(a) Find the expression for L(s).

(b) How many branches does the root-locus have?

(c) What is the value of K at the poles?

(d) What is the value of K at the zeros?

(e) Sketch and label the real-axis parts of the locus.

(f) Compute, mark and label the asymptote center.

(g) Compute, sketch and label the asymptotes.

(h) Compute the departure angles from the poles.

(i) Compute the arrival angles at the zeros.

(j) How does the arrival angle help you sketch the root-locus?

(k) If the locus crosses the imaginary axis, find ω and K at the crossing point.

(l) Compute K for real values of s in the range where you suspect a break point.

(m) Sketch the general shape of the locus. See Figure 1.

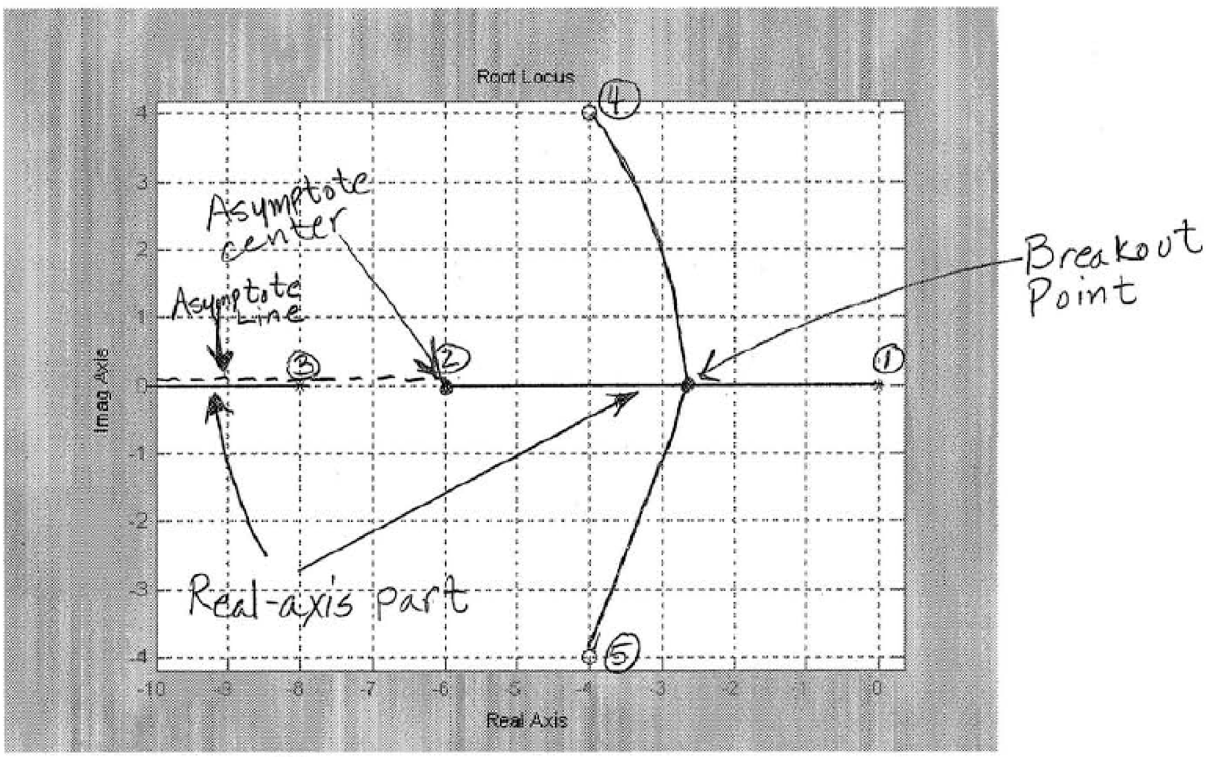

Figure 1

(a) $L(s) = \frac{(s+4-4j)(s+4+4j)}{s(s+6)(s+8)} = \frac{(s+4)^2+16}{s(s+6)(s+8)}$

(b) There are 3 poles, therefore there are 3 branches.

(c) $K = 0$ at the poles

(d) $K = \infty$ at the zeros.

(e) See the graph marked Figure 1.

(f) $\alpha = \frac{-6-8-(-4)(-4)}{1} = -6$ see graph marked Figure 1.

