7. THE ROOT LOCUS METHOD [CONT.]

Example 1

Plot the root locus for the characteristic equation of a system as $0 < K < \infty$

$$1+\frac{K}{s^4+12s^3+64s^2+128s}=0$$

Solution

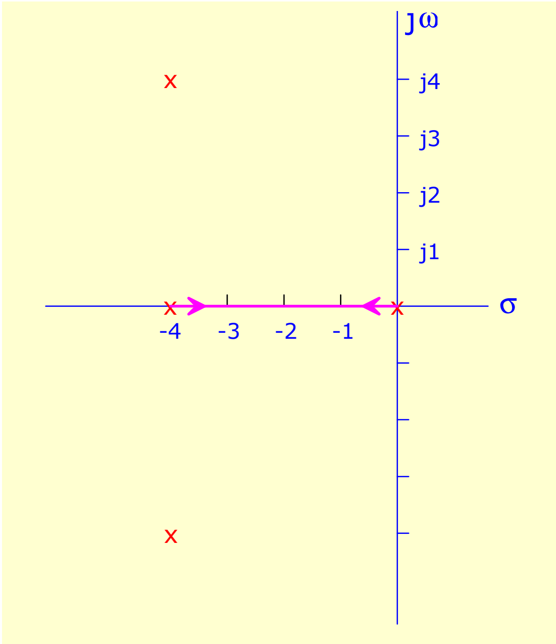
Steps 1 and 2:

$$1+K\frac{1}{s(s+4)(s+4\pm j4)}=0$$

Step 3:

We have

- 4 open-loop poles at $s=0$, $s=-4$, $s=-4+j4$ & $s=-4-j4$
- no open-loop zeros



We locate the poles as shown.

Step 4:

Locate the root locus segments that lie on the real axis as shown . A segment of the root locus exists on the real axis between $s=0$ and $s=-4$.

Step 5

The number of separate loci is equal to $n_p=4$
The number of loci branches proceeding to zeros at infinity is $n_p-n_z=4$

Step 6

The root loci are symmetrical with respect to the real axis

Step 7:

$n_p=4\,;n_z=0$

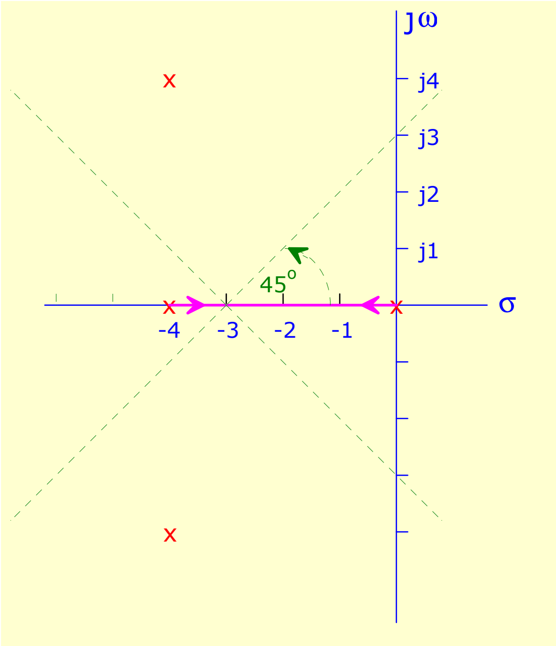
$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z}$$

$$= \frac{(0-4-4-4))}{4} = -3$$

$$\phi_A = \frac{(2q+1)}{n_p - n_z}180^\circ \;, q = 0,1,2,3$$

$$\phi_A = \pm 45^\circ, \pm 135^\circ$$

Then the asymptotes are drawn as shown.



Step 8:

To determine the imaginary axis crossing, we write the C.E.,

$$s(s+4)(s+4\pm j4)+K=0 \quad \Rightarrow \quad s^4+12s^3+64s^2+128s+K=0$$

s^4	1	64	K
s^3	12	128	0
s^2	b_1	K	
s	c_1	0	
s^0	K		

$$b_1 = \frac{12x64-128}{12} = 53.33 \;;\; c_1 = \frac{53.33x128-12K}{53.33}$$

The limiting value of the gain for stability is $K = \frac{53.33x128}{12} = 568.89$

To find the points where the locus crosses the imaginary axis, we find the roots of the auxiliary equation,

$$53.33s^2 + 568.89 = 0 \quad \Rightarrow \quad s = \pm j\sqrt{\frac{568.89}{53.33}} = j3.26$$

Step 9:

To determine the breakaway point [must be between $s=-4$ & $s=0$], we have

$$K = -(s^4+12s^3+64s^2+128s)$$

$$\frac{dK}{ds} = -(4s^3+36s^2+128s+128) = 0 \quad \rightarrow \; s^3+9s^2+32s+32=0$$

The roots are $-1.5767\;;-3.7117\pm j2.5532$

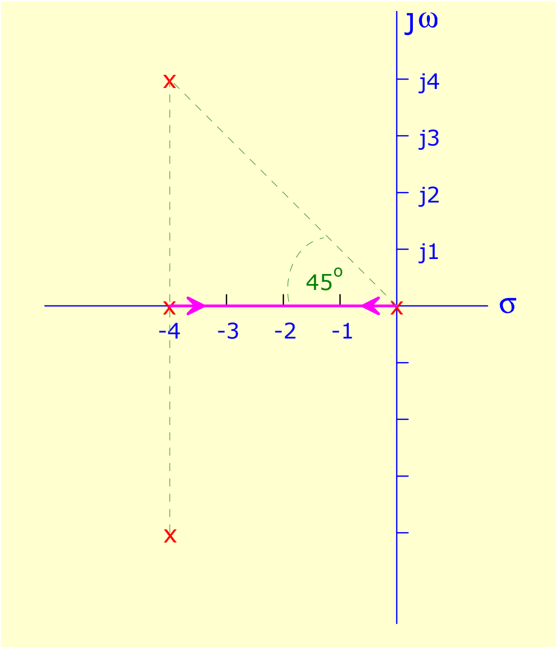
Breakaway point: $s=-1.5767$

Step 10:

To determine the angle of departure at the complex pole

$$-(\theta_d + 90^\circ + 90^\circ + 135^\circ) = -180^\circ$$

$$\theta_d = -135^\circ$$



Step 11:

To determine the gain at the breakaway point

$$\left| K \frac{1}{s(s+4)(s+4 \pm j4)} \right|_{s=-1.5767} = 1$$

$$K_{ba} = 83.57$$

The complete root locus plot is shown