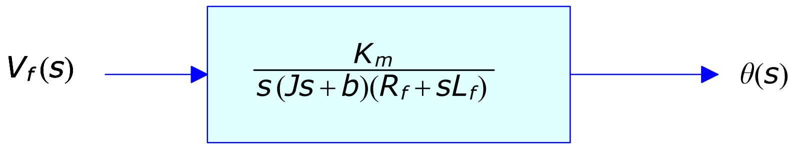
2. **Mathematical Models of Systems (cont.)**

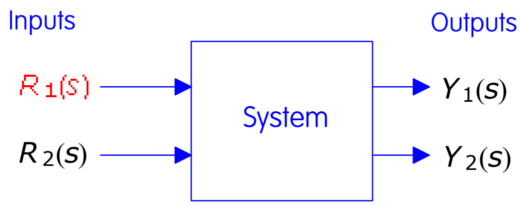
BLOCK DIAGRAM MODELS

The block diagram representation of the system relationships is prevalent in control system engineering. A block diagram consists of unidirectional operational blocks that represent the transfer function of the variables of interest. A block diagram of the field controlled dc motor is shown below



To represent a system with several variables under control, an interconnection of blocks is utilized. Consider the two-input variables, two output variables system, we can write

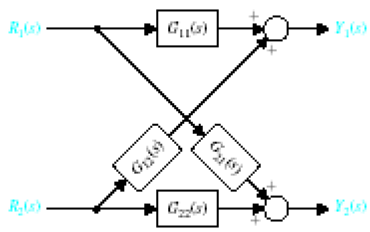
$$Y_1(s) = G_{11}(s)R_1(s) + G_{12}(s)R_2(s)$$
$$Y_2(s) = G_{21}(s)R_1(s) + G_{22}(s)R_2(s)$$



where

$$G_{11}(s) = \left.\frac{Y_1(s)}{R_1(s)}\right|_{R_2(s)=0};\ G_{12}(s) = \left.\frac{Y_1(s)}{R_2(s)}\right|_{R_1(s)=0}$$

$$G_{21}(s) = \left.\frac{Y_2(s)}{R_1(s)}\right|_{R_2(s)=0};\ G_{22}(s) = \left.\frac{Y_2(s)}{R_2(s)}\right|_{R_1(s)=0}$$



In matrix form

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} R_1(s) \\ R_2(s) \end{bmatrix}; \quad \Rightarrow \boldsymbol{Y} = \boldsymbol{GR}$$

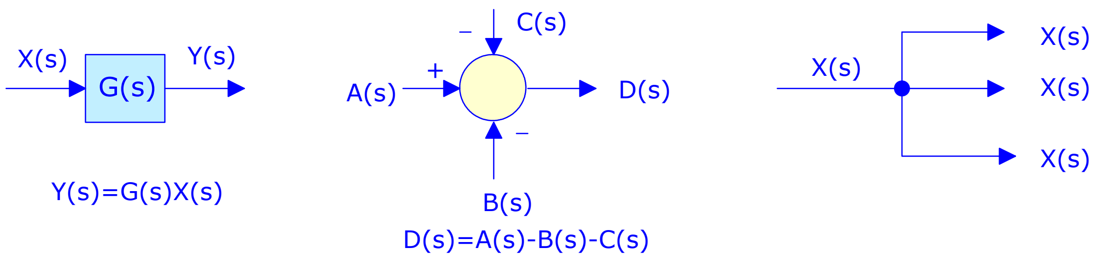
Block Diagram Elements

(a) *Block*: Indicates a proportional relationship between two Laplace -transformed signals

(b) *Summing Point*: indicates addition and subtraction of signals

(c) *Pickoff Point*: indicates that the same signal go to several places

Examples of each of these elements are shown

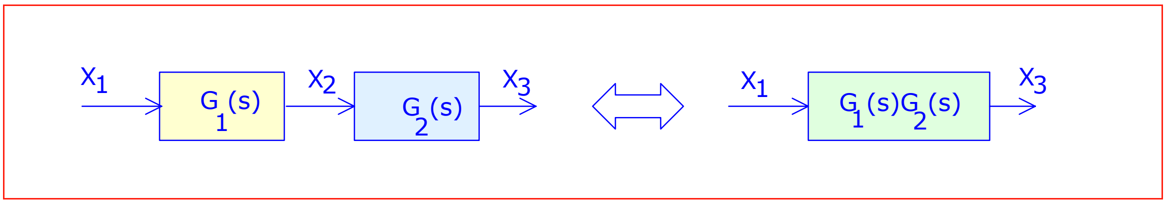


(a) (b) (c)

