Electric Circuits II

Frequency Selective Circuits (Filters)
Bandstop Filters

Lecture #39

The material to be covered in this lecture is as follows:

- Introduction to Bandstop filters
- Center Frequency, Bandwidth and Quality factor
- Bandstop Series RLC Circuit Analysis
- Relationship between different parameters

After finishing this lecture you should be able to:

- Understand the behavior of Bandstop filters
- Determine the different Bandstop filter parameters
- Relate each parameter in terms of other parameters
- Analyze Bandstop filters
- Design a simple Bandstop filter

Introduction to Bandstop filters

- Bandstop filters are those that pass voltages outside the band between the two cutoff frequencies to the output while attenuating voltages at frequencies between the two cutoff frequencies.
- Bandpass filters and Bandstop filters thus perform complementary functions in the frequency domain.
- Bandstop filters are also characterized by same parameters
  - Two cutoff frequencies ω_{C1}, ω_{C2}
  - The center frequency ω_0
  - The bandwidth β
  - The quality factor Q
- Only two of these five parameters can be specified independently
- These parameters are defined in the same way for both types of filters. So we examine Bandstop filters in the same way as we did with the Bandpass filters.

Bandstop Filter: Center Frequency, Bandwidth, and Quality Factor

- The center frequency ω_0 is defined as the frequency for which a circuit's TF is purely real. It is also referred to as the resonant frequency or corner frequency
- The center frequency is the geometric center of the stopband, that is, $\omega_0 = \sqrt{\omega_{C1}\omega_{C2}}$ (39-1)
- The magnitude of the TF is minimum at the center frequency $H_{\max} = \left|H\left(j\omega_0\right)\right|$ (39-2)
- The bandwidth β is the width of the stopband $\beta = \omega_{C2} - \omega_{C1}$ (39-3)
- The quality factor Q is the ratio of the center frequency to the bandwidth $Q = \dfrac{\omega_0}{\beta}$ (39-4)

Analysis of the Bandstop Series RLC Circuit

Consider the Bandstop Circuit of Fig. 39-1. Using the same arguments used for Low-pass and High-pass filters, so as before changing the frequency of the source results in changes to the impedance of the capacitor and the inductor.

- At $\omega = 0$ the impedance of the capacitor is $\frac{1}{j\omega C} = \infty$ so the capacitor behaves like an open circuit, and the impedance of the inductor is $j\omega L = 0$ so the inductor behaves like a short circuit. Thus $V_o = V_i$ as depicted in Fig. 39-1b

