7. THE ROOT LOCUS METHOD [CONT.]

STEP 7:

The loci proceed to the zeros at infinity along linear asymptotes centered at σ_A on the real axis, and with angles ϕ_A. The number of loci branches that end at zeros at infinity is equal to $n_p - n_z$.

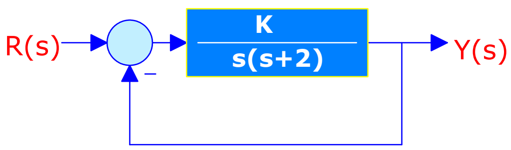
$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z} \quad ;$$

$$\phi_A = \frac{(2q+1)}{n_p - n_z}180^\circ \quad , q = 0,1,2,\ldots(n_p - n_z - 1)$$

Two examples will further illustrate the process of utilizing the asymptotes

Example 1

$$n_p = 2 ; n_z = 0$$



$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z} = \frac{(0-2)}{2} = -1$$

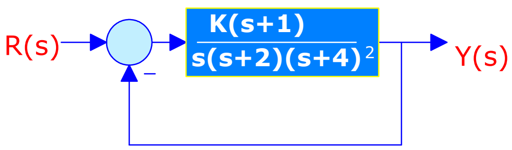
$$\phi_A = \frac{(2q+1)}{n_p - n_z}180^\circ \quad , q = 0,1$$

$$\phi_A = \pm 90^\circ$$

Example 2

Steps 1 and 2:

$$1 + K\frac{(s+1)}{s(s+2)(s+4)^2} = 0$$



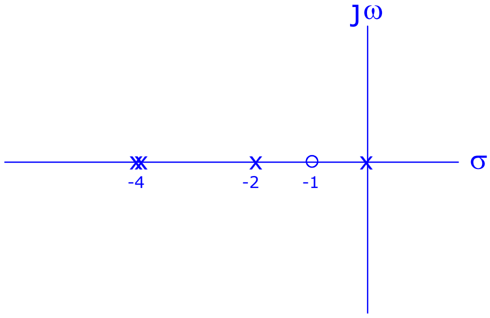
Step 3:

We have

- 4 open-loop poles at $s = 0$, $s = -2$, $s = -4$ & $s = -4$

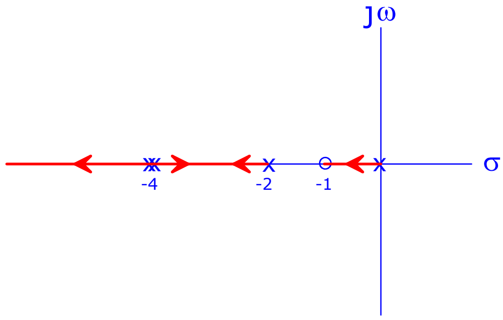
- 1 open-loop zero at $s = -1$

We locate the poles and zeros as shown.



Step 4:

Locate the root locus segments that lie on the real axis



Step 5

The number of separate loci is equal to $n_p = 4$

The number of loci branches proceeding to zeros at infinity is $n_p - n_z = 3$

Step 7:

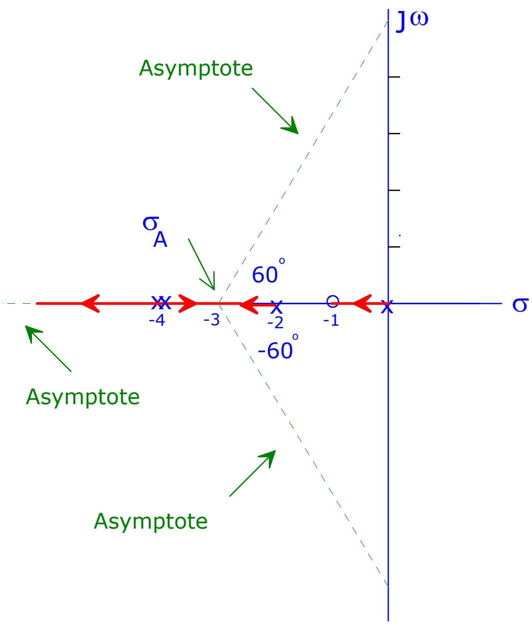
$n_p = 4\,; n_z = 1$

$$\sigma_A = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{M}(-z_i)}{n_p - n_z}$$

$$= \frac{(0-2-4-4)-(-1)}{3} = -3$$

$$\phi_A = \frac{(2q+1)}{n_p - n_z}180^\circ \;\; , q = 0,1,2$$

$$\phi_A = \pm 60^\circ , -180^\circ$$



We now return to developing the remaining steps.

STEP 8:

Determine the point at which the locus crosses the imaginary axis (if it is applicable). You may use one of the following methods:

- Routh-Hurwitz Criterion. The roots of the auxiliary equation determine the imaginary-axis crossover points.

- Let $s = j\omega$ in the characteristic equation. This will result in two algebraic equations that will yield the imaginary-axis crossover point, and the relevant gain.

STEP 9:

Determine the breakaway and breakin points on the real axis (if any). The root locus leaves (enters) the real axis at a breakaway point (breakin point).

