بسم الله الرحمن الرحيم

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF ELECTRICAL ENGINEERING

EE 380 **CONTROL ENGINEERING**

MAJOR EXAM I

TIME: 5:15 p.m. **November 8, 2009**

NAME :	
I.D. # :	
SEC # :	

QUESTION #	SCORE	MAXIMUM
1.		20
2.		30
3.		25
4		25
TOTAL		100

Instructor: Dr. J. M. BAKHASHWAIN

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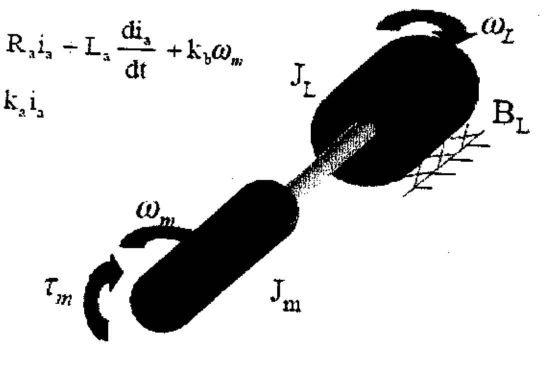
2. Draw the block diagram for armature controlled dc motor. Show that the transfer function is given by $\dfrac{\omega_m(s)}{V_a(s)} = \dfrac{a}{s^2+bs+c}$

(i) Find a, b, and c

(ii) Find the state space representation when the state vector $x = (i_a \;\; \omega_m \;\; \theta_m)^T$

$V_a(t) = R_a i_a + L_a \dfrac{di_a}{dt} + k_b\omega_m$

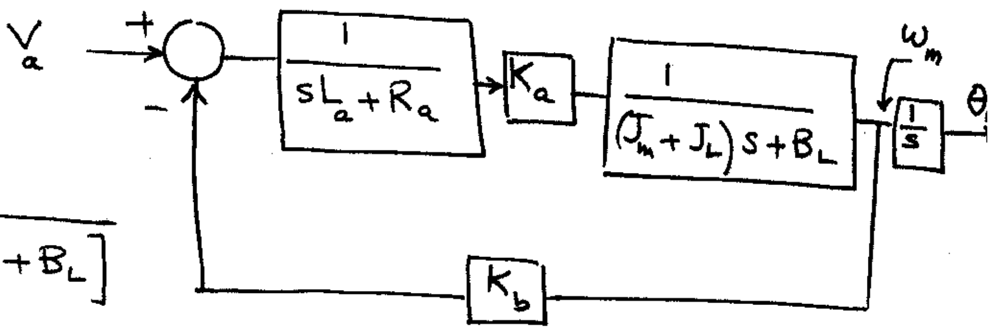
$\tau_m = k_a i_a$



$$V_a(t) = R_a i_a(t) + L_a \frac{di_a}{dt} + K_b\omega_m$$

$$\tau_m = K_a i_a = (J_m + J_L)\ddot{\theta}_m + B_L\dot{\theta}_m$$

$$\frac{\omega_m}{V_a} = \frac{G}{1+GH}$$



$$G = \frac{K_a}{(sL_a+R_a)\left[(J_m+J_L)s+B_L\right]}$$

$$H = K_b$$

$$\therefore \frac{\omega_m(s)}{V_a(s)} = \frac{K_a}{(sL_a+R_a)\left[(J_m+J_L)s+B_L\right] + K_aK_b}$$

$$= \frac{K_a}{L_a(J_m+J_L)s^2 + \left[R_a(J_m+J_L)+L_aB_L\right]s + (R_aB_L+K_aK_b)}$$

$$= \frac{\dfrac{K_a}{L_a(J_m+J_L)}}{s^2 + \left(\dfrac{R_a}{L_a}+\dfrac{B_L}{J_m+J_L}\right)s + \left(\dfrac{R_aB_L+K_aK_b}{L_a(J_m+J_L)}\right)}$$

$$\therefore\ a = \frac{K_a}{L_a(J_m+J_L)}, \quad b = \frac{R_a}{L_a} + \frac{B_L}{J_m+J_L}$$

$$c = \frac{R_aB_L + K_aK_b}{L_a(J_m+J_m)}$$

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a}\omega_m + \frac{V_a}{L_a} \quad \text{— (1)}$$

$$K_a i_a = (J_m + J_L)\dot{\omega}_m + B_L \omega_m$$

$$\dot{\omega}_m = \frac{K_a}{J_m + J_L} i_a - \frac{B_L}{J_m + J_L}\omega_m \quad \text{— (2)}$$

$$\dot{\theta}_m = \omega_m$$

$$X = \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix}$$

$$\therefore \quad \dot{X}_1 = -\frac{R_a}{L_a} X_1 - \frac{K_b}{L_a} X_2 + \frac{1}{L_a} V_a$$

