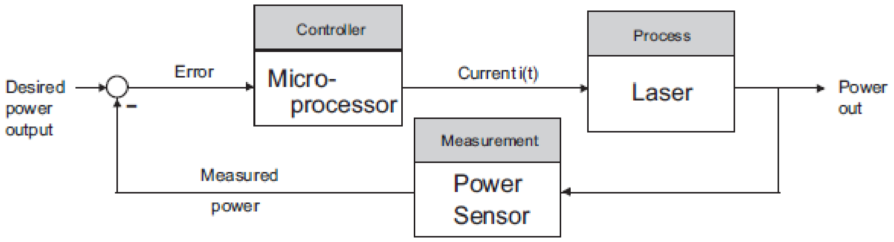
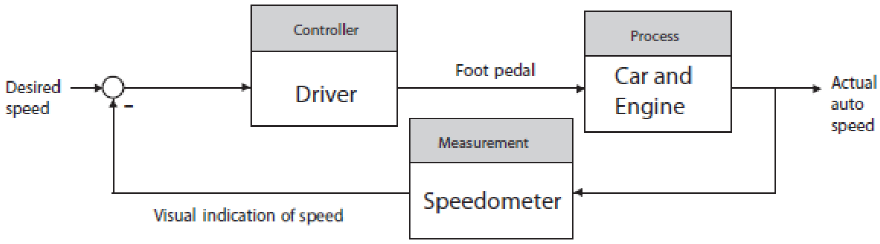
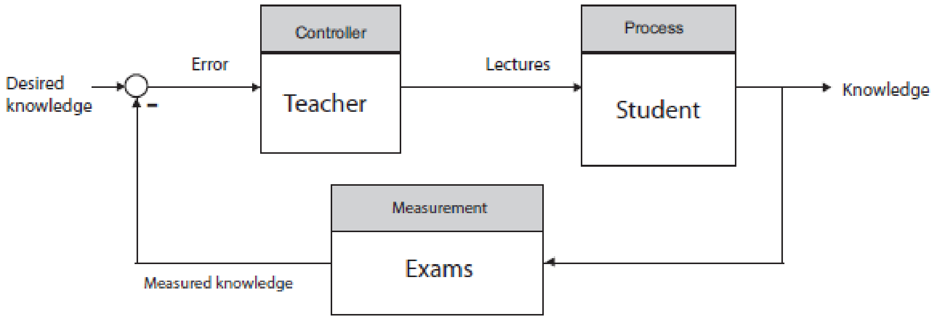
E1.1 A microprocessor controlled laser system:



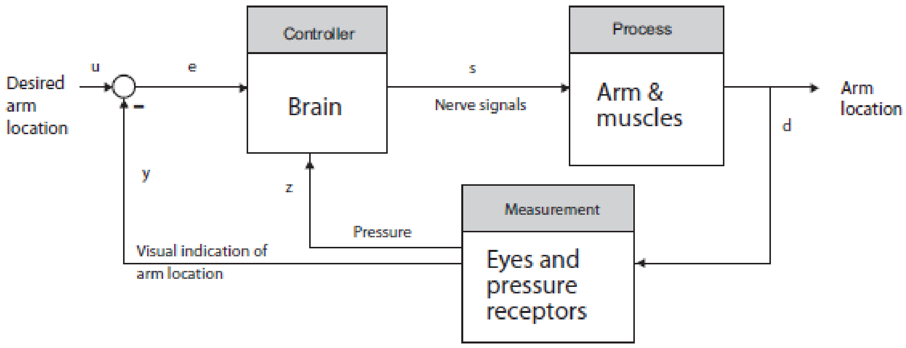
E1.2 A driver controlled cruise control system:



P1.8 The student-teacher learning process:



P1.9 A human arm control system:



E2.4 Since

$$R(s) = \frac{1}{s}$$

we have

$$Y(s) = \frac{4(s+50)}{s(s+20)(s+10)} \ .$$

The partial fraction expansion of $Y(s)$ is given by

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+20} + \frac{A_3}{s+10}$$

where

$$A_1 = 1 \ , \quad A_2 = 0.6 \text{ and } \quad A_3 = -1.6 \ .$$

Using the Laplace transform table, we find that

$$y(t) = 1 + 0.6e^{-20t} - 1.6e^{-10t} \ .$$

The final value is computed using the final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \left[\frac{4(s+50)}{s(s^2+30s+200)} \right] = 1 \ .$$

E2.8 The block diagram is shown in Figure E2.8.