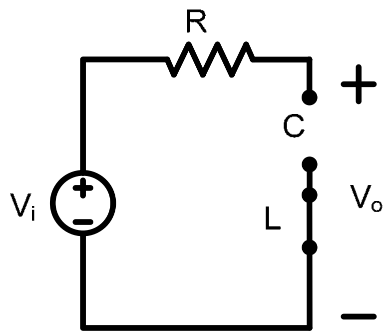

Fig. 39-1b The equivalent Circuit for $\omega = 0$

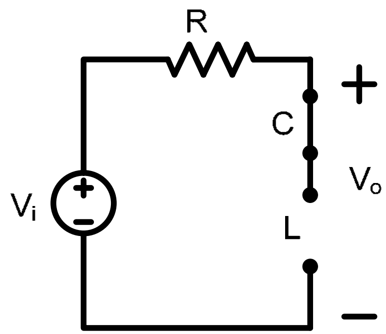

Fig. 39-1c The equivalent Circuit for $\omega = \infty$

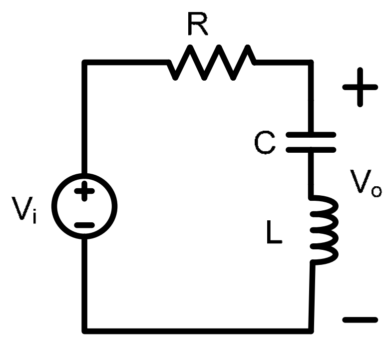

Fig. 39-1a A Series RLC Bandstop filter

- At $\omega = \infty$ the impedance of the capacitor is $\frac{1}{j\omega C} = 0$ so the capacitor behaves like a short circuit, and the impedance of the inductor is $j\omega L = \infty$ so the inductor behaves like an open circuit. Thus $V_o = V_i$ also, as depicted in Fig. 39-1c
- Between these two passbands, both the inductor and capacitor have finite impedances of opposite signs.
- As the frequency is increased from zero, $j\omega L$ increases and $\frac{1}{j\omega C}$ decreases. The phase shift between the input and the output approaches -90° as ωL approaches $\frac{1}{\omega C}$. As soon as ωL exceeds $\frac{1}{\omega C}$, the phase jumps to +90° and then approaches zero as ω continues to increase as seen in Fig. 39-2b
- At some frequency in between the two passbands, the center frequency ω_0, the two impedances cancel out causing the output voltage to be zero $V_o = 0$. At ω_0 the series combination of L and C appears as a short circuit.
- The plot of the voltage magnitude ratio is shown in Fig. 39-2a. Note the ideal bandstop filter plot is also overlaid on the plot of the RLC circuit.

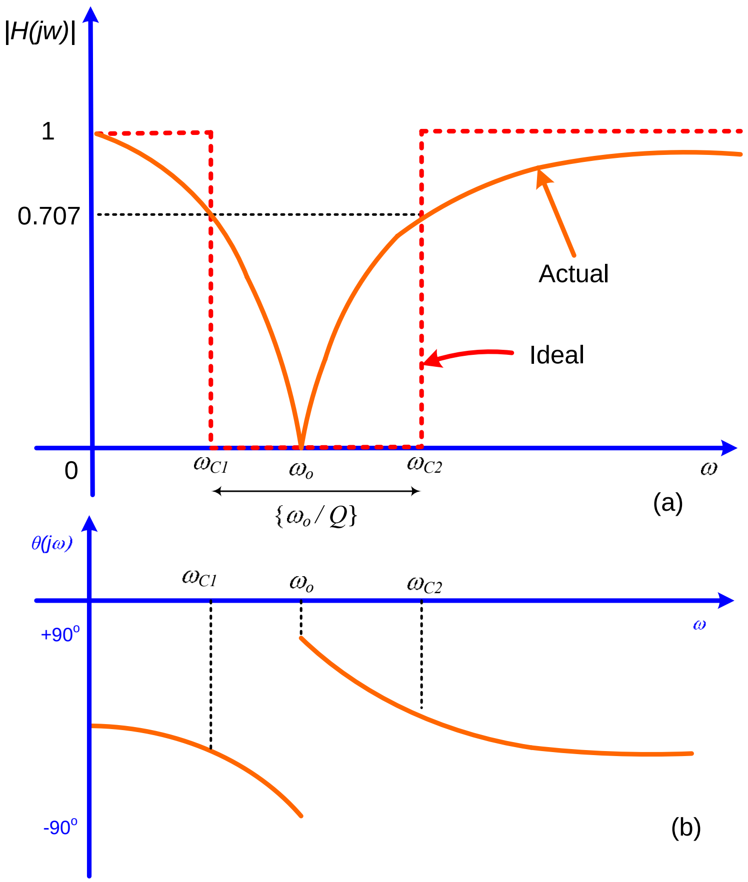

Fig. 39-2 The frequency response plot for the series RLC bandstop filter circuit in Fig. 39-1

Bandstop Series RLC Circuit Analysis Cont.

Let’s now examine the circuit quantitatively. Consider the s-domain equivalent circuit for the series RLC shown in Fig. 39-3 below

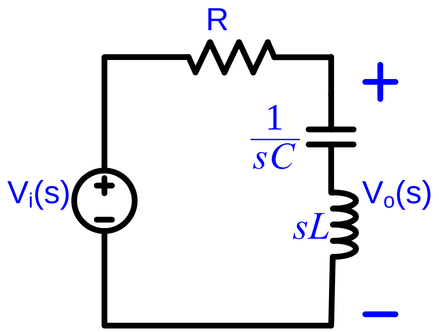

Fig. 39-3 s-domain equivalent of the Circuit of Fig. 39-1a

Transfer function of the circuit:

$$H(s)=\frac{sL+1/sC}{R+sL+1/sC}=\frac{s^2+1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}} \qquad (39\text{-}5)$$

This equation gives magnitude and phase as before if we express it in polar form and substituting $s=j\omega$ we get:

Bandstop Series RLC Circuit Analysis Cont.

