

## 5. THE PERFORMANCE OF FEEDBACK CONTROL SYSTEMS

### INTRODUCTION

To analyze and design a control system, we must define and measure its performance. Then based on the desired performance of the control system, the system parameters may be adjusted to provide the desired response. The performance of control systems is usually specified in terms of :

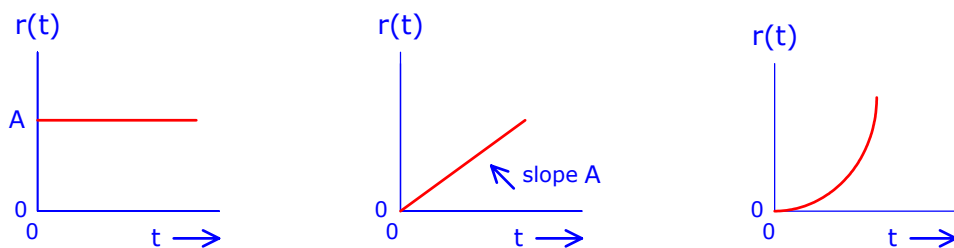
- **the transient response** disappears with time
- **the steady-state response** exists a long time after input is applied

Design specifications for control systems normally include several time-response indices for a specific input command [transient response], as well as a desired steady-state accuracy [steady-state]

### TEST INPUT SIGNALS

The time-domain performance specifications are important indices because control systems are inherently time-domain systems. That is, the system transient or time performance is the response of prime interest for control systems. If the system is stable, the response to a standard test input signal will provide several measures of performance.

The standard test input signals commonly used are the step input, the ramp input, and the parabolic input. These input are shown below



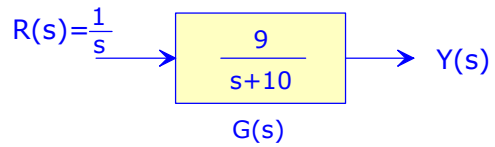
### Test Signal Inputs

Test Signal	$r(t)$	$R(s)$
Step	$r(t) = A ; t \geq 0$ $= 0 ; t < 0$	$R(s) = \frac{A}{s}$
Ramp	$r(t) = At ; t \geq 0$ $= 0 ; t < 0$	$R(s) = \frac{A}{s^2}$
Parabolic	$r(t) = At^2 ; t \geq 0$ $= 0 ; t < 0$	$R(s) = \frac{2A}{s^3}$

The step input signal is the easiest to generate and evaluate and is usually chosen for performance tests.

Example

$$Y(s) = G(s)R(s) = \frac{9}{s(s+10)}$$



Taking the inverse Laplace transform, the step-response is given by

$$y(t) = 0.9(1 - e^{-\frac{t}{10}}) = 0.9(1 - e^{-\frac{t}{\tau}})$$

The steady-state response is  $y(\infty) = 0.9$

Let us now examine the significance of the parameter  $\tau$ , the only parameter needed to describe the transient response. When  $t = \tau = \frac{1}{10}$ ,

$$e^{-\frac{t}{\tau}} \Big|_{t=\tau} = 0.37 \Rightarrow y(t) = 0.9(1 - 0.37) = 0.9(63\%)$$

The time constant is then the time it takes for the step response to rise to 63% of its final value.

If the error is  $E(s) = R(s) - Y(s)$ , then the steady-state error is

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = 0.1 \quad \text{or} \quad e_{ss} = r(\infty) - y(\infty) = 1 - 0.9 = 0.1$$