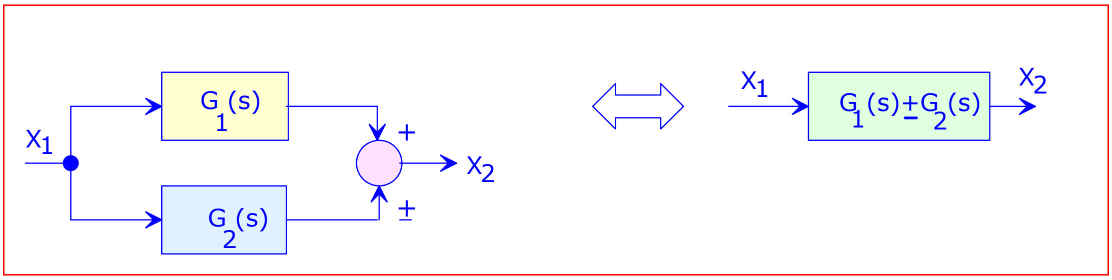
Block Diagram Reduction

For a single-input, single output system, block diagram reduction means simplifying the composite diagram to the point where it is a single block, displaying the transfer function relating the output to the input. Block diagram transformation and reduction is done by considering the algebra of the diagram variables. The following are some useful simplifications:

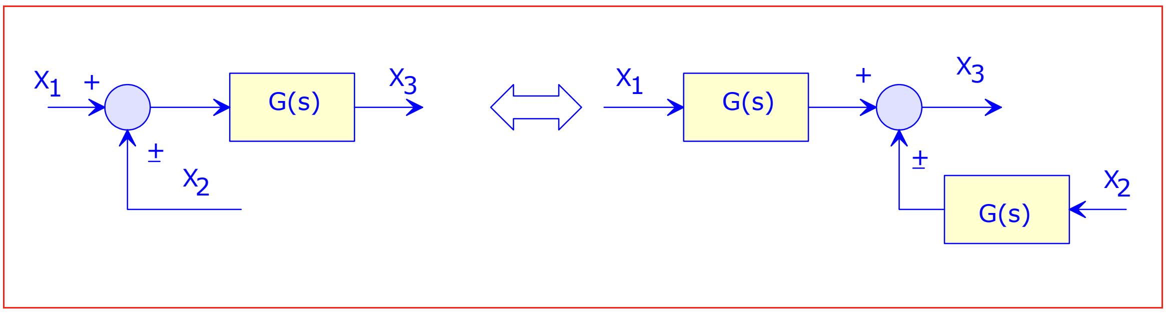
1. *Combining blocks in cascade*



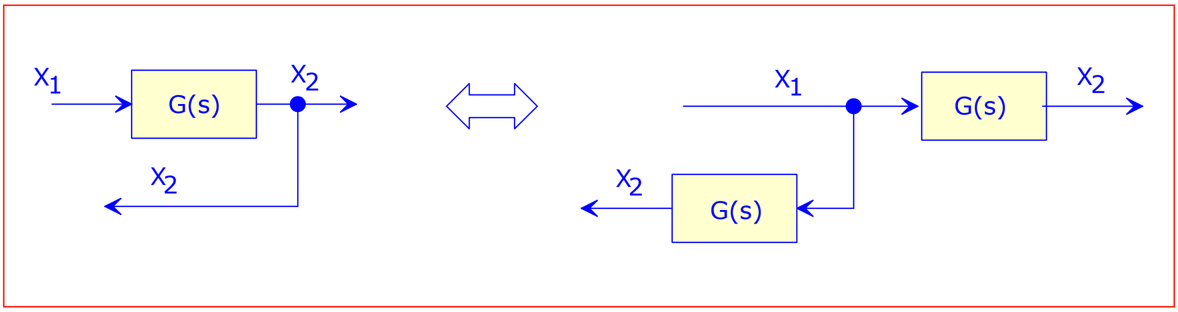
2. *Combining blocks in parallel*



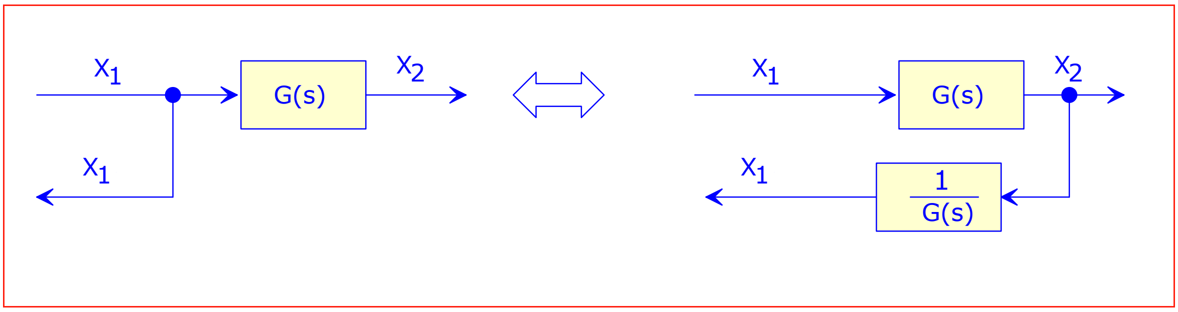
3. Moving a summing point behind a block



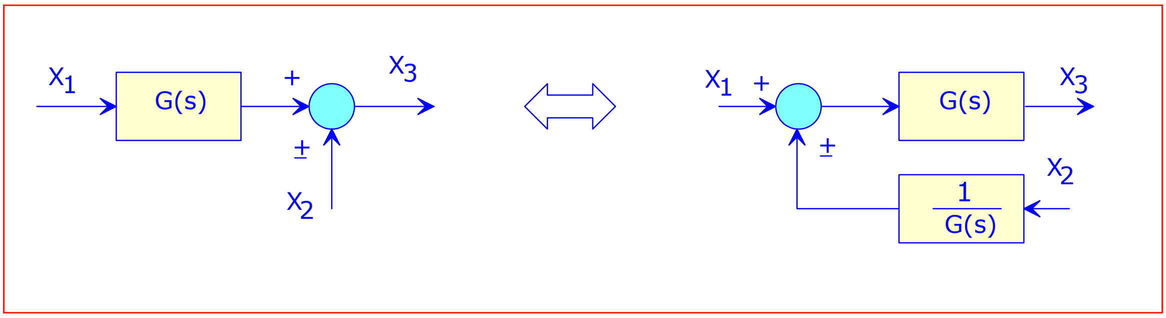
4. Moving a pickoff point ahead of a block



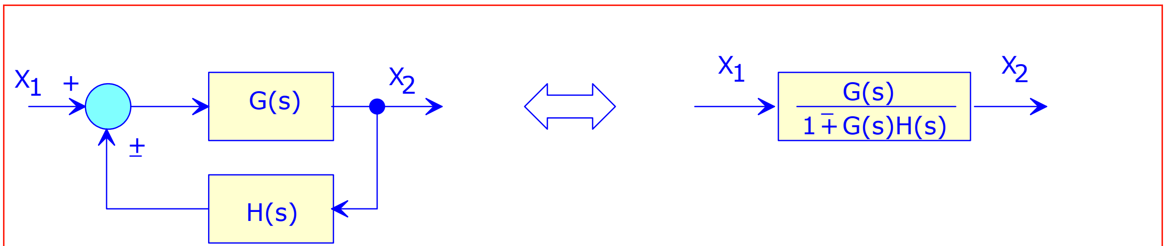
5. Moving a pickoff point behind a block



