3. STATE VARIABLE MODELS (cont.)

Method 2 Input Feed forward format (Observer canonical form OCF)

Consider the transfer function:

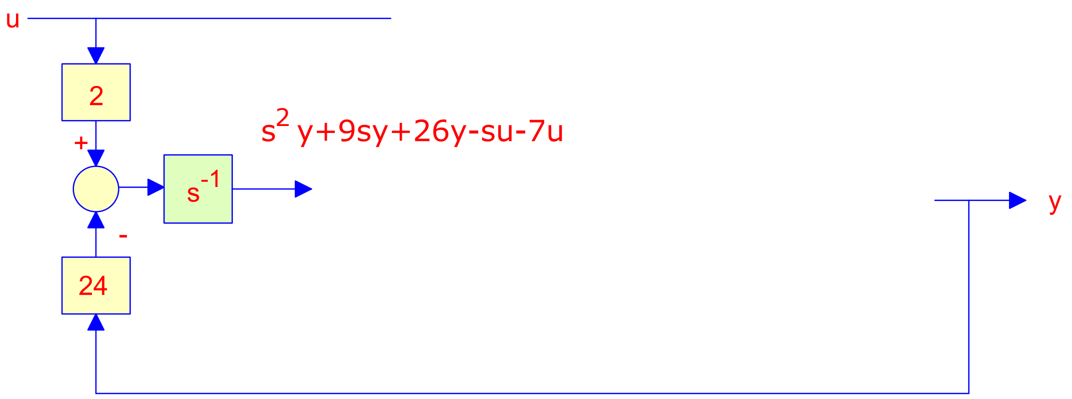
$$\frac{Y(s)}{U(s)} = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

1. Express the transfer function in the form

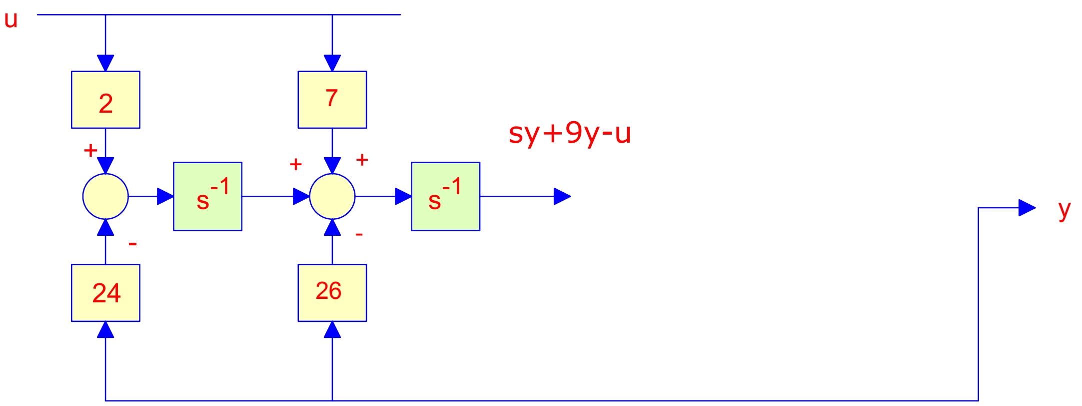
$$s^2u + 7su + 2u = s^3y + 9s^2y + 26sy + 24y$$

2. Rewrite the above equation so that every term on the right is multiplied by at least one power of s. This is done so that we can operate on the lot by s^{-1},

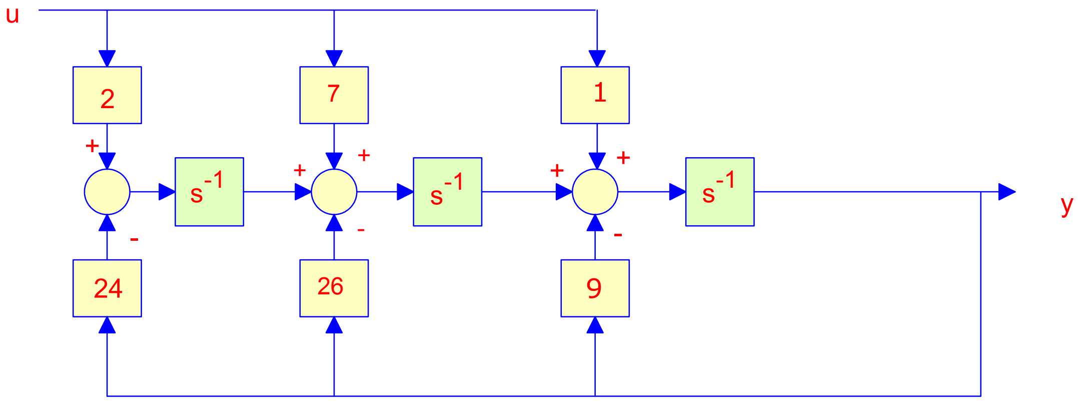
$$2u - 24y = s^3y + 9s^2y + 26sy - s^2u - 7su$$



