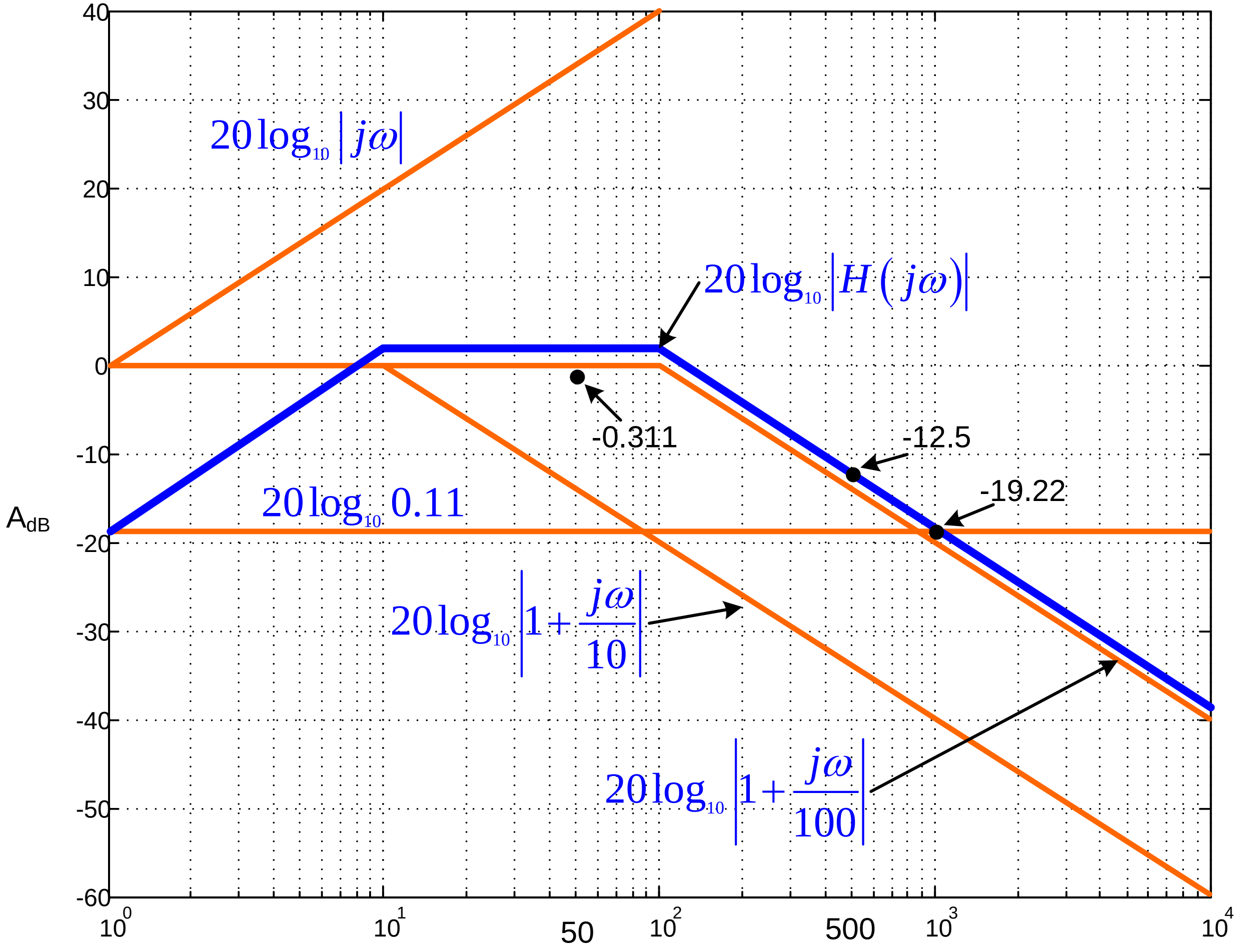

Fig. 40-5 A Straight-line Approximation of the amplitude plot
for the Circuit in Fig. 40-4

e) we can see from the Bode plot in Fig. 40-5, the value of $A_{dB} \simeq -12.5dB$ at $\omega = 500 \, rad/s$ therefore $|A| = 10^{(-12.5/20)} = 0.24$ and $V_{mo} = |A| V_{mi} = 0.24 \times 5 = 1.19V$

We can compute the actual value of $|H(j\omega)|$ by substituting $\omega = 500 \, rad/s$ into the equation for $|H(j\omega)|$ therefore $H(j500) = \dfrac{0.11(j500)}{(1+j50)(1+j5)} = 0.22\angle -77.54^\circ$ and $V_{mo} = |A| V_{mi} = 0.22 \times 5 = 1.1V$

More Accurate Amplitude Plots

We can make the straight-line plots for first order poles and zeros more accurate by correcting the amplitude values at the corner frequency.