$$|H(j\omega)| = \frac{\left| \frac{1}{LC} - \omega^2 \right|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega R/L\right)^2}} \qquad (39\text{-}6)$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega R/L}{\frac{1}{LC} - \omega^2}\right) \qquad (39\text{-}7)$$

This confirms the frequency response shape pictured in Fig. 39-2 from qualitative analysis.
We now calculate the five parameters that characterize this RLC bandstop filter:

Center Frequency: The TF in (39-5) will be purely real when $j\,\omega_0 L + \frac{1}{j\,\omega_0 C} = 0$ (39-8)

solving for ω_0 we get:

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad (39\text{-}9)$$

Cutoff Frequencies ω_{C1} and ω_{C2}: Substituting (39-9) into (39-6) and solving in exactly the same way as for the bandpass filter we get:

Bandstop Series RLC Circuit Analysis Cont.

$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \qquad (39\text{-}10)$$

$$\omega_{C2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \qquad (39\text{-}11)$$

We can use these last two equations to confirm that ω_0 is the geometric mean of the two cutoff frequencies $\omega_0 = \sqrt{\omega_{C1}\omega_{C2}} = \sqrt{\frac{1}{LC}}$ (39-12)

We can compute also the bandwidth $\beta = \omega_{C2} - \omega_{C1} = \frac{R}{L}$ (39-13)

And the Quality factor $Q = \frac{\omega_0}{\beta} = \frac{\sqrt{1/LC}}{\left(R/L\right)} = \sqrt{L/CR^2}$ (39-14)

Again we can express the cutoff frequencies in terms of center frequency and bandwidth as:

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} \qquad (39\text{-}15) \qquad\qquad \omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} \qquad (39\text{-}16)$$

Example 39-1

Using the series RLC circuit in Fig. 39-1a compute the component values that yield a bandreject filter with $\beta = 250Hz$ a center frequency $\omega_0 = 750Hz$, using $100nF$ capacitor. Compute values of R, L, the cutoff frequencies ω_{C1} and ω_{C2}, and the Quality factor Q.

Solution:

We begin by Quality factor Q, consider the s-domain circuit in Fig. 39-3:

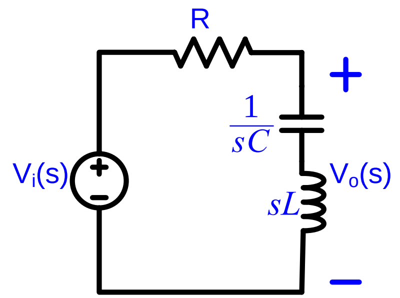

Fig. 39-3 s-domain equivalent of the Circuit of Fig. 39-1a

$$Q = \omega_0 / \beta = 3$$

To compute L we convert ω_0 to radians per second and

$$L = \frac{1}{\omega_0^2 C}$$

$$= \frac{1}{\left[2\pi(750)\right]^2 \left(100 \times 10^{-9}\right)} = 450mH$$

To compute R

$$R = \beta L$$

Or $R = 2\pi(250)\left(450 \times 10^{-3}\right) = 707\Omega$

Cutoff frequencies: we can use the values of the ω_0 and β to compute ω_{C1} and ω_{C2}:

$$\omega_{C1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} = 3992.0\, rad/s \qquad \omega_{C2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} = 5562.8\, rad/s$$

Check

$$\omega_{C2} - \omega_{C1} = 885.3 - 635.3 = 250\, Hz$$

$$\omega_0 = \sqrt{\omega_{C2} \times \omega_{C1}} = \sqrt{885.3 \times 635.3} = 750\, Hz$$

Self Test 39:

If the bandstop filter in Fig. 39-1a, is to reject a $200 Hz$ sinusoid while passing other frequencies, calculate the values of L and C. Take $R = 150\Omega$, and the bandwidth $\beta = 100 Hz$.

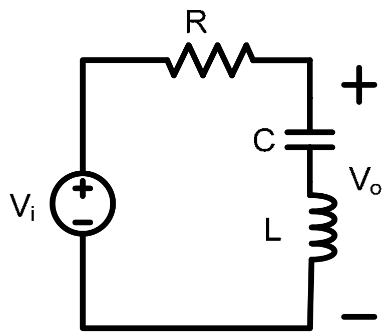

Fig. 39-1a A Series RLC Bandstop filter

Answer:

$L = 0.2387 H$, $C = 2.66 \mu F$