(g) $\phi_1 = 180^\circ$ see graph marked Figure 1.

(h) $\phi_{1Dep} = 180^\circ$ by inspection and Rule #1

$\phi_{2Dep} = 0^\circ$ by inspection and Rule #1

$\phi_{3Dep} = 180^\circ$ by inspection and Rule #1

(i) $\psi_{4Arr} = \phi_{14} + \phi_{24} + \phi_{34} - \psi_{54} - 180^\circ$

$\phi_{14} = 135^\circ$

$\phi_{24} = \tan^{-1}\left(\frac{4}{2}\right) = 63.43^\circ$

$\phi_{34} = \tan^{-1}\left(\frac{4}{4}\right) = 45^\circ$

$\psi_{54} = 90^\circ$

$\psi_{4Arr} = 135 + 63.43^\circ + 45 - 90 - 180^\circ = -26.57^\circ$

$\psi_{5Arr} = 26.57^\circ$ by symmetry

(j) It tells you that the break-out point occurs for $s < 4$ otherwise the locus would arrive at the zeros at an angle $< -90°$.

(k) The locus does not cross the imaginary axis because the only asymptote is at 180°.

(l) The locus breaks-out of the real-axis somewhere between $s = -2$ and $s = -3$, as shown in Figure 2.

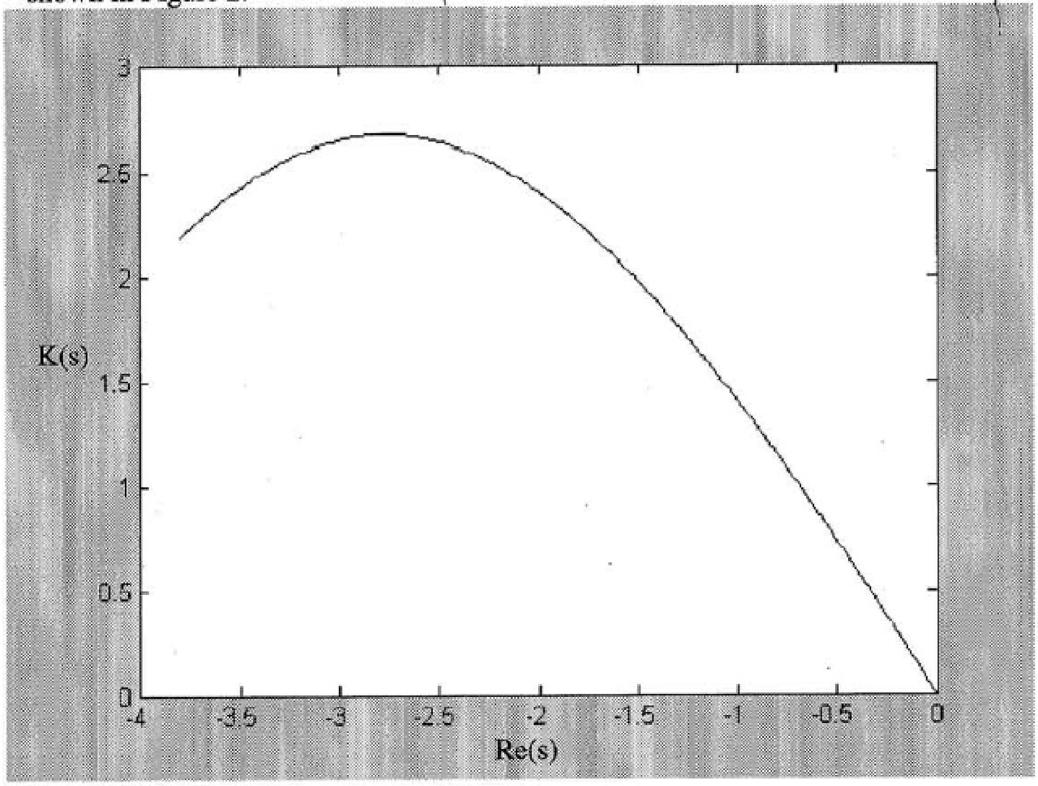

Figure 2

(m) A sketch of the root-locus of the system is shown in Figure 1.

(a) Find the expression for L(s).

(b) Sketch and label the real-axis parts of the locus.