The actual value at the corner frequency in decibels is: $A_{dB_C} = \pm 20\log_{10}|1+j1| = \pm 20\log_{10} 2 \simeq \pm 3dB$

The actual value at half the corner frequency is: $A_{dB_{c/2}} = \pm 20\log_{10}\left|1+j\frac{1}{2}\right| = \pm 20\log_{10}\sqrt{5/4} \simeq \pm 1dB$

The actual value at twice the corner frequency is: $A_{dB_{2c}} = \pm 20\log_{10}|1+j2| = \pm 20\log_{10}\sqrt{5} \simeq \pm 7dB$

A 2-to-1 change in frequency is called an octave. A slope of 20 dB/decade is equivalent to 6dB/octave.

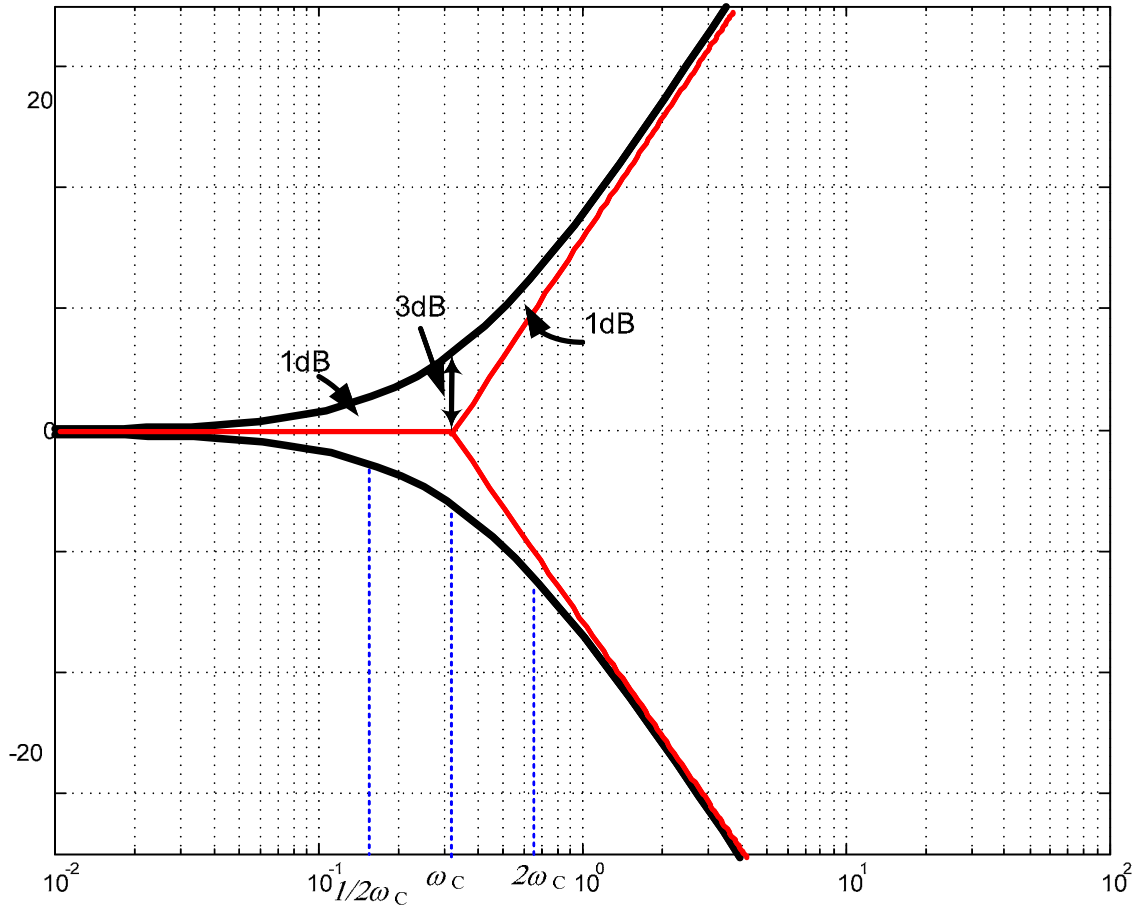

Fig. 40-6 Corrected amplitude plots for a first order zero and pole

Straight-Line Phase Angle Plots

We can also make phase angle plots by using straight-line approximation

- Phase angle for $K_0 > 0$ is 0 degree
- Phase angle for $K_0 < 0$ is 180 degree