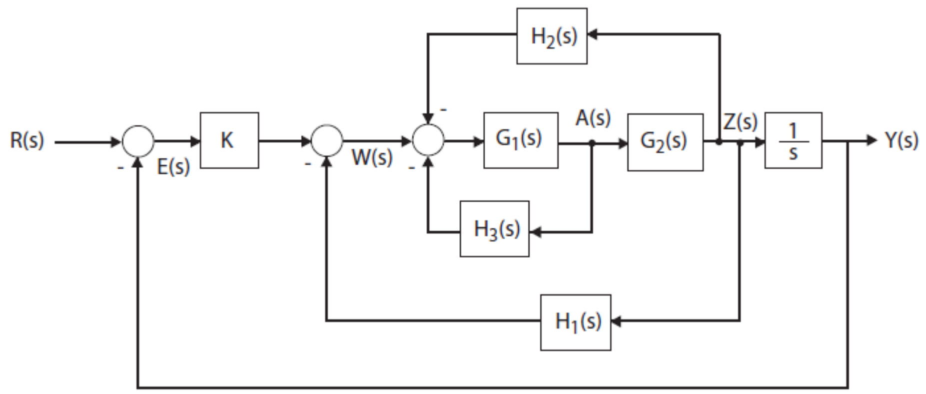


FIGURE E2.8
Block diagram model.

Starting at the output we obtain

$$Y(s) = \frac{1}{s}Z(s) = \frac{1}{s}G_2(s)A(s).$$

But $A(s) = G_1(s)\left[-H_2(s)Z(s) - H_3(s)A(s) + W(s)\right]$ and $Z(s) = sY(s)$, so

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)W(s).$$

Substituting $W(s) = KE(s) - H_1(s)Z(s)$ into the above equation yields

$$\begin{aligned} Y(s) = &-G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) \\ &+ \frac{1}{s}G_1(s)G_2(s)\left[KE(s) - H_1(s)Z(s)\right] \end{aligned}$$

and with $E(s) = R(s) - Y(s)$ and $Z(s) = sY(s)$ this reduces to

$$\begin{aligned} Y(s) = &\left[-G_1(s)G_2(s)\left(H_2(s) + H_1(s)\right) - G_1(s)H_3(s)\right. \\ &\left. - \frac{1}{s}G_1(s)G_2(s)K\right]Y(s) + \frac{1}{s}G_1(s)G_2(s)KR(s). \end{aligned}$$

Solving for $Y(s)$ yields the transfer function

$$Y(s) = T(s)R(s),$$

where

$$T(s) = \frac{KG_1(s)G_2(s)/s}{1 + G_1(s)G_2(s)\left[(H_2(s) + H_1(s)\right] + G_1(s)H_3(s) + KG_1(s)G_2(s)/s}.$$

E2.19 The input-output relationship is

$$\frac{V_o}{V} = \frac{A(K-1)}{1 + AK}$$

where

$$K = \frac{Z_1}{Z_1 + Z_2}.$$

Assume $A \gg 1$. Then,

$$\frac{V_o}{V} = \frac{K-1}{K} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = \frac{R_1}{R_1C_1s + 1} \quad \text{and} \quad Z_2 = \frac{R_2}{R_2C_2s + 1}.$$

Therefore,

$$\frac{V_o(s)}{V(s)} = -\frac{R_2(R_1C_1s + 1)}{R_1(R_2C_2s + 1)} = -\frac{2(s+1)}{s+2}.$$

E2.20 The equation of motion of the mass m_c is

$$m_c\ddot{x}_p + (b_d + b_s)\dot{x}_p + k_d x_p = b_d\dot{x}_{in} + k_d x_{in} \ .$$

Taking the Laplace transform with zero initial conditions yields

$$[m_c s^2 + (b_d + b_s)s + k_d]X_p(s) = [b_d s + k_d]X_{in}(s) \ .$$

So, the transfer function is

$$\frac{X_p(s)}{X_{in}(s)} = \frac{b_d s + k_d}{m_c s^2 + (b_d + b_s)s + k_d} = \frac{0.7s + 2}{s^2 + 2.8s + 2} \ .$$

E2.22 The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{K_1 K_2}{s^2 + (K_1 + K_2 K_3 + K_1 K_2)s + K_1 K_2 K_3} \ .$$

E2.25 The equations of motion are

$$m_1\ddot{x}_1 + k(x_1 - x_2) = F$$
$$m_2\ddot{x}_2 + k(x_2 - x_1) = 0 \ .$$

Taking the Laplace transform (with zero initial conditions) and solving for $X_2(s)$ yields

$$X_2(s) = \frac{k}{(m_2 s^2 + k)(m_1 s^2 + k) - k^2} F(s) \ .$$

Then, with $m_1 = m_2 = k = 1$, we have

$$X_2(s)/F(s) = \frac{1}{s^2(s^2 + 2)} \ .$$

P2.2 The differential equations describing the system can be obtained by using a free-body diagram analysis of each mass. For mass 1 and 2 we have

$$M_1\ddot{y}_1 + k_{12}(y_1 - y_2) + b\dot{y}_1 + k_1 y_1 = F(t)$$
$$M_2\ddot{y}_2 + k_{12}(y_2 - y_1) = 0 \ .$$

Using a force-current analogy, the analagous electric circuit is shown in Figure P2.2, where $C_i \to M_i$, $L_1 \to 1/k_1$, $L_{12} \to 1/k_{12}$, and $R \to 1/b$.

