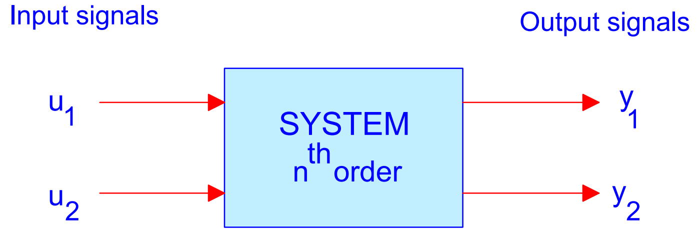
3. STATE VARIABLE MODELS

INTRODUCTION

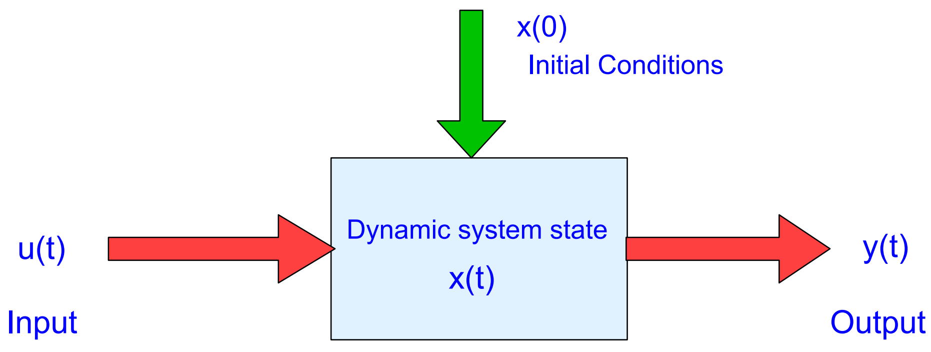
- The state of a system is set of variables such that the knowledge of these variables and the input functions will, with the equations describing the dynamics, provide the future state and output of the system.

STATE VARIABLE OF A DYNAMIC SYSTEMS

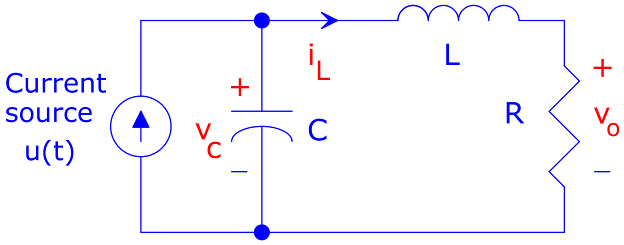
Consider the system shown below where $y_1(t)$ and $y_2(t)$ are the output signals and $u_1(t)$ and $u_2(t)$ are the input signals. A set of state variables $(x_1, x_2, \ldots x_n)$ for the system shown is a set such that knowledge of the initial values of the state variables $[x_1(t_0), x_2(t_0), \ldots x_n(t_0)]$ at the initial time t_0, and of the input signals $u_1(t)$ and $u_2(t)$ for $t \geq t_0$ suffices to determine the future values of the outputs and the state variables.



The general form of a dynamic system is shown



We proceed now to establish the state-variable (also known as state-space) approach as an alternate method for representing physical systems. Consider the RLC circuit shown



1. Since the network is of a second order, we need two state variables. Select the capacitor voltage $v_c(t)$ and the inductor current $i_L(t)$ as the two sate variables.
2. KCL at the junction leads to
$C\frac{dv_c}{dt} = -i_L + u(t)$
3. KVL for the right-hand loop leads to
$L\frac{di_L}{dt} = -Ri_L + v_c$
4. The output of the system is
$v_o = Ri_L(t)$
5. Let $x_1 = v_c$ and $x_2 = i_L$, then

$$\frac{dx_1}{dt} = -\frac{1}{C}x_2 + \frac{1}{C}u(t)$$

$$\frac{dx_2}{dt} = \frac{1}{L}x_1 - \frac{R}{L}x_2$$

$$y(t) = Rx_2$$

For a passive RLC network, the number of state variables is equal to the number of independent energy-storage elements

Note that the state variables that describe a system are not a unique set, and several alternative sets can be chosen.

State Differential Equation

The above set of simultaneous first order differential equations can be written in matrix form as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} u(t)$$

Similarly, the output can be expressed as $y(t) = \begin{bmatrix} 0 & R \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Summary

now that we have represented a physical network in state space and have a good idea of the terminology and concept, let us summarize some of the definitions we came across .

<u>*State equations*</u> $\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$;
A set of n simultaneous, first-order differential equations with n variables, where the n variables to be solved are the state

<u>Output equation</u> $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + D\boldsymbol{u}$;
The algebraic equation that expresses the output variables of a system as linear combinations of the state variables and the inputs.

Where

- $\boldsymbol{x}$ state vector $[nx1]$
- $\dot{\boldsymbol{x}}$ derivative of state vector $[nx1]$
- $\boldsymbol{y}$ output vector $[rx1]$
- $\boldsymbol{u}$ input or control $[mx1]$
- A system matrix $[nxn]$
- B input matrix $[nxm]$
- C output matrix $[rxn]$
- D feedforward matrix $[rxm]$