- Phase angle associated with a first order zero or a pole at origin is a constant $\pm 90^\circ$ degree
- For a first order pole or zero not at origin, the straight line approximation are as follows;
  - For frequencies $\omega \le \frac{\omega_C}{10}$ $\quad \rightarrow \phi = \pm \tan^{-1} 0.1 = 0^\circ$
  - For frequencies $\omega = \omega_C$ $\quad \rightarrow \phi = \pm \tan^{-1} 1 = \pm 45^\circ$
  - For frequencies $\omega \ge 10\omega_C$ $\quad \rightarrow \phi = \pm \tan^{-1} 10 = \pm 90^\circ$

  (The signs are + for zeros and – for poles in all the above)

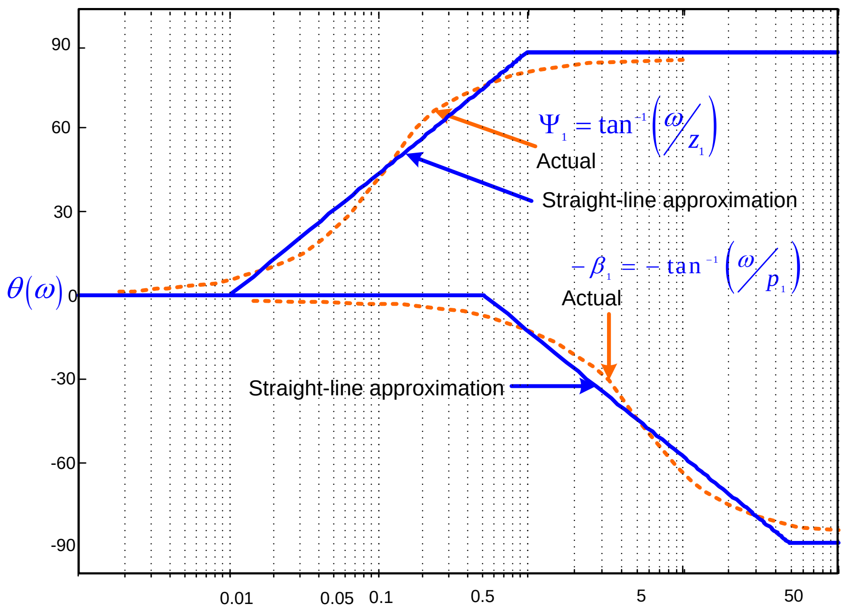

Fig. 40-7 a straight-line approximation of the Phase angle plot for equ. (40-1)

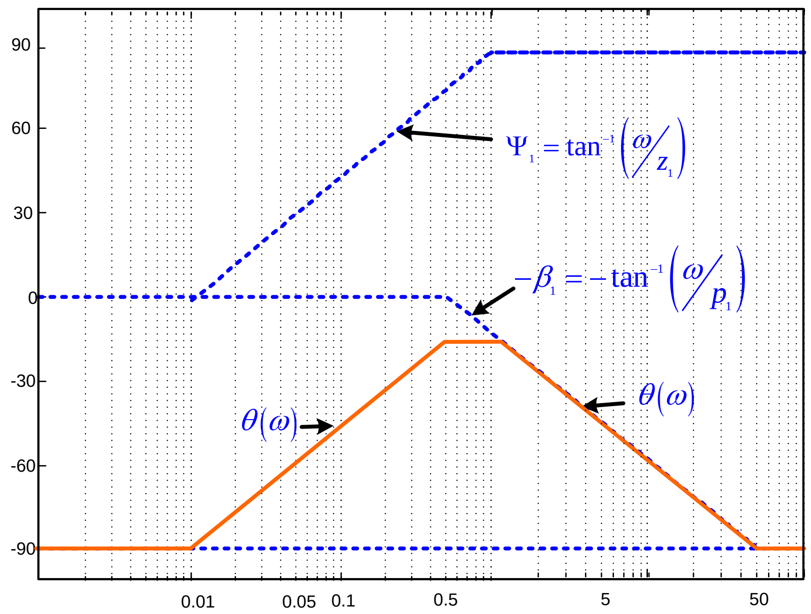

Fig. 40-8 a straight-line approximation of the Phase angle plot for equ. (40-1)

- Fig. 40-7 depicts the straight-line approximation for a first order zero and pole. The dashed curves show the exact variation of the phase angle as ω varies.

