Q1 $s^4 + 2s^3 + 10s^2 + 20s + 5 = 0$

s^4	1	10	5	
s^3	2	20	0	
s^2	$\frac{20-20}{2} = \cancel{0}^{*}\ \epsilon$	5	0	$*$ (replace by ϵ)
s^1	$\frac{20\epsilon - 10}{\epsilon} \simeq -\frac{10}{\epsilon}$	0		
s^0	5			

Two sign changes in 1^{st} column $\Rightarrow$ Unstable

Two roots in RHP

Q2 $s^4 + 25s^3 + 15s^2 + 20s + K = 0$

s^4	1	15	K
s^3	25	20	0
s^2	$\frac{25\times15-20}{25} = 14.2$	K	
s^1	$\frac{14.2\times20-25K}{14.2}$	0	
s^0	K		

✓ for stable system : $K > 0$ & $K < \frac{20}{1.76}$

$K < 11.36$

$\therefore \quad 0 < K < 11.36$

✓ For a marginally stable system :

$K = 11.36$

A.E. $\quad 14.2 s^2 + 11.36 = 0$

$\therefore \quad s^2 = -\frac{11.36}{14.2} = -0.8$

$s = \mp j\sqrt{0.8} = \mp j\,0.894$

$\therefore \quad \omega = 0.894$ rad/s ; $f = 0.142$ Hz

Q3. C.E. $s^2(s+2) + K(s+10)(s+20) = 0$

$$s^3 + (2+K)s^2 + 30Ks + 200K = 0$$

s^3	1	$30K$	
s^2	$2+K$	$200K$	$\rightarrow K > -2$
s^1	$\dfrac{30K^2 - 140K}{2+K}$	0	$\rightarrow K > \dfrac{140}{30} = 4.667$
s^0	$200K$		$\rightarrow K > 0$

✓ For stability: $K > \dfrac{14}{3} = 4.667$

✓ For Sustained Oscillation $K = \dfrac{14}{3}$

A.E. $\Rightarrow \left(\dfrac{14}{3} + 2\right)s^2 + 200 \times \dfrac{14}{3} = 0$

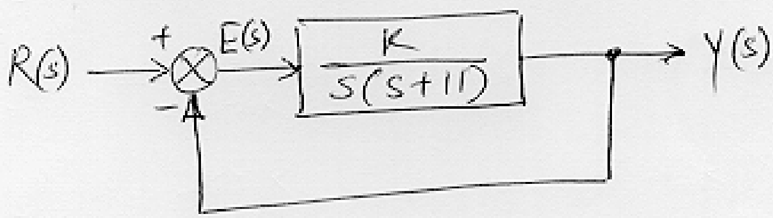
$$s^2 = -140$$

$$s = \mp j\,11.83$$

$\therefore\ \omega = 11.83$ rad/s and $f = 1.88$ Hz

Q4: Reduce the block diagram to the standard form

$$\frac{\dfrac{K}{s(s+1)}}{1+\dfrac{10s}{\cancel{K}}\times\dfrac{\cancel{K}}{s(s+1)}} = \frac{K}{s(s+1)+10s} = \frac{K}{s(s+11)}$$



type 1 system

$$K_v = \lim_{s \to 0} s\,G(s) = \frac{K}{11}$$

$$e_{ss} = \frac{100}{K_v} = \frac{100}{K}\times 11 = \frac{1100}{K}$$

for $e_{ss} = 0.01 \Rightarrow K = 110000$

$$\underline{K = 11\times 10^4}$$

Q5 C.E. $1 + K \dfrac{100}{0.2s + (1 + 100K_t)} \cdot \dfrac{1}{20s} = 0$

$$s^2 + 5(1 + 100K_t)s + 25K = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad \text{standard form}$$

$$MOS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.043$$

$$\zeta = \frac{1}{\sqrt{2}} = 0.707$$

from C.E. $\begin{cases} \omega_n^2 = 25K \\ 2 \times 0.707 \times \cancel{5}\sqrt{K} = \cancel{5}(1 + 100K_t) \end{cases}$

$$\sqrt{2K} = 1 + 100K_t$$

but $\tau_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$

$$\therefore \quad \omega_n = \frac{2.16 \times 0.707 + 0.6}{0.2} = 10.64 \text{ rad/s}$$

$$\therefore \quad K = \frac{\omega_n^2}{25} = \frac{(10.64)^2}{25} = \underline{\underline{4.52}}$$

$$K_t = \frac{\sqrt{2K} - 1}{100} = \underline{\underline{0.02006}}$$

Q6.