(c) Compute, mark and label the asymptote center.

(d) Compute, sketch and label the asymptotes.

(e) If the locus crosses the imaginary axis, find ω and K at the crossing point.

(f) Sketch the general shape of the locus.

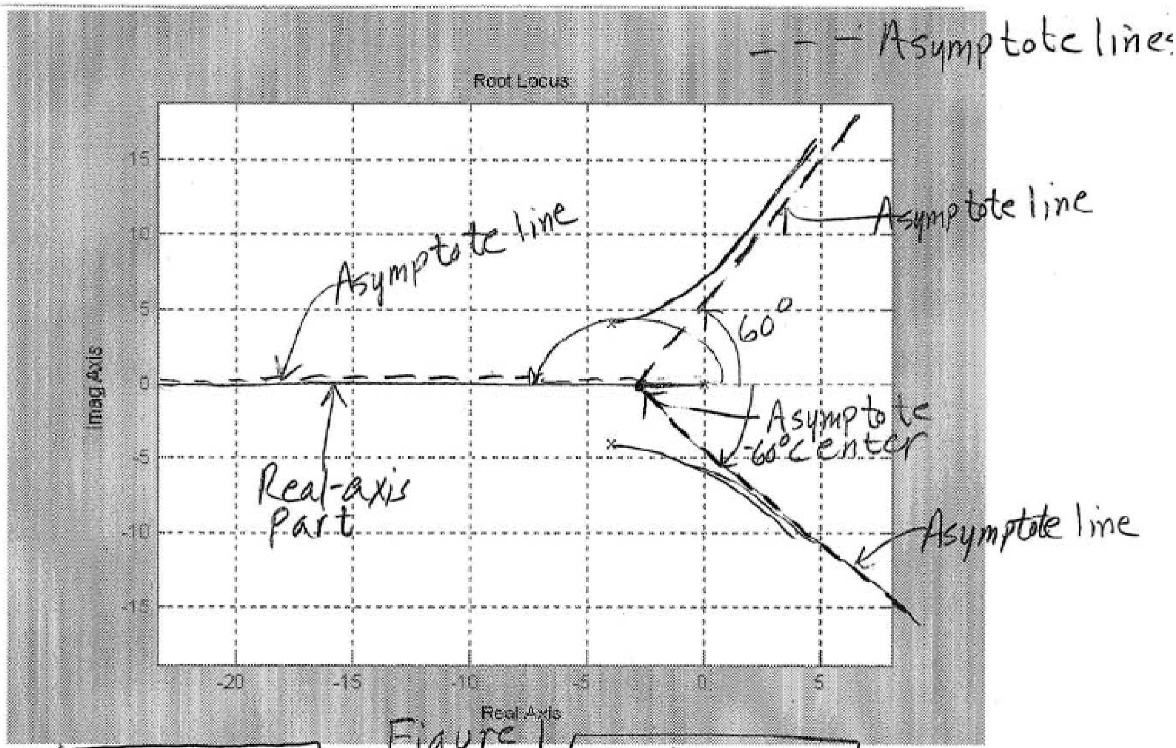

Figure 1

(a) $L(s) = \frac{1}{s(s+4+4j)(s+4-4j)} = \frac{1}{s(s^2+8s+32)}$

(b) See graph marked Figure 1.

(c) $\alpha = \frac{-4-4}{3} = -\frac{8}{3}$ see graph marked Figure 1.

(d) $\phi_1 = 60^\circ \quad \phi_3 = -60^\circ$

$\phi_2 = 180^\circ$

Cont'd.

(e) The locus crosses the $j\omega$ axis.

$$1 + KL(j\omega) = 0$$

$$(j\omega)^3 + 8(j\omega)^2 + 32j\omega + K = 0$$

$$-j\omega^3 - 8\omega^2 + 32j\omega + K = 0$$

$$8\omega^2 - K = 0$$

$$\omega^2 - 32 = 0$$

$$\omega^2 = 32$$

$$\boxed{K = (32)(8) = 256}$$

$$\boxed{\omega = \pm\sqrt{32}}$$

(f) See Figure 1

9. Using the pole-zero plot shown below, do the following for the positive root-locus:

(a) Find the expression for L(s).

(b) Sketch and label the real-axis parts of the locus.