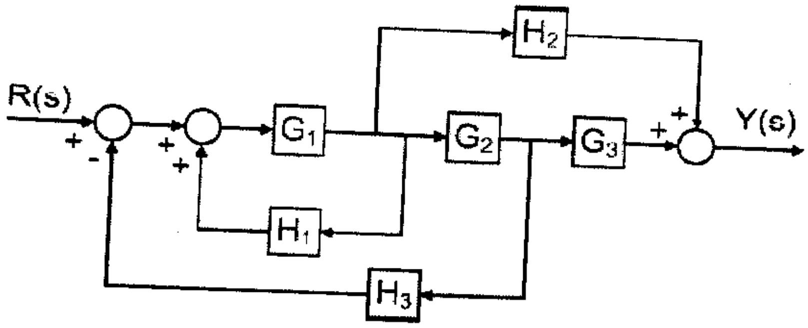
$$\dot{X}_2 = \frac{K_a}{J_m + J_L} X_1 - \frac{B_L}{J_m + J_L} X_2$$

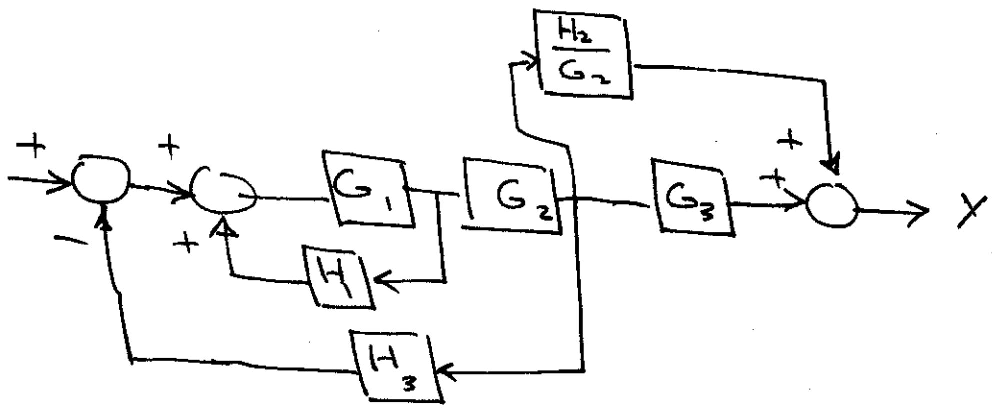
$$\dot{X}_3 = X_2$$

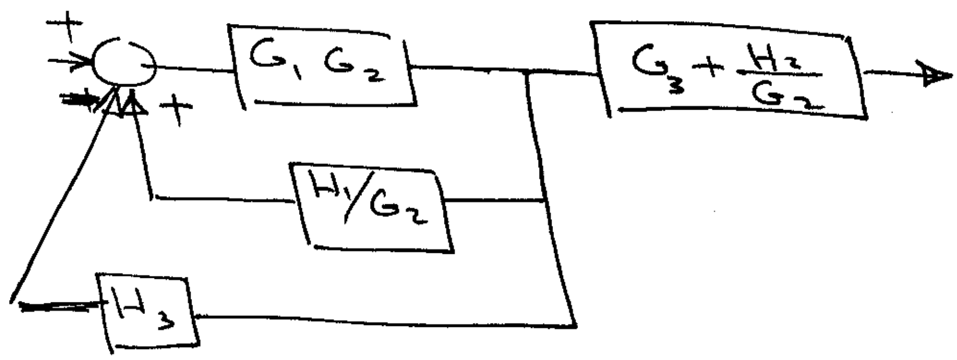
$$\therefore \quad \underline{\dot{X}} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_a}{J_m + J_L} & -\frac{B_L}{J_m + J_L} & 0 \\ 0 & 1 & 0 \end{bmatrix} \underline{X} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} V_a$$