$$G(s) = K \frac{100}{(0.2s + 1 + 100K_t)} \cdot \frac{1}{20s}$$

Type $\underline{\underline{1}}$

$$K_p = \lim_{s \to 0} G(s) = \infty$$

$$K_v = \lim_{s \to 0} sG(s) = \frac{5K}{1 + 100K_t}$$

$$K_a = \lim_{s \to 0} s^2 G(s) = 0$$

(a) $$e_{ss} = \frac{1}{1 + K_p} = 0$$

(b) $$e_{ss} = \frac{1}{K_v} = \frac{1 + 100K_t}{5K}$$

(c) $$e_{ss} = \frac{1}{K_a} = \infty$$

Q7 From Q5

$$\text{C.E.} \quad s^2 + 5(1 + 100K_t)s + 25K = 0$$

$$2\zeta\omega_n = 5(1 + 100K_t)$$

$$\omega_n^2 = 25K$$

$$t_s \cong \frac{4}{\zeta\omega_n} \Rightarrow \omega_n = \frac{4}{\zeta t_s} = \frac{4}{0.6 \times 0.1}$$

$$\omega_n = \underline{\underline{66.66}} \text{ rad/s}$$

$$K = \frac{\omega_n^2}{25} = \frac{(66.66)^2}{25} = \underline{\underline{177.74}}$$

$$K_t = \frac{1}{100}\left(\frac{2 \times 0.6 \times 66.66}{5} - 1\right) = \underline{\underline{0.15}}$$

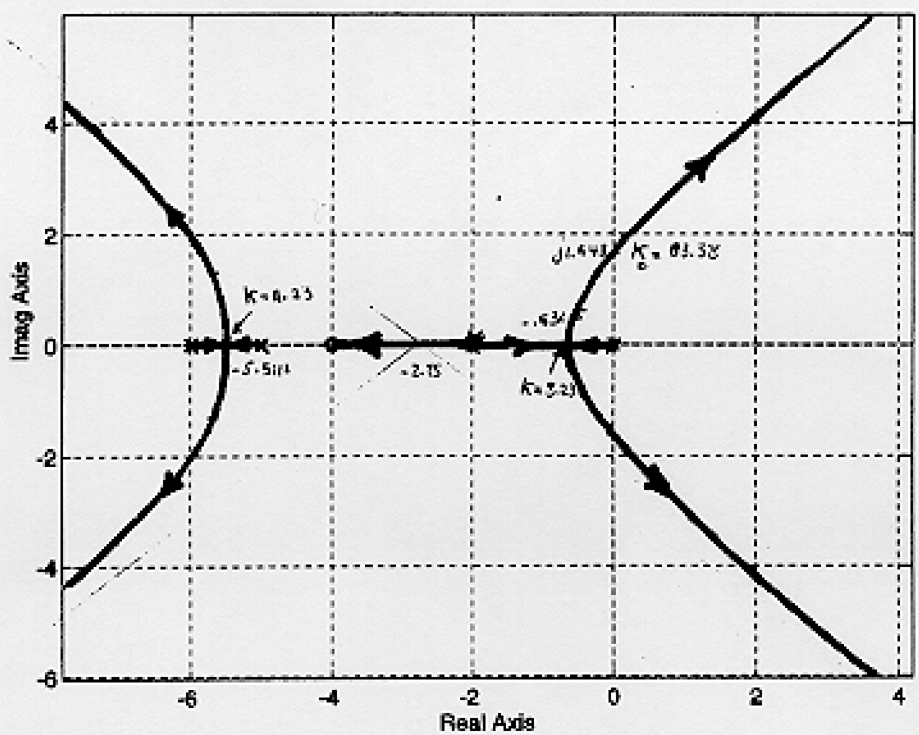
Q.8a $G(s) = K \dfrac{s+4}{s(s^2+4s+4)(s+5)(s+6)}$; 5 o.l. poles at 0, −2, −2, −5, −6
; 1 o.l. zero at −4

4 Asymptotes at ±45°, ±135° starting at $\sigma_a = -\dfrac{(0+2+2+5+6)-(4)}{5-1} = -2.7$

Break away eqn : $\dfrac{dK}{ds} = 0 \Rightarrow 4s^5 + 65s^4 + 395s^3 + 1100s^2 + 1312s + 420 = 0$

-5.5112 & -0.6326 (on RL)

K at breakaway : $\left| \dfrac{K(s+4)}{s(s^2+4s+4)(s+5)(s+6)} \right|_{\substack{s=-0.6326 \\ s=-5.5112}} = 1 \Rightarrow 8.23$ & 11.23



Imaginary axis crossing C.E. $s^5 + 15s^4 + 78s^3 + 164s^2 + (120+K)s + 4K = 0$

$j\omega^5 + 15\omega^4 - 78j\omega^3 - 164\omega^2 + (120+K)j\omega + 4K = 0$

$15\omega^4 - 164\omega^2 + 4K = 0$ & $\omega^5 - 78\omega^3 + (120+K)\omega = 0$

$\omega = 1.5435$ rad/sec , $K_c = 85.38$

Q.3(b) $\quad G(s) = K \cdot \dfrac{s^2+2s+10}{s(s+5)(s+10)}$; 3 o.l. poles at $0, -5, -10$
2 o.l. zeros at $-1 \pm j3$

1 asymptote at 180°

Breakaway eqn. $\dfrac{dK}{ds} = 0 \Rightarrow s^4 + 4s^3 + 10s^2 + 300s + 500 =$

Breakaway point $\quad -1.727$

K at Breakaway point: $\quad \left| \dfrac{K\,(s^2+2s+10)}{s\,(s+5)(s+10)} \right|_{s=-1.727} = 1$

$K = 4.9$