Electric Circuits II

Mutual Inductance

Lecture #31

The material to be covered in this lecture is as follows:

- Review of Self Inductance
- Concept of Mutual Inductance
- Magnetic coupling
- Dot Convention

After finishing this lecture you should be able to:

- Understand the concept of Mutual Inductance
- Determine the polarity of Induced Voltages
- Write Mesh Equations for a Circuit with mutual Inductance

Self Inductance

- A magnetic field consists of force surrounding the current carrying conductor.
- The voltage induced in the conductor is proportional to the number of lines that collapse into or cut the conductor.

Faraday's Law is $$v = \frac{d\lambda}{dt}$$

λ is the flux linkage and is measured in Wb –turns

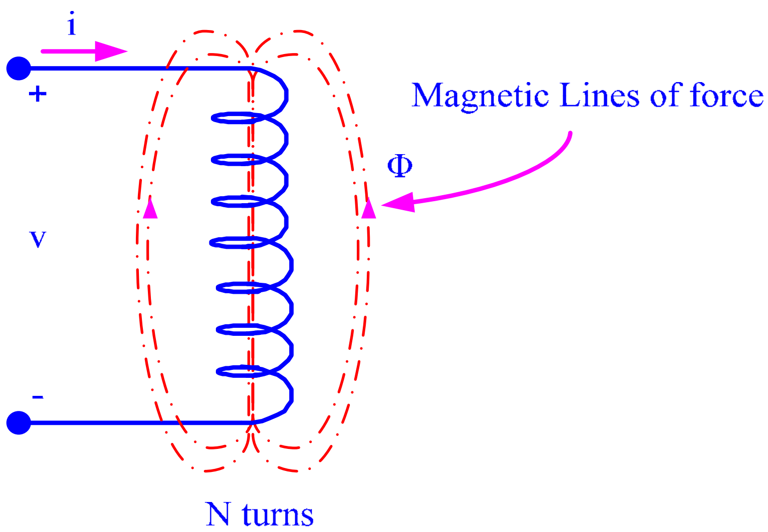


Fig.31-1 Representation of a magnetic field linking an N turns coil

Self Inductance (Cont)

- The flux linkage λ is the product of magnetic field (Φ) measured in (Wb) and the number of turns linked by the field (N).

$$\lambda = N\,\Phi$$

The magnetic flux Φ is related to the magnitude of the coil current by the relationship:

$$\Phi = pNi$$

where

p is the permeance of the space occupied by the flux
N is the number of turns
i is the current

Self Inductance (Cont)

Assuming the core material is non-magnetic then:

$$v = \frac{d\lambda}{dt} = \frac{d(N\Phi)}{dt}$$

$$= N\frac{d\Phi}{dt} = N\frac{d(pNi)}{dt}$$

$$= pN^{2}\frac{di}{dt} = L\frac{di}{dt}$$

L is the self inductance, given by $L = \frac{N\Phi}{i}$

Example 31-1:

- If an inductor with 100 turns and a current of 5 A produces a flux of 10 Wb the flux linkage equals NΦ = 100x10 = 1000 Wb
- The inductance is therefore: $L = \frac{N\Phi}{i} = \frac{1000}{5} = 200 \ Wb/A$

Concept of Mutual Inductance (Cont):

Mutual Inductance (M): Circuit Parameter that relates the voltage induced in one circuit to a time-varying current in a neighboring circuit, measured in henrys (H).

Consider Two Magnetically Coupled Coils

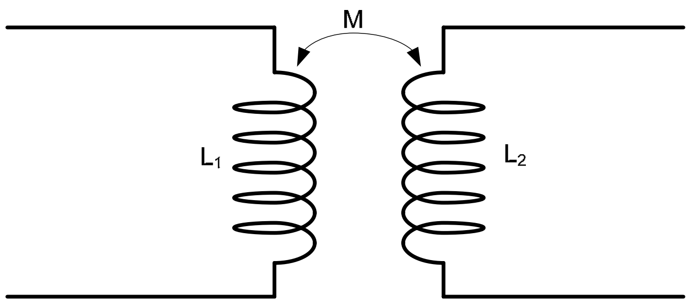


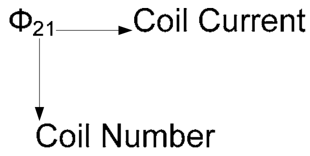
Fig31-2 Circuit of Two Magnetically Coupled coils