6. Moving a summing point ahead of a block



6. Eliminating a feedback loop

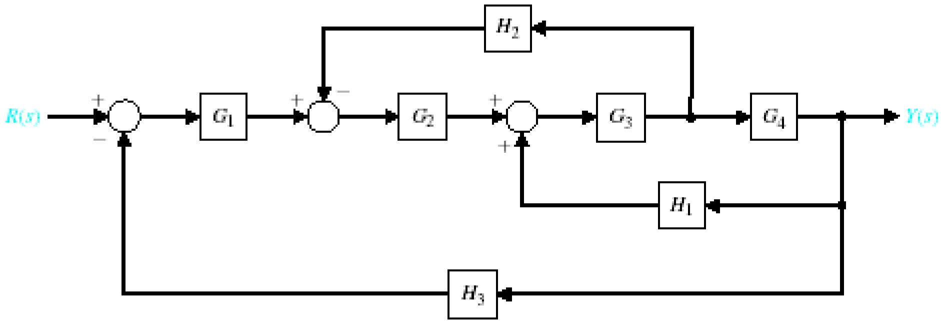


$$[X_1(s) \pm H(s)X_2(s)]\,G(s) = X_2(s)$$

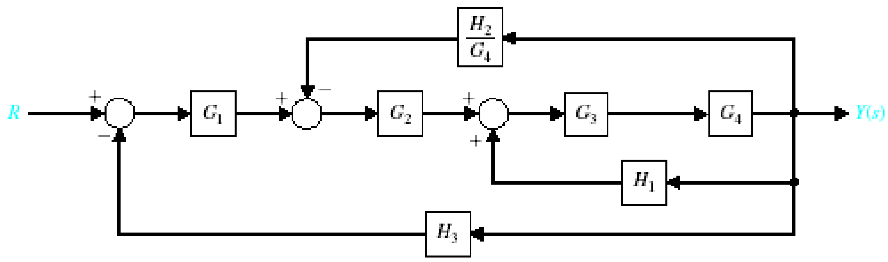
The utility of the block diagram transformation will be illustrated by an example using block diagram reduction.

Example

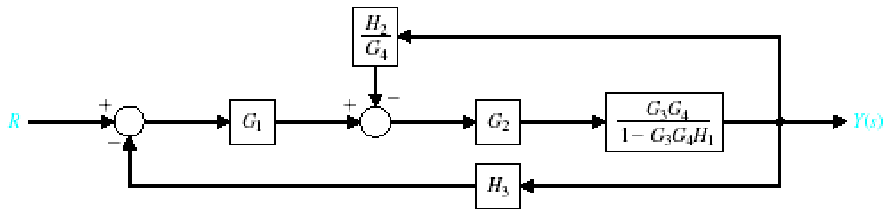
Reduce the following block diagram to a single block



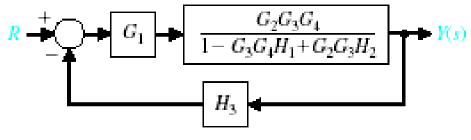
Solution



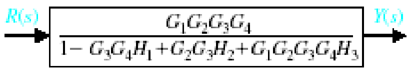
(a)



(b)



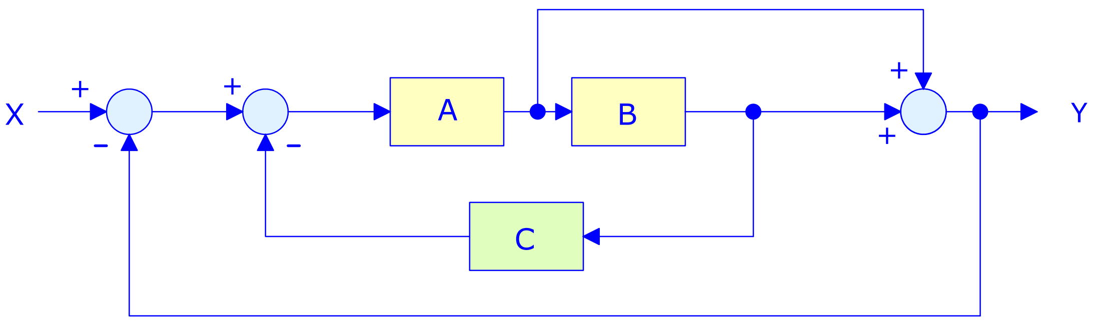
(c)



(d)

Drill Problem

Find $\frac{Y}{X}$ in the given block diagram.



Answer $\left\{\frac{A(1+B)}{1+ABC+AB+A}\right\}$