(c) Compute, mark and label the asymptote center.

(d) Compute, sketch and label the asymptotes.

(e) If the locus crosses the imaginary axis, find ω and K at the crossing point.

(f) Plot K for real-values of s. Where is the breakaway point?

(g) Sketch the general shape of the locus.

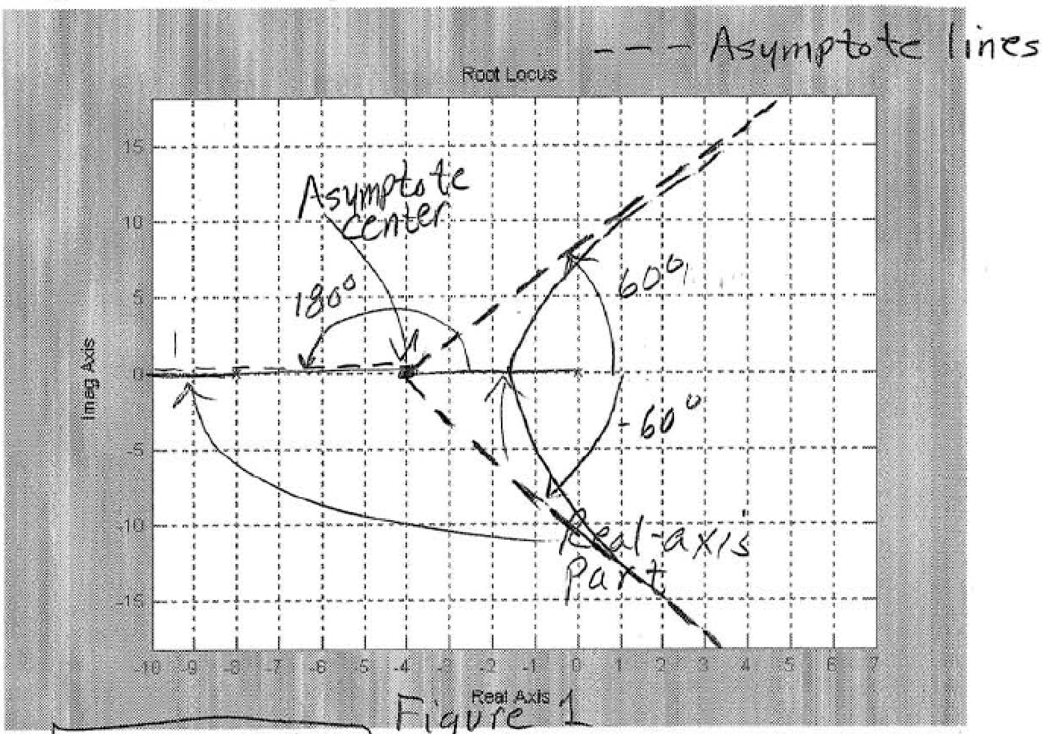

Figure 1

(a) $L(s) = \frac{1}{s(s+8)(s+4)}$

(b) See graph marked Figure 1.

(c) $\alpha = \frac{-4-8}{3} = -4$

(d) $\phi_1 = 60°$ $\phi_3 = -60°$
$\phi_2 = 180°$

Cont'd.

(e) It crosses the $j\omega$ axis.

$$1 + KL(j\omega) = 0$$

$$(j\omega)^3 + 12(j\omega)^2 + 32j\omega + K = 0$$

$$-j\omega^3 - 12\omega^2 + 32j\omega + K = 0$$

$$K = 12\omega^2$$

$$\omega^2 = 32$$

$$\boxed{\omega = \pm\sqrt{32}}$$

$$\boxed{K = (12)(32)}$$

(f) See attached graph marked Figure 2.

(g) See graph marked Figure 1.

. Cont'd.