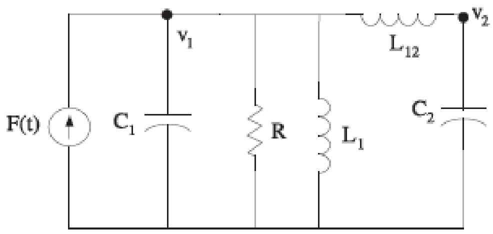


FIGURE P2.2
Analagous electric circuit.

P2.11 The transfer functions from $V_c(s)$ to $V_d(s)$ and from $V_d(s)$ to $\theta(s)$ are:

$$V_d(s)/V_c(s) = \frac{K_1K_2}{(L_qs + R_q)(L_cs + R_c)} \text{ , and}$$

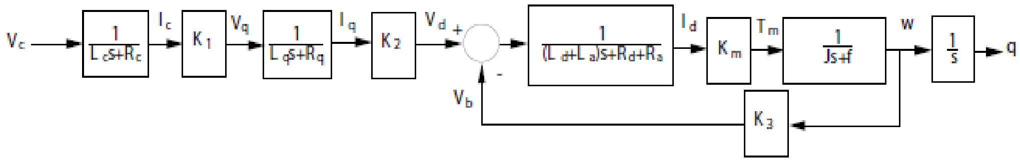
$$\theta(s)/V_d(s) = \frac{K_m}{(Js^2 + fs)((L_d + L_a)s + R_d + R_a) + K_3K_ms} \ .$$

The block diagram for $\theta(s)/V_c(s)$ is shown in Figure P2.11, where

$$\theta(s)/V_c(s) = \frac{\theta(s)}{V_d(s)} \frac{V_d(s)}{V_c(s)} = \frac{K_1K_2K_m}{\Delta(s)} \ ,$$

where

$$\Delta(s) = s(L_cs + R_c)(L_qs + R_q)((Js + b)((L_d + L_a)s + R_d + R_a) + K_mK_3) \ .$$



P2.13 The motor torque is given by

$$\begin{aligned} T_m(s) &= (J_ms^2 + b_ms)\theta_m(s) + (J_Ls^2 + b_Ls)n\theta_L(s) \\ &= n((J_ms^2 + b_ms)/n^2 + J_Ls^2 + b_Ls)\theta_L(s) \end{aligned}$$

where

$$n = \theta_L(s)/\theta_m(s) = \text{ gear ratio } .$$

But

$$T_m(s) = K_mI_g(s)$$

and

$$I_g(s) = \frac{1}{(L_g + L_f)s + R_g + R_f} V_g(s) \ ,$$

and

$$V_g(s) = K_gI_f(s) = \frac{K_g}{R_f + L_fs} V_f(s) \ .$$

Combining the above expressions yields

$$\frac{\theta_L(s)}{V_f(s)} = \frac{K_gK_m}{n\Delta_1(s)\Delta_2(s)} \ .$$

where

$$\Delta_1(s) = J_Ls^2 + b_Ls + \frac{J_ms^2 + b_ms}{n^2}$$

and

$$\Delta_2(s) = (L_gs + L_fs + R_g + R_f)(R_f + L_fs) \ .$$

P2.33 The signal flow graph shows three loops:

$$\begin{aligned} L_1 &= -G_1G_3G_4H_2 \\ L_2 &= -G_2G_5G_6H_1 \\ L_3 &= -H_1G_8G_6G_2G_7G_4H_2G_1 \ . \end{aligned}$$

The transfer function Y_2/R_1 is found to be

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_1G_8G_6\Delta_1 - G_2G_5G_6\Delta_2}{1-(L_1+L_2+L_3)+(L_1L_2)} \ ,$$

where for path 1

$$\Delta_1 = 1$$

and for path 2

$$\Delta_2 = 1 - L_1 \ .$$

Since we want Y_2 to be independent of R_1, we need $Y_2/R_1 = 0$. Therefore, we require

$$G_1G_8G_6 - G_2G_5G_6(1+G_1G_3G_4H_2) = 0 \ .$$

AP2.2 The closed-loop transfer function form $R_1(s)$ to $Y_2(s)$ is

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_1G_4G_5(s) + G_1G_2G_3G_4G_6(s)}{\Delta}$$

