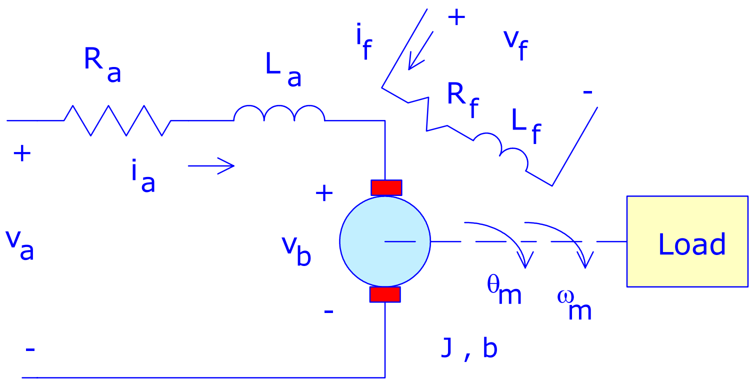
2. Mathematical Models of Systems (cont.)

ELECTROMECHANICAL SYSTEM TRANSFER FUNCTIONS

dc motor

The dc motor is a component that converts direct current (dc) electrical energy into rotational mechanical energy, which is available to drive an external load. In the figure,



- The air-gap flux of the motor is proportional to the field current, provided the field is not saturated, so that

$$\Phi = K_f \, i_f ;$$

- The torque developed by the motor is assumed to be related linearly to Φ and the armature current as follows:

$$T_m = K_1 \Phi \, i_a(t) = K_1 K_f \; i_f(t) \; i_a(t)$$

- The back electromotive-force voltage which is induced to oppose the input voltage to the armature is

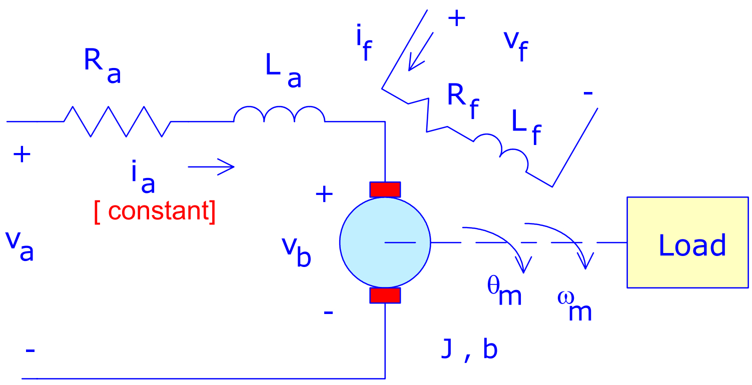
$$v_b = K \, \omega \phi ; \quad \rightarrow \quad v_b = K_b \, \omega \;\; (\text{ constant field})$$

- A dc motor with a fixed armature current may be controlled by the field current and is known as **Field-controlled dc motor**

- A dc motor with a fixed field current may be controlled by the armature current and is known as **Armature-controlled dc motor**

We shall derive the transfer functions of both types.

Field-controlled dc motor [i_a : *constant*]



In Laplace Transform notation

$$T_m(s) = \{K_1 K_f \quad I_a(s)\} I_f(s) = K_m I_f(s)$$

The field current is related to the field voltage as

$$V_f(s) = (R_f + sL_f)\, I_f(s) \quad \Rightarrow \quad I_f(s) = \frac{V_f(s)}{(R_f + sL_f)}$$

The motor torque $T_m(s)$ is equal to the torque delivered to the load. This relation may be expressed as

$$T_m(s) = T_L(s) + T_d(s)$$

where $T_L(s)$ is the load torque and $T_d(s)$ is the disturbance torque, which is often negligible ($\simeq 0.$) [**However, the disturbance torque often must considered in systems subjected to external forces such as antenna win-gust forces].**