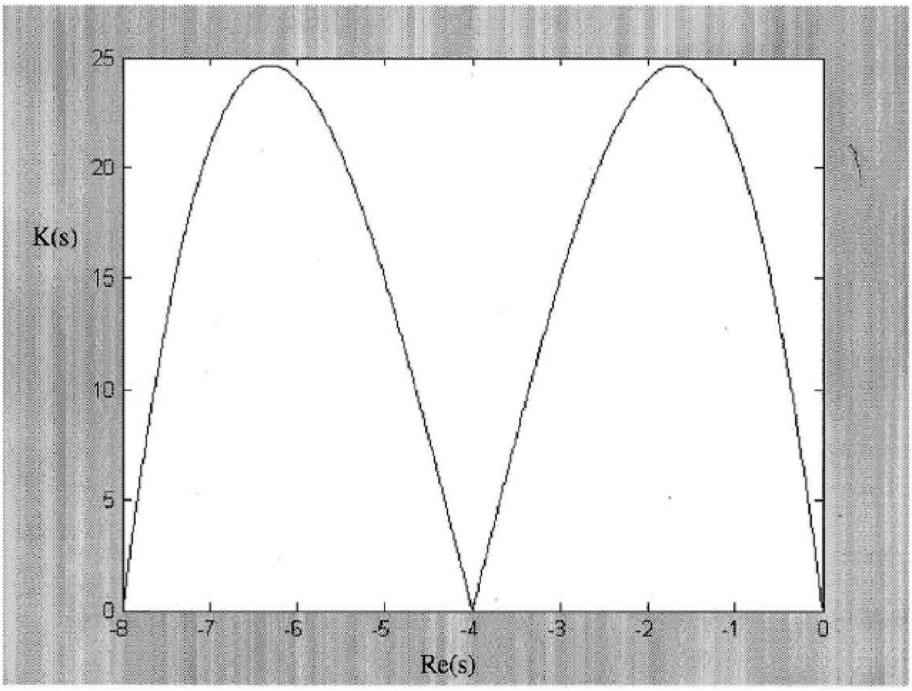

Figure 2

The plot shows two maxima.
For positive K, the real-axis part of the locus lies in the segment [0,-4]. The maximum in this segment indicates a break-out point at about -1.75.
For negative K, the real-axis part of the locus lies in the segment [-4,-8]. The maximum in this segment indicates a break-out point at about -6.25.

EE 380: Home Work #5

E7.1 (a) For the characteristic equation

$$1 + K\frac{s(s+4)}{s^2+2s+2} = 0 ,$$

the root locus is shown in Figure E7.1.

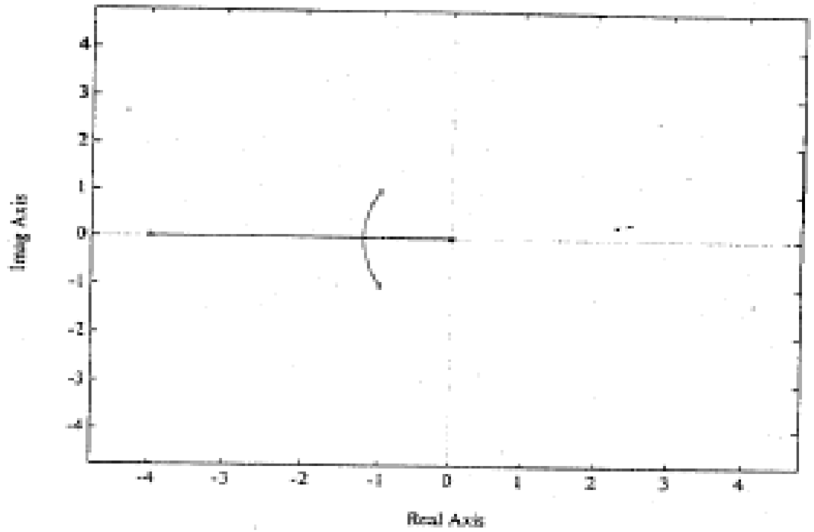

FIGURE E7.1
Root locus for $1 + K\frac{s(s+4)}{s^2+2s+2} = 0$.

(b) The system characteristic equation can be written as

$$(1+K)s^2 + (2+4K)s + 2 = 0 .$$

Solving for s yields

$$s = \frac{-(1+2K)}{(1+K)} \pm \frac{\sqrt{(2+4K)^2 - 8(1+K)}}{2(1+K)} .$$

When

$$(2+4K)^2 - 8(1+K) = 0 ,$$

then we have two roots at $s_{1,2} = -\frac{(1+2K)}{1+K}$. Solving for K yields $K = 0.31$.