where

$$\Delta = [1+G_3G_4H_2(s)][1+G_1G_2H_3(s)] \ .$$

If we select

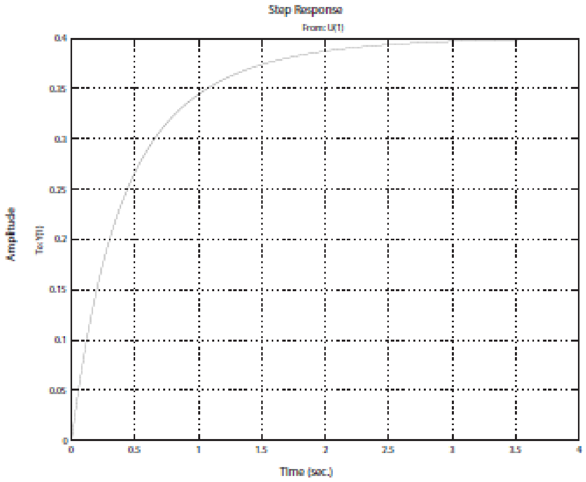
$$G_5(s) = -G_2G_3G_6(s)$$

then the numerator is zero, and $Y_2(s)/R_1(s) = 0$. The system is now decoupled.

CP2.2 The m-file script and step response is shown in Figure CP2.2.

```
numc = [1]; denc = [1 1]; sysc = tf(numc,denc)
numg = [1 2]; deng = [1 3]; sysg = tf(numg,deng)
% part (a)
sys_s = series(sysc,sysg);
sys_cl = feedback(sys_s,[1])
% part (b)
step(sys_cl); grid on
```

```
Transfer function:
  s + 2
-------------
s^2 + 5 s + 5
```



CP2.3 Given

$$\ddot{y} + 4\dot{y} + 4y = u$$

with $y(0) = \dot{y} = 0$ and $U(s) = 1/s$, we obtain (via Laplace transform)

$$Y(s) = \frac{1}{s(s^2 + 4s + 4)} = \frac{1}{s(s+2)(s+2)} .$$

Expanding in a partial fraction expansion yields

$$Y(s) = \frac{0.25}{s} - \frac{0.25}{s+2} - \frac{0.5}{(s+2)^2} .$$

Taking the inverse Laplace transform we obtain the solution

$$y(t) = 0.25 - 0.25e^{-2t} - 0.5te^{-2t} .$$

The m-file script and step response is shown in Figure CP2.3.

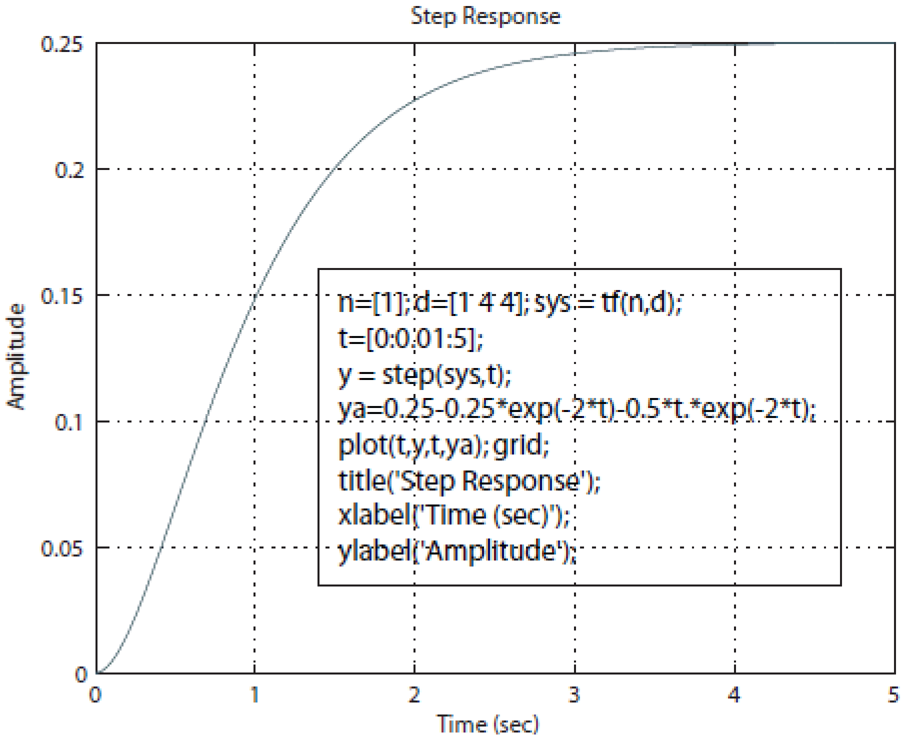


FIGURE CP2.3
Step response.