KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE380 [081]	Quiz # 1
Name: Key Solution ID: _____	Grade: _____

Given the following d.e., find its solution using L.T. Assume zero initial conditions

$\ddot{y}+4\dot{y}+3y=1$ By L.T $\Rightarrow (s^2+4s+3)\,Y(s)=\frac{1}{s}$

$\therefore Y(s)=\frac{1}{s(s^2+4s+3)}=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{(s+3)}$

$A=\lim_{s\to 0} sY(s)=\frac{1}{3}$, $B=\lim_{s\to -1}(s+1)Y(s)=-\frac{1}{2}$

$C=\lim_{s\to -3}(s+3)Y(s)=\frac{1}{6}$

$\therefore y(t)=\frac{1}{3}-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}$ $t\geq 0$ (*)

Ch Eqn: $s^2+4s+3=0 \Rightarrow (s+1)(s+3)=0 \Rightarrow s=-1,-3$

$\therefore$ It is overdamped case

$y_n(t)=A_1e^{-t}+A_2e^{-3t}$ natural response

$y_f(t)=B \Rightarrow$ from d.e $\Rightarrow 3y_f(t)=1 \Rightarrow y_f=\frac{1}{3}$

$\therefore y=y_n(t)+y_f=\frac{1}{3}+A_1e^{-t}+A_2e^{-3t}$

$y(0)=0 \Rightarrow A_1+A_2=-1/3$ — ①

$\dot{y}(0)=0 \Rightarrow -A_1-3A_2=0$ — ②

From ① & ② $\Rightarrow A_2=+1/6 \Rightarrow A_1=-\frac{1}{2}$

$\therefore y(t)=\frac{1}{3}-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}$ (**)

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EE380 [081] sec# _____ Quiz # 1

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Given the following d.e., find its solution using L.T. Assume zero initial conditions

$\ddot{y} + 3\dot{y} + 2y = 1$

Verify your result by finding the solution using the method of EE 205 (i.e. finding the characteristic equation and assuming the solution accordingly)

$$s^2 Y(s) + 3sY(s) + 2Y(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s(s+1)(s+2)}$$

$$Y(s) = \frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2} \Rightarrow y(t) = 1 - e^{-t} + \frac{1}{2}e^{-2t}$$

Ch Eqn: $s^2 + 3s + 2 = 0 \Rightarrow s = -1, -2$

$y_n(t) = A_1 e^{-t} + A_2 e^{-2t}$

$y_f(t) = \frac{1}{2}$

$y(t) = y_n(t) + y_f(t) = \frac{1}{2} + A_1 e^{-t} + A_2 e^{-2t}$

$y(0) = \frac{1}{2} + A_1 + A_2 = 0$

$\dot{y}(0) = -A_1 - 2A_2 = 0$ $\Rightarrow \frac{1}{2} - A_2 = 0 \Rightarrow A_2 = \frac{1}{2}$

$\therefore A_1 = -2A_2 = -1$

$\therefore y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$