(c) When $K = 0.31$, the roots are

$$s_{1,2} = \frac{-(1+0.62)}{(1.31)} = -1.24 .$$

(d) When $K = 0.31$, the characterisitc equation is

$$s^2 + 2.472s + 1.528 = (s+1.24)^2 = 0 .$$

Thus, $\omega_n = 1.24$ and $\zeta = 1$, the system is critically damped. The settling time is $T_s \approx 4$ sec.

E7.2 (a) The root locus is shown in Figure E7.2. When $K = 6.5$, the roots of the characteristic equation are

$$s_{1,2} = -2.65 \pm j1.23 \quad \text{and} \quad s_{3,4} = -0.35 \pm j0.8 \ .$$

The real part of the dominant root is 8 times smaller than the other two roots.

(b) The dominant roots are

$$(s + 0.35 + j0.8)(s + 0.35 - j0.8) = s^2 + 0.7s + 0.7625 \ .$$

From this we determine that

$$\omega_n = 0.873 \quad \text{and} \quad \zeta = \frac{0.7}{2(0.873)} = 0.40 \ .$$

Thus, the settling time is

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.35} = 11.43 \text{ sec} \ .$$

The percent overshoot is $P.O. = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 25.4\%$.

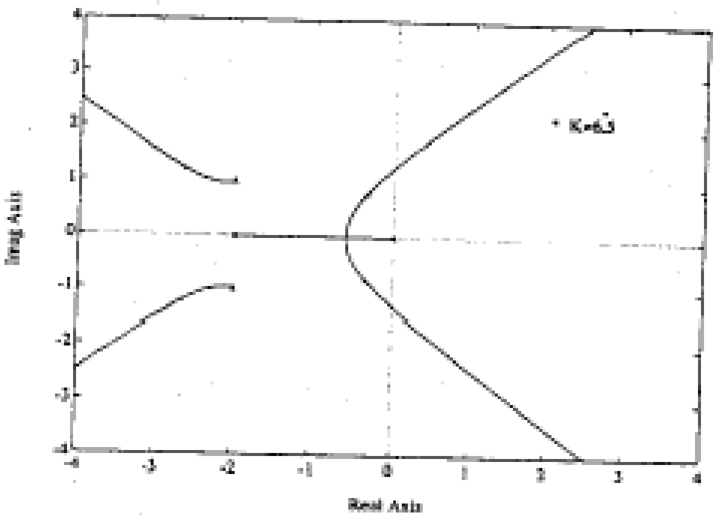

FIGURE E7.2
Root locus for $1 + K\frac{1}{s(s+2)(s^2+4s+5)} = 0$.

E 7.8

The characteristic equation is

$$1 + K\frac{(s+1)}{s^2(s+9)} = 0 ,$$

or

$$s^3 + 9s^2 + Ks + K = 0 .$$

For all the roots to be equal and real, we require

$$(s+r)^3 = s^3 + 3rs^2 + 3r^2 s + r^3 = 0 .$$

Equating terms and solving for K yields $K = 27$. All three roots are equal at $s = -3$, when $K = 27$. The root locus is shown in Figure E7.8.

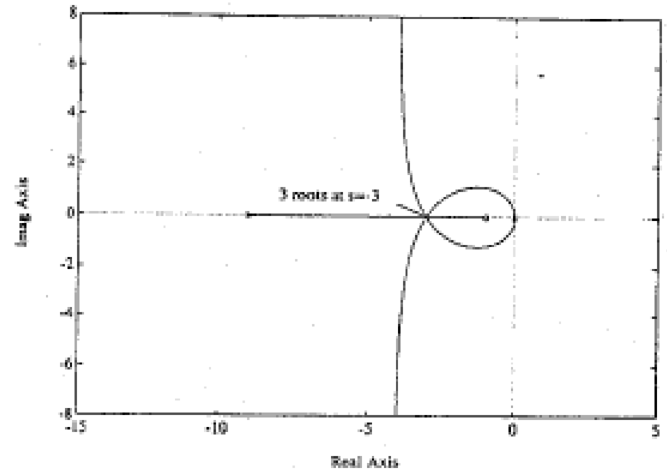

FIGURE E7.8
Root locus for $1 + K\frac{s+1}{s^2(s+9)} = 0$.