The magnetic field of the first coil also passes through the second coil and induces a voltage across it as well. We say that the two coils are magnetically coupled

Concept of Mutual Inductance (Cont):

Case I:

- Coil 1 is energized by a time –varying current source that establishes a current i_1 in N_1 turns.
- Coil 2 is not energized and open.
- The Coils are wound on a magnetic core.
- The flux produced by i_1 can be divided into two components:
  - Φ_{11} is the flux produced by i_1 that links only the N_1 turns.
  - Φ_{21} is the flux produced by i_1 that links N_2 and N_1 turns.
- The Total flux linking coil 1 is $\Phi_1 = \Phi_{11} + \Phi_{21}$



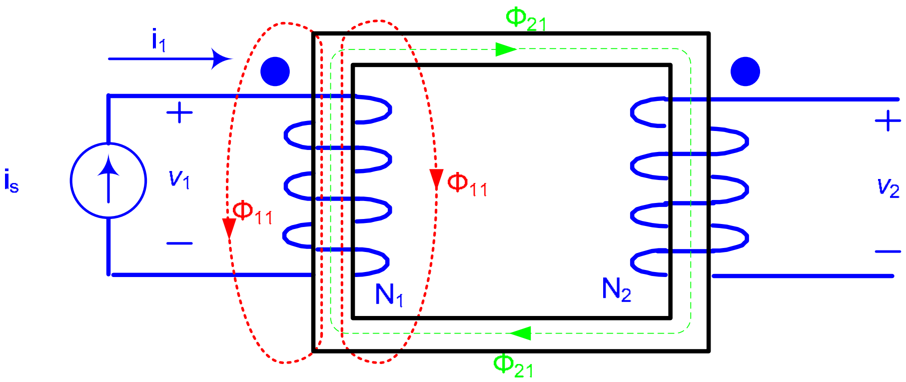


Fig31-3 Magnetically Coupled coils with coil 1 excited and coil 2 open.

$$\Phi_1 = p_1 N_1 i_1$$

$$\Phi_{11} = p_{11} N_1 i_1$$

$$\Phi_{21} = p_{21} N_1 i_1 \qquad p_1 = p_{11} + p_{21}$$

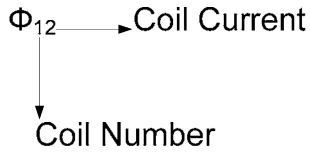
The Voltages V_1 and V_2 are

$$v_1 = \frac{d\lambda_1}{dt} = p_1 N_1^2 \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = p_{21} N_2 N_1 \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

Concept of Mutual Inductance (Cont):

Case II:



- Coil 2 is energized by a time –varying current source that establishes the current i_2 in N_2 turns.
- Coil 1 is not energized and open.
- The Coils are wound on a magnetic core.
- The flux produced by i_2 can be divided into two components
  - Φ_{22} is the flux produced by i_2 that links only the N_2 turns.
  - Φ_{12} is the flux produced by i_2 that links N_1 and N_2 turns.
- The Total flux linking coil 2 is $\Phi_2 = \Phi_{22} + \Phi_{12}$

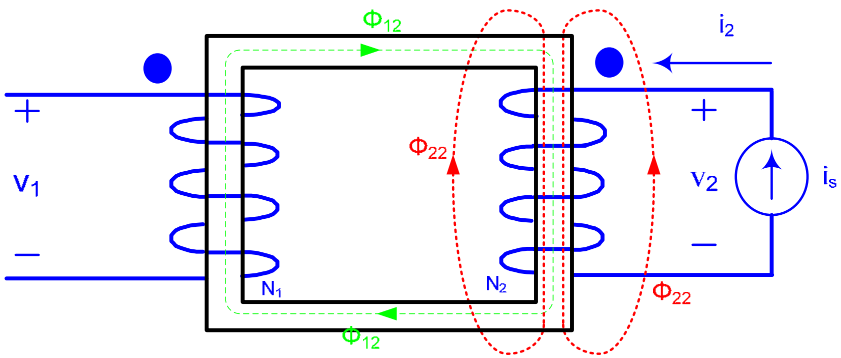


Fig31-4 Magnetically Coupled coils with coil 2 excited and coil 1 open.