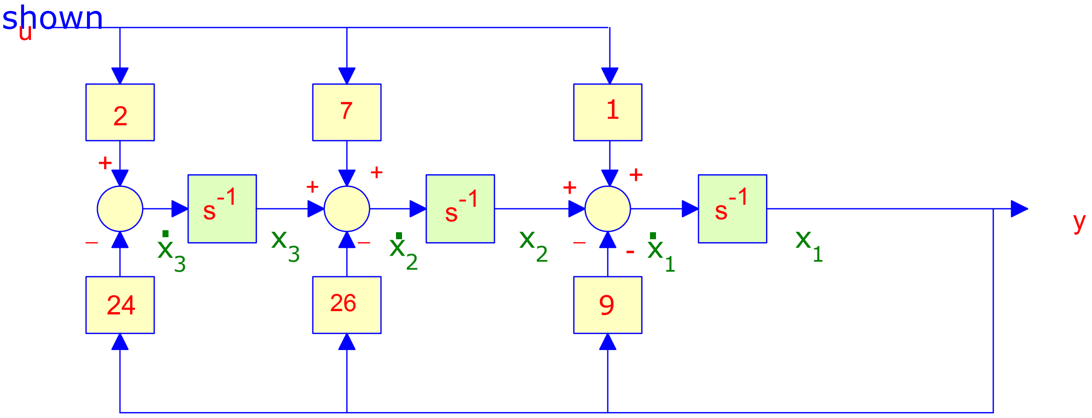
3. Now in this internal result there appear terms that do not have any power of s, which can be canceled by adding proper multiple of u and y



4. If we continue this process of subtracting out the terms having no power of z and operating on the rest with s^{-1}, we finally arrive at the place where all is left is y alone.



5. The state variables are now assigned to each integrator output as shown



6. Using the above block diagram to derive the set of first-order differential equations, we obtain:

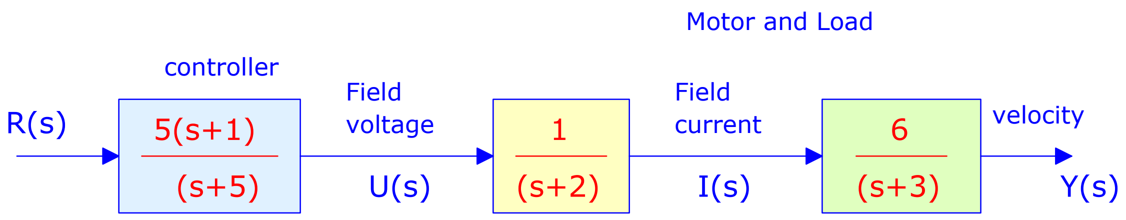
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

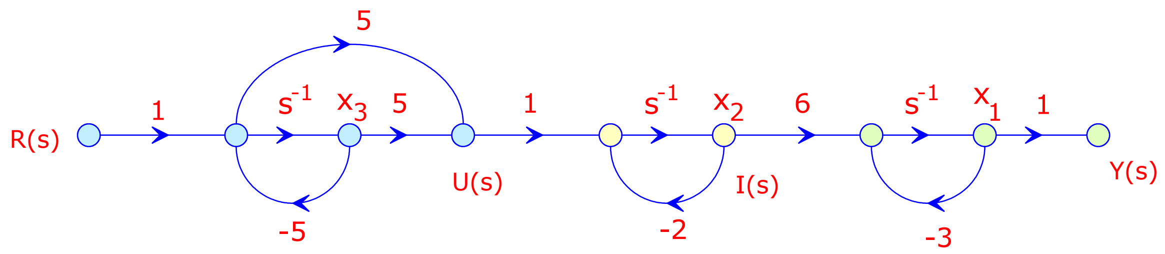
Alternative Signal-Flow Graph Models

Physical variables form

Often the control system designer studies an actual control system block diagram that represent physical devices and variables. An example of a model of a dc motor with shaft velocity as the output is shown. We wish to select the physical variables as state variables.



First, we are going to draw the physical state variable flow graph model. Model. This is done by considering each block separately.



The state variables are identified on the signal flow graph. Note that x_1 is the velocity, x_2 is the field current, and x_3 determines the field voltage ($u = 5r - 20x_3$).

The state variable differential equation is directly obtained from the signal flow graph as :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 0 \\ 0 & -2 & -20 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} r(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$