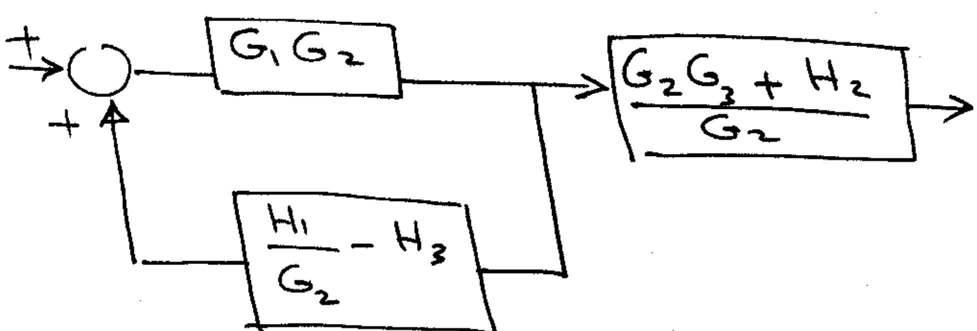
$$\underline{Y} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} X$$

3. Simplify the block diagram shown and obtain the closed-loop transfer function $\frac{Y(s)}{R(s)}$.





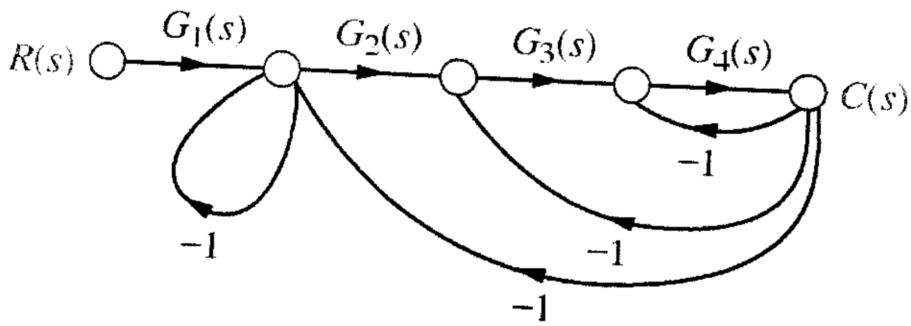




$$\frac{G_1 \cancel{G_2}}{1 - G_1 G_2\left(\frac{H_1}{G_2} - H_3\right)} * \left[\frac{G_2 G_3 + H_2}{\cancel{G_2}}\right]$$

$$\frac{G_1 (G_2 G_3 + H_2)}{1 - G_1 H_1 + G_1 G_2 H_3} \quad *$$

4. Using Mason's gain formula, find the transfer function for the shown signal flow graph



$P_1 = G_1 G_2 G_3 G_4$, $\Delta_1 = 1$

$L_1 = -1$

$L_2 = -G_2 G_3 G_4$

$L_3 = -G_3 G_4$

$L_4 = -G_4$

L_1 & L_3 nontouching

L_1 & L_4 "

$$\therefore \quad \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_3 + L_1 L_4$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + [1 + G_2 G_3 G_4 + G_3 G_4 + G_4] + (-1)(-G_3 G_4) + (-1)(-G_4)}$$

$$= \frac{G_1 G_2 G_3 G_4}{2 + G_2 G_3 G_4 + 2 G_3 G_4 + 2 G_4} \quad *$$

1. Find the value of a for the following system

$$\dot{x} = \begin{pmatrix} a & a \\ a & a \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

If $y(t) = 0.5(1 - e^t + te^t)$ for an input $u(t) = e^t$. Assume zero initial conditions

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$$

$$= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s-a & -a \\ -a & s-a \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad , D = 0$$

$$= \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{1}{\Delta(s)} \begin{bmatrix} s-a & a \\ a & s-a \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{a}{\Delta(s)}$$

$$\Delta(s) = |sI - A| = (s-a)^2 - a^2 = s^2 - 2as$$

$$\therefore \quad \frac{Y(s)}{U(s)} = \frac{a}{s^2 - 2as} = \frac{a}{s(s-2a)} \quad *$$

$$y(t) = \frac{1}{2}(1 - e^t + te^t) \Rightarrow Y(s) = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s-1} + \frac{1}{(s-1)^2}\right]$$

$$Y(s) = \frac{1}{2}\left[\frac{(s-1)^2 - s(s-1) + s}{s(s-1)^2}\right] = \frac{1}{2}\left[\frac{s^2 - 2s + 1 - s^2 + s + s}{s(s-1)^2}\right] = \frac{1/2}{s(s-1)^2}$$

$$u(t) = e^t \Rightarrow U(s) = \frac{1}{s-1}$$

$$\therefore \quad \frac{Y(s)}{U(s)} = \frac{1/2}{s(s-1)} \quad (**)$$

From $(*)$ & $(**)$ $\quad \dfrac{a}{s(s-2a)} = \dfrac{1/2}{s(s-1)}$

$$\Rightarrow \boxed{a = 1/2}$$