KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE380 CONTROL ENGINEERING 081

November 26, 2008 Time: 5:15-6:45 PM

[MAJOR EXAM # 1]

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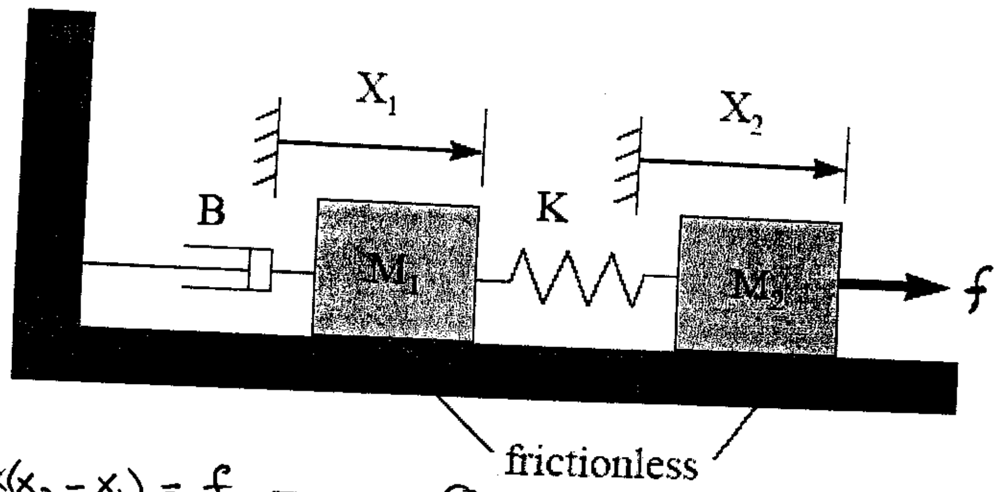
Name:	Key Solution
ID #:	
Section	

PROBLEM #	SCORE	MAXIMUM
1		25
2		25
3		20
4		30
TOTAL		**100**

Problem#1: Let $Z = [x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2]$ be the state vector. Find the state variable equations and the transfer function $X_1(s)/F(s)$ for the mechanical system.



$$M_2 \ddot{x}_2 + K(x_2 - x_1) = f \quad \text{——— ①}$$

$$M_1 \ddot{x}_1 + B\dot{x}_1 + K(x_1 - x_2) = 0 \quad \text{— ②}$$

$$\left.\begin{aligned}
\dot{z}_1 &= z_3 \\
\dot{z}_2 &= z_4 \\
\dot{z}_3 &= \ddot{x}_1 = -\frac{K}{M_1} z_1 + \frac{K}{M_1} z_2 - \frac{B}{M_1} z_3 \\
\dot{z}_4 &= \ddot{x}_2 = \frac{K}{M_2} z_1 - \frac{K}{M_2} z_2 + \frac{1}{M_2} f
\end{aligned}\right\} \Rightarrow \dot{Z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{M_1} & \frac{K}{M_1} & -\frac{B}{M_1} & 0 \\ \frac{K}{M_2} & -\frac{K}{M_2} & 0 & 0 \end{bmatrix} Z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} u$$

$$Y = x_1 = z_1 = [1 \quad 0 \quad 0 \quad 0] Z \qquad Y = [1 \quad 0 \quad 0 \quad 0] Z$$

Taking L.T of Eqn ① $\Rightarrow (M_2 s^2 + K) X_2(s) - K X_1(s) = F(s) \quad *$

" " " " ② $\Rightarrow (M_1 s^2 + Bs + K) X_1(s) = K X_2(s) \quad **$

From $(**)$ $X_2(s) = \frac{1}{K}(M_1 s^2 + Bs + K) X_1(s)$

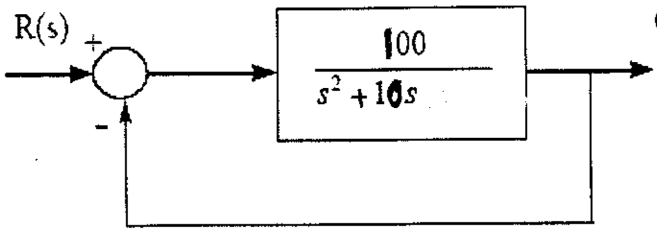
Substitute into $(*)$ $\Rightarrow \left[(M_2 s^2 + K)(M_1 s^2 + Bs + K)\frac{1}{K} - K\right] X_1(s) = F(s)$

$\therefore \left[(M_2 s^2 + K)(M_1 s^2 + Bs + K) - K^2\right] X_1(s) = K F(s)$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{K}{s\left[M_1 M_2 s^3 + M_2 B s^2 + K(M_1 + M_2)s + BK\right]}$$

Problem #2:

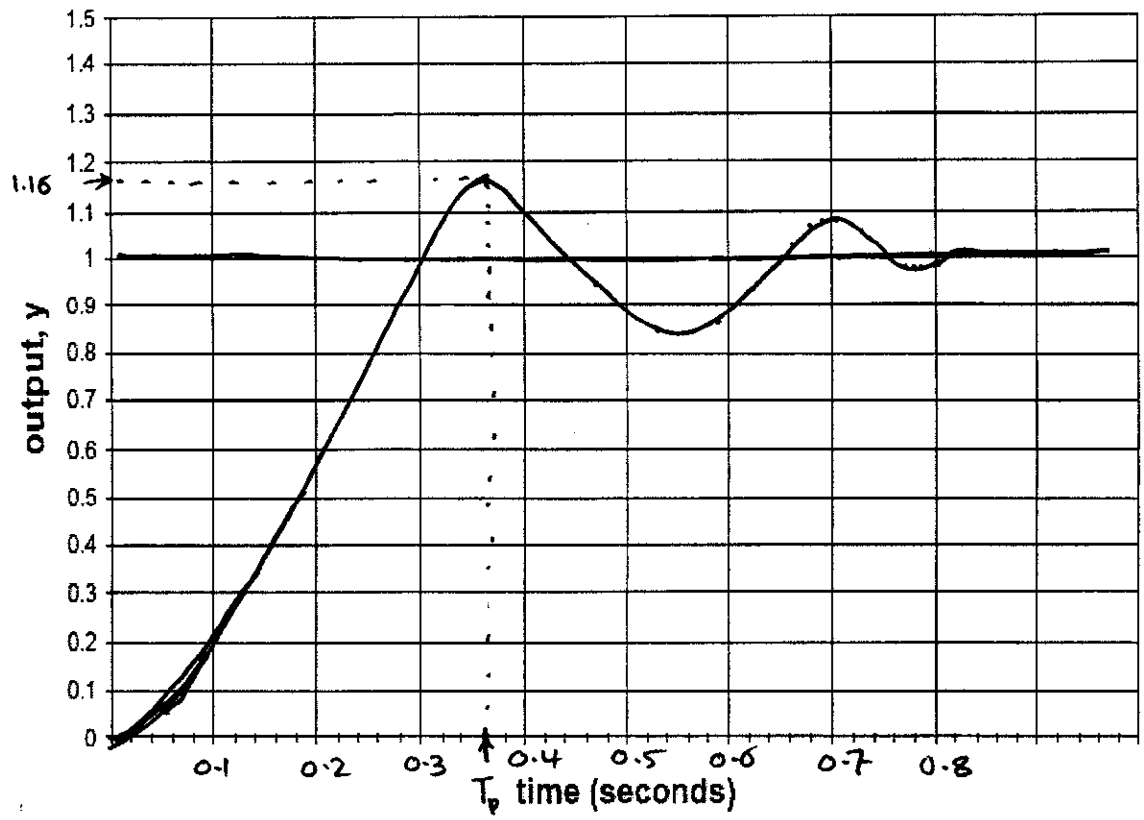
A unity feedback control system is shown below:



a) find the natural frequency, damping ratio, and damped natural frequency of the closed loop system

b) determine the %OS, T_P and T_S for a step input to the closed loop system

c) sketch the unit step response of the closed loop system on the graph below



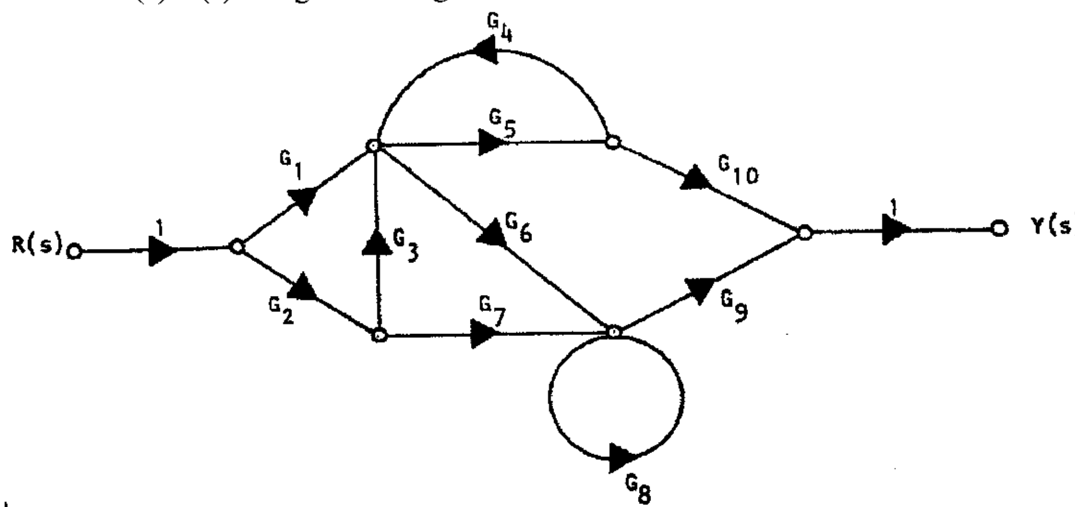
$$\frac{C(s)}{R(s)} = \frac{100}{s^2+10s+100} \Rightarrow \boxed{\omega_n = 10} \;,\; 2\zeta\omega_n = 10 \Rightarrow \boxed{\zeta = 0.5}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{3} = \beta \;,\; \boxed{\alpha = \zeta\omega_n = 5}$$