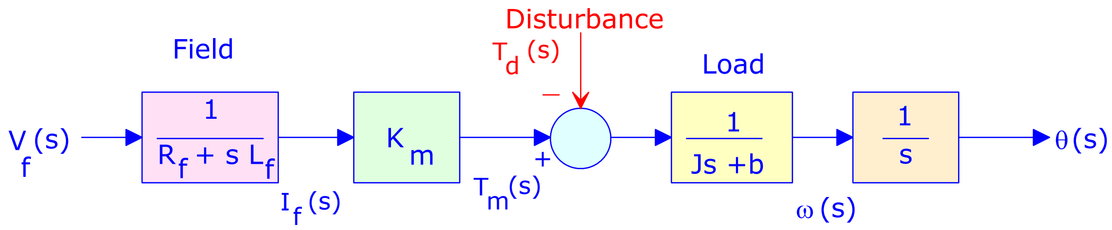
The load torque for rotating inertia is

$$T_L(s) = Js^2\theta(s) + bs\theta(s) = s(Js + b)\,\theta(s)$$

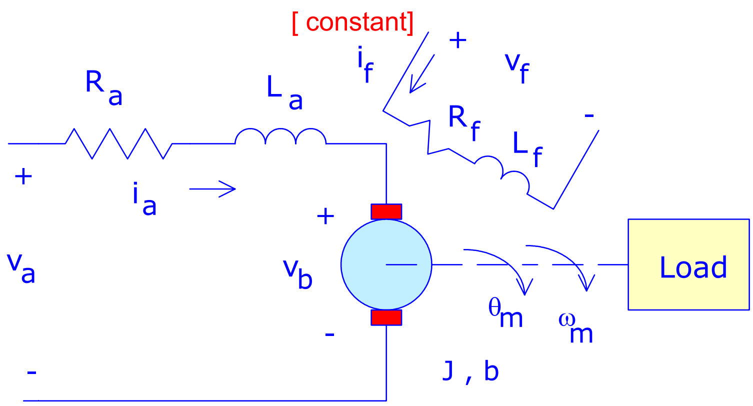
Equating $T_m(s)$ to $T_L(s)$ and making use of the above equations result in the transfer function

$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s\,(Js + b)(R_f + sL_f)}$$

The block-diagram model of the field-controlled dc motor is shown next



Armature-controlled dc motor [i_f : *constant]*



In this case, the motor torque is

$$T_m(s) = \{K_1 K_f \ I_f(s)\ \} I_a(s) = K_m I_a(s)$$

The armature current is related to the armature voltage as

$$\begin{aligned} V_a(s) &= (R_a + sL_a)\ I_a(s) + V_b(s) \\ &= (R_a + sL_a)\ I_a(s) + K_b \omega(s) \end{aligned}$$

Hence the armature current is given by

$$I_a(s) = \frac{V_a(s) - K_b \omega(s)}{(R_a + sL_a)} \quad \Rightarrow \quad T_m(s) = K_m I_a(s) = K_m \frac{V_a(s) - K_b \omega(s)}{(R_a + sL_a)}$$

The load torque for rotating inertia is

$$T_L(s) = Js^2 \theta(s) + bs\theta(s) = s(Js + b)\,\theta(s)$$

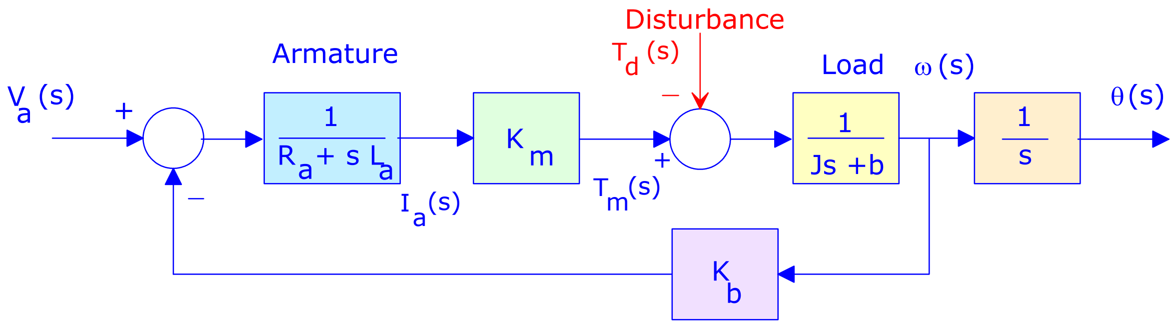
Equating $T_m(s)$ to $T_L(s)$ result in the transfer function

$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[\,(R_a + sL_a)\,(Js + b) + K_b K_m]}$$

However, for many dc motors, the time constant of the armature $\tau_a = \frac{L_a}{R_a}$, is negligible, and therefore

$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[R_a(Js+b)+K_bK_m]}$$

The block-diagram model of the armature-controlled dc motor is shown next



Comment on the relation between K_b and K_m

At steady state,

Power input to the rotor = power delivered to the shaft

$$(K_b\omega)i_a = (K_m i_a)\omega$$

Which means that $K_b = K_m$ [numerically]