- The maximum deviation between the straight-line and the actual plot is approximately 6°. Fig. 40-8 depicts the straight-line approximation of the phase angle equ. (40-7) of the TF of equ. (40-1) with

$z_1 = 0.1 rad/s$

, $p_1 = 5 rad/s$

$p_1 = 5 rad/s$

Self Test 40:

The numerical expression for a transfer function is $H(s)=\frac{(10s+3)}{15s(s+2)}$ Make a straight-line amplitude and phase angle plots for H(s).

Answer:

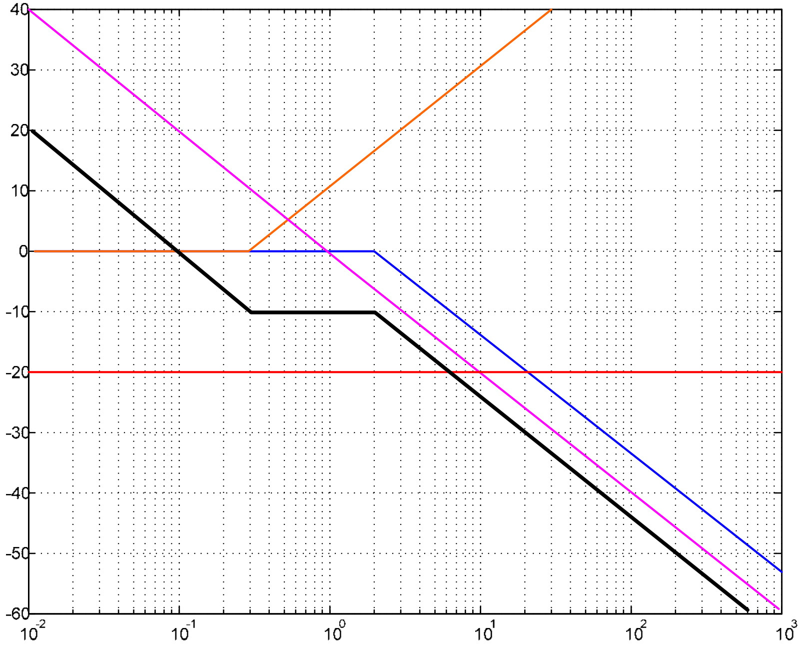

Fig. 40-9 A Straight-line Approximation of the amplitude plot for self test 40

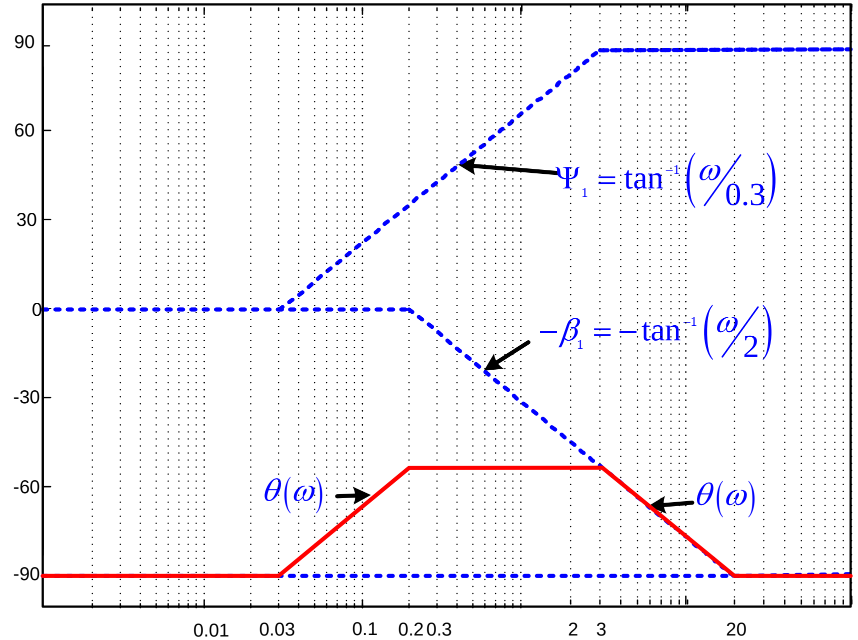

Fig. 40-10 a straight-line approximation of the Phase angle plot for self test 40