Problem 1

Consider the following second-order system with an extra pole:

$$H(s) = \frac{\omega_n^2 p}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

Show that the unit step response is

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta),$$

where

$$A = \frac{-\omega_n^2}{\omega_n^2 - 2\zeta\omega_n p + p^2},$$

$$B = \frac{p}{\sqrt{(p^2 - 2\zeta\omega_n p + \omega_n^2)(1-\zeta^2)}},$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{p - \zeta\omega_n}.$$

(a) Which term dominates $y(t)$ as p gets large?

(b) Give approximate values for A and B for small values of p.

(c) Which term dominates as p gets small? (Small with respect to what?)

(d) Using the preceding explicit expression for $y(t)$ or the step command in MATLAB, and assuming that $\omega_n = 1$ and $\zeta = 0.7$, plot the step response of the preceding system for several values of p ranging from very small to very large. At what point does the extra pole cease to have much effect on the system response?

Problem 2

Suppose that unity feedback is to be applied around the listed open-loop systems. Use Routh's stability criterion to determine whether the resulting closed-loop systems will be stable.

(a) $KG(s) = \frac{4(s+2)}{s(s^3+2s^2+3s+4)}$

(b) $KG(s) = \frac{2(s+4)}{s^2(s+1)}$

(c) $KG(s) = \frac{4(s^3+2s^2+s+1)}{s^2(s^3+2s^2-s-1)}$

Problem 3

Use Routh's stability criterion to determine how many roots with positive real parts the following equations have:

(a) $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$.

(b) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$.

(c) $s^4 + 2s^3 + 7s^2 - 2s + 8 = 0$.

(d) $s^3 + s^2 + 20s + 78 = 0$.

(e) $s^4 + 6s^2 + 25 = 0$.

Problem 4

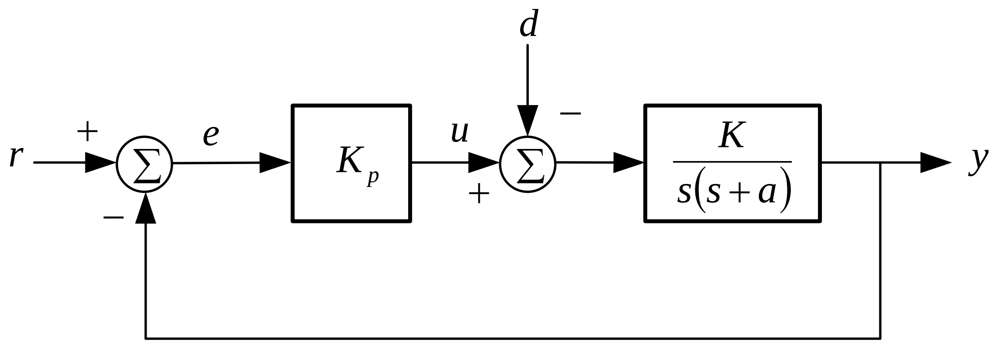
Find the range of K for which all the roots of the following polynomial are in the LHP:

$$s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + K = 0.$$

Use MATLAB to verify your answer by plotting the roots of the polynomial in the s-plane for various values of K.

Problem 5

Steady State Error Analysis for a P-Controlled System



Consider the P-controlled system in the above figure.

a) Express the output $y(s)$ as a function of the command input $r(s)$ and the disturbance $d(s)$.

b) Express the following error $e(s)$ as a function of the command input $r(s)$ and the disturbance $d(s)$.

c) Determine the steady-state value of the following error $e_{ss} = \lim_{t \to 0} e(t)$ when the command is a ramp input $r(s) = A/s^2$, and the disturbance is a step $d(s) = D/s$. (*Use the Final Value Theorem*).

d) Discuss how the natural frequency ω_n and damping ratio ζ of the closed loop poles affects the steady state error.