E7.24 The transfer function is

$$\begin{aligned} G(s) &= \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \\ &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 2 & s+k \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{s^2 + ks + 2} . \end{aligned}$$

Therefore, the characteristic equation is

$$s^2 + ks + 2 = 0 ,$$

or

$$1 + k\frac{s}{s^2 + 2} = 0 .$$

The root locus for $0 < k < \infty$ is shown in Figure E7.24. The closed-loop system is stable for all $0 < k < \infty$.

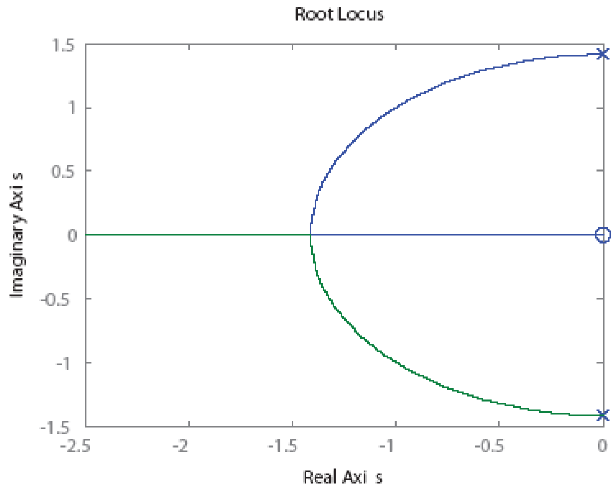

FIGURE E7.24
Root locus for $1 + k\frac{s}{s^2+2} = 0$.

P7.1

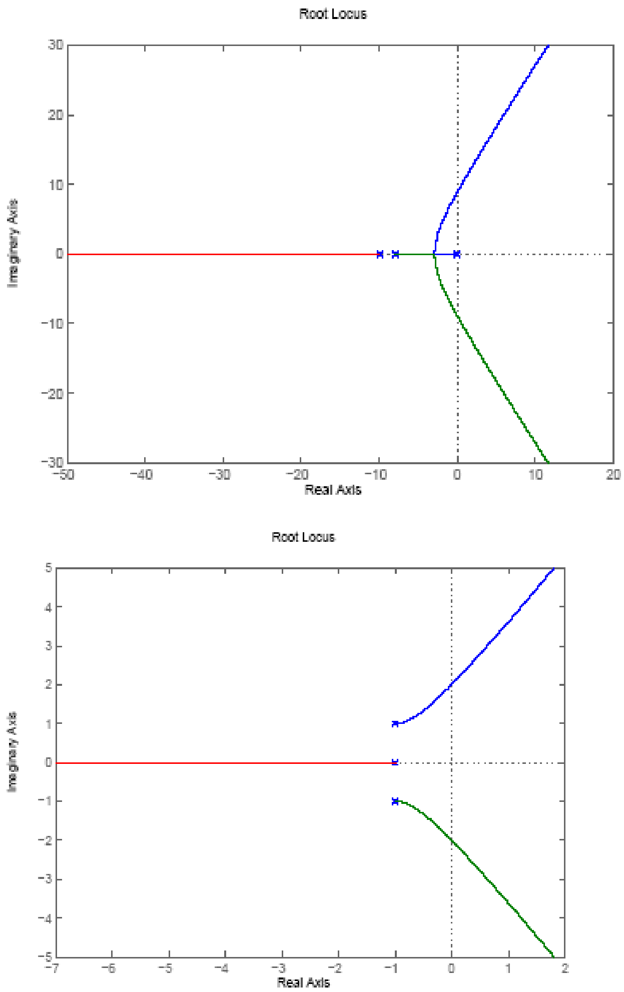

FIGURE P7.1
(a) Root locus for $1 + \frac{K}{s(s+10)(s+8)} = 0$, and (b) $1 + \frac{K}{(s^2+2s+2)(s+1)} = 0$.