$$\Phi_2 = p_2 N_2 i_2$$

$$\Phi_{22} = p_{22} N_2 i_2$$

$$\Phi_{12} = p_{12} N_2 i_2$$

The Voltages V_2 and V_1 are

$$v_2 = \frac{d\lambda_2}{dt} = p_2 N_2^2 \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = \frac{d\lambda_1}{dt} = p_{12} N_1 N_2 \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

Concept of Mutual Inductance (Cont.)

Therefore

$$v_2 = M_{21}\frac{di_1}{dt} \qquad if \qquad i_2 = 0, i_1 \neq 0$$

$$v_1 = M_{12}\frac{di_2}{dt} \qquad if \qquad i_1 = 0, i_2 \neq 0$$

For Nonmagnetic materials, the permeance p_{12} and p_{21} are equal and Therefore $M_{12} = M_{21} = M$

The Polarity of Mutually Induced Voltages (The Dot Convention)

- When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

- When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

➢ Example 31-2

Consider the Circuit of Fig.32-5:

- Given the circuit with the dotted markings as shown in Fig.31-5 (a)
- Arbitrarily choose the reference direction for each current Fig.31-5 (b)
- Determine the polarity of the mutually induced voltage, using the dot convention rule stated earlier Fig.31-5 (c)
- Write the mesh equations for i_1 and i_2

$$-v_g + R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = 0 \qquad \text{(Mesh 1)}$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0 \qquad \text{(Mesh 2)}$$

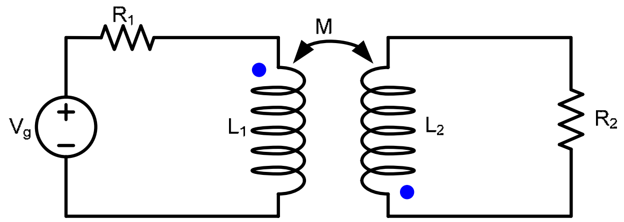


Fig.31-5 (a) Two Magnetically Coupled Coils

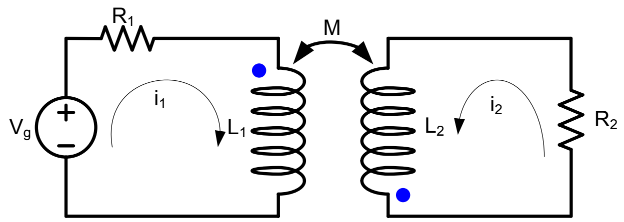


Fig.31-5 (b) Coil Currents i_1 and i_2 and Dot Marking to Indicate Polarity of Induced Voltages for Circuit of Fig.31-4 (a)

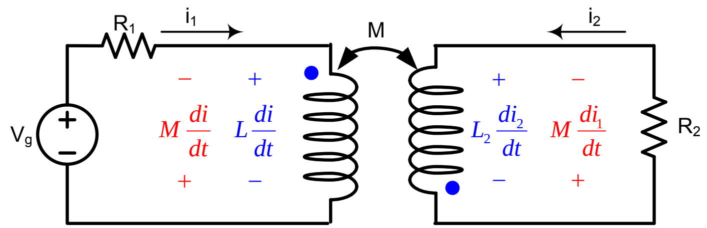


Fig.31-5 (c) The Self and Mutually Induced Voltages appearing across the coils shown in Fig.31-4 (a)

Self Test:

- Which is closest in meaning to large inductance?
  a) large flux b) low current c) large flux linkage d) large flux linkage per ampere
  answer: d

- If you double the number of turns but keep the flux and the current the same; the inductance will
  a) stay the same b) halve c) double d) triple
  answer: c