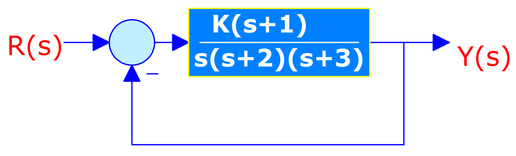
The breakaway and breakin points are obtained by solving for the roots of the equation:

$$\frac{dK}{ds} = 0$$

An example will further illustrate step 9.

Example

$$1 + K\frac{(s+1)}{s(s+2)(s+3)} = 0$$

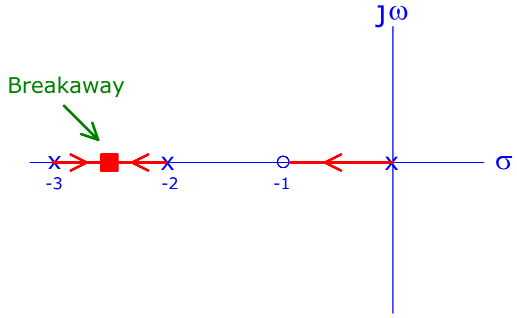


$$K = \frac{-s(s+2)(s+3)}{(s+1)}$$

$$\frac{dK}{ds} = 0$$

$$s(s+2)(s+3) = (s+1)(3s^2 + 10s + 6)$$

$$s^3 + 8s^2 + 5s + 3 = 0$$

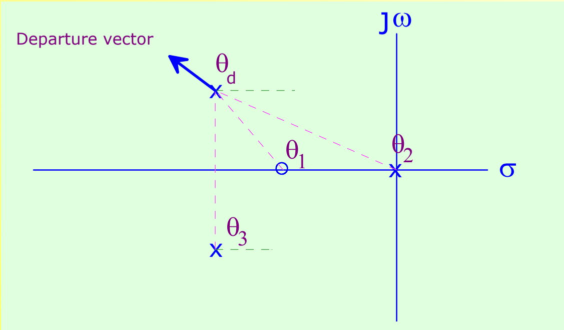


Roots are at: $-2.47, -0.767 \pm j0.793$

Breakaway point is s=-2.47

STEP 10:

Determine the angle of departure (arrival) of the locus from (to) a complex pole (zero), using the angle criterion.



θ_d angle of departure

$$\angle\text{zeros} - \angle\text{poles} - 180^\circ \Rightarrow \theta_1 - (\theta_d + \theta_3 + \theta_2) = -180^\circ$$

STEP 11:

To prove that a point is on the root locus, the angle condition must be satisfied:

$$\angle G(s)H(s) = -180^\circ$$

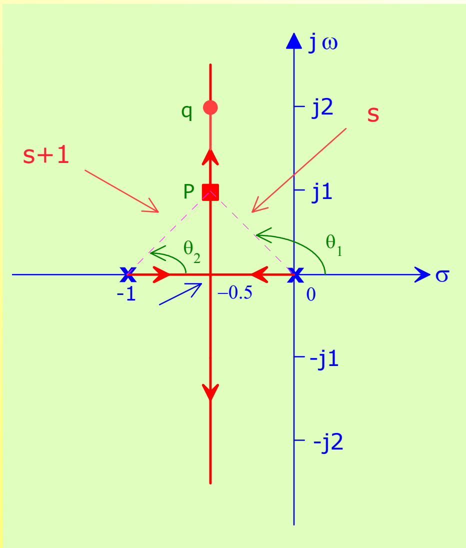
Consider point P

$$\angle G(s)H(s) = \angle \frac{K}{s(s+1)}$$

$$= -[\angle s + \angle s + 1)]$$

$$= -[\theta_1 + \theta_2] = \textit{always} - 180^\circ$$

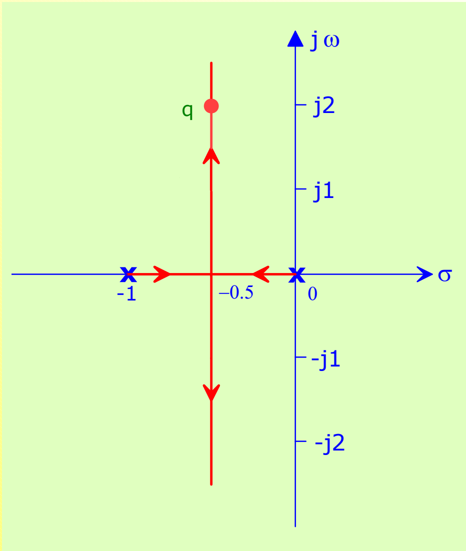
Hence, the point P is on the root locus.



STEP 12:

To find the parameter value K_x at a specified root s_x, use the magnitude condition.

Consider point q on the root locus. It is required to find the gain at that point.
The coordinates of point q are : $-0.5+j2$



$$\left|\frac{K}{s(s+1)}\right|_{s=-0.5+j2} = 1$$

$$\left|\frac{K}{(-0.5+j2)(-0.5+j2+1)}\right| = 1$$

$$K = \frac{17}{4}$$

We are now going to illustrate the use of the twelve steps in some examples.