$$P.O = e^{-\frac{\alpha}{\beta}\pi} \times 100 = e^{-\frac{\pi}{\sqrt{3}}} \times 100 = 16.3\,\% \;,\; T_s = \frac{4}{\alpha} = 0.8 \text{ sec}$$

$$T_p = \frac{\pi}{\beta} = \frac{\pi}{5\sqrt{3}} = 0.36 \text{ sec}$$

Problem #3
Find the transfer function Y(s)/R(s) using Mason's gain formula



Forward Paths:

$P_1 = G_1 G_5 G_{10}$ $\Delta_1 = 1 - G_8$

$P_2 = G_1 G_6 G_9$ $\Delta_2 = 1$

$P_3 = G_2 G_3 G_5 G_{10}$ $\Delta_3 = 1 - G_8$

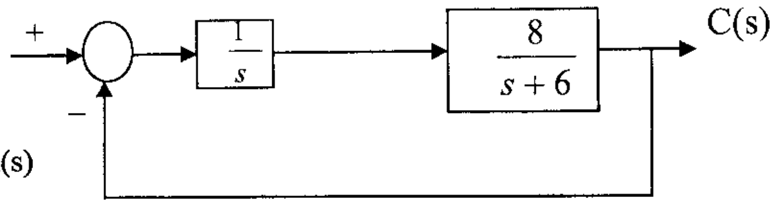
$P_4 = G_2 G_7 G_9$ $\Delta_4 = 1 - G_4 G_5$

$P_5 = G_2 G_3 G_6 G_9$ $\Delta_5 = 1$

$$\Delta = 1 - (G_8 + G_4 G_5) + \underbrace{G_8 \; G_4 G_5}_{\text{non touching Loops}}$$

$$\therefore \quad \frac{Y(s)}{R(s)} = \frac{\sum_{i=1}^{5} P_i \Delta_i}{\Delta}$$

Problem#4:



For the system shown find the following:

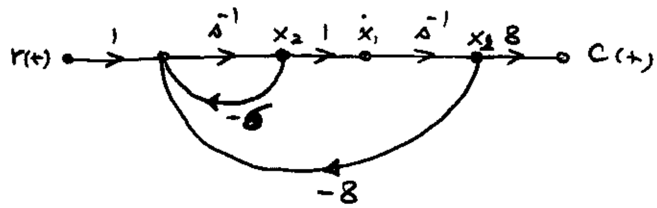
a) The closed loop transfer function C(s)/R(s)
b) The differential equation for the system
c) The state variables flow graph
d) The phase variable state equations
e) The state transition matrix $\Phi(t)$
f) c (t) for a unit step input when I.C = zero

a) $\frac{C(s)}{R(s)} = \frac{8}{s^2+6s+8}$ $\Rightarrow$ b) $\ddot{c}(t) + 6\dot{c}(t) + 8c(t) = 8r(t)$

d) Phase - variable form $\dot{x} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$

$c = y = [8 \quad 0]\, x$

c)



e) $\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 8 & s+6 \end{bmatrix}^{-1} = \frac{1}{s^2+6s+8} \begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}$

$$\Phi(s) = \begin{bmatrix} \frac{2}{s+2} - \frac{1}{s+4} & \frac{1/2}{s+2} - \frac{1/2}{s+4} \\ \frac{-4}{s+2} + \frac{4}{s+4} & \frac{-1}{s+2} + \frac{2}{s+4} \end{bmatrix} \Rightarrow \Phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-4t} & \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t} \\ -4e^{-2t} + 4e^{-4t} & -e^{-2t} + 2e^{-4t} \end{bmatrix}$$

f) $Y(s) = c\,\Phi(s)\,B\,U(s) = [8 \quad 0] \frac{1}{\Delta(s)} \begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s}$

$$Y(s) = \frac{8}{s(s^2+6s+8)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} = \frac{1}{s} + \frac{-2}{s+2} + \frac{1}{s+4}$$

$\therefore \; c(t) = 1 - 2e^{-2t} + e^{-4t} \qquad t \geq 0$

Alternative Solution: from T.F $C(s) = \frac{8}{s(s^2+6s+8)}$, then proceed as above.