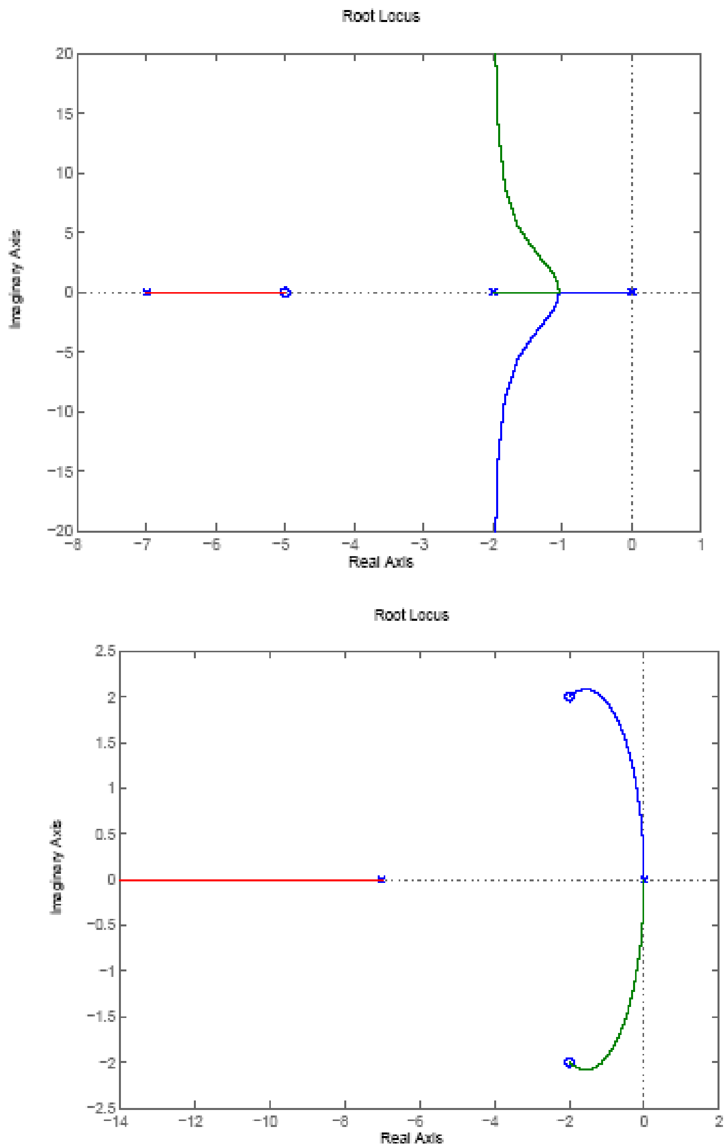

FIGURE P7.1
CONTINUED: (c) Root locus for $1 + \frac{K(s+5)}{s(s+2)(s+7)} = 0$, and (d)$1 + \frac{K(s^2+4s+8)}{s^2(s+7)} = 0$.

MP7.4 The characteristic equation is

$$1 + p\frac{s-1}{s^2+4s+6} = 0 .$$

The root locus is shown in Figure MP7.4. The closed-loop system is stable for

$$0 < p < 6 .$$

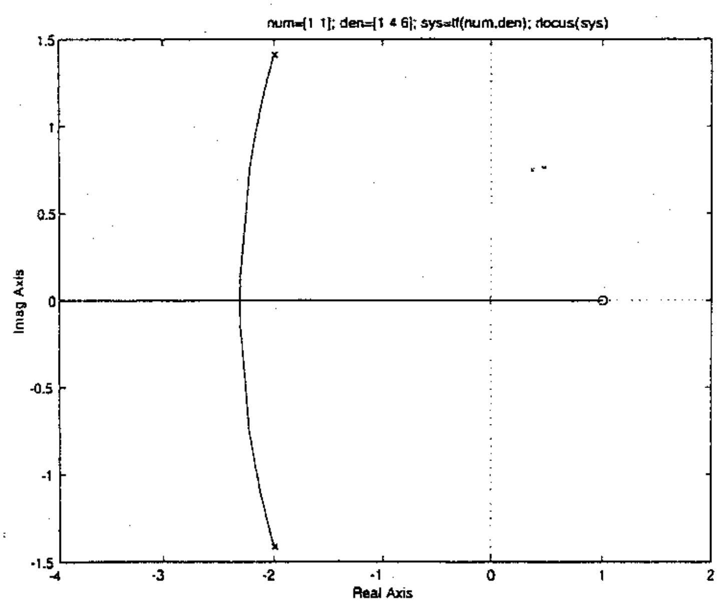

FIGURE MP7.4
Root locus for $1 + p\frac{s-1}{s^2+4